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LOCAL CHANGES IN THE GRAVITY AND MAGNETIC FIELD DUE TO TECTONIC STRAIN

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FINAL TECHNICAL REPORT

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Introduction

Surface displacements and tilts and local changes in the gravity and magnetic fields provide complementary sources of information about tectonic events at depth. The changes in these parameters measured at the surface before and after an earthquake, for example, are all the result of the tectonic strain which occurred, but each depends on different components of the strain field. I have been studying the local changes in the earth's gravitational and magnetic fields using an elastic half-space as a model of the earth. My goal is to derive the influence functions relating the changes in gravity and magnetism measured at the surface to the strain source at depth for realistic models. I currently have four analyses in progress in this general field.

Magnetic field

I am analyzing the changes in the earth's magnetic field due to a dislocation in an isotropic and homogeneous half-space. The half-space contains randomly-oriented paramagnetic minerals magnetized by the earth's field. The change in the stress caused by the dislocation changes the magnetic properties of the crystals, and the integrated effect is measured at the surface.

I have been studying this problem following the approach that I used [Walsh and Rice, 1979] to derive the influence functions relating changes in gravity to a dislocation in an elastic half-space. The first step involves the use of Betti's reciprocal theorem, and the derivation is completely analogous for the two cases. Consider Figure 1a where we want to find the change $\Delta M_\alpha{}^\beta$ in magnetic moment (about an axis $-\alpha$) acting on a unit dipole (oriented parallel to an axis $-\beta$) due to displacements u_i on a surface S in a magnetic elastic half-space. Betti's reciprocal theorem demonstrates that we can find $\Delta M_\alpha{}^\beta$ by considering the elasticity problem in Figure 1b; there, the unit dipole in direction β is rotated through a small angle $d\beta$, thereby changing the stress σ_{ij} in a continuous half-space. Applying the reciprocal theorem to the tractions and displacements in Figures 1a and 1b, I find

$$\Delta M_\alpha{}^\beta = \int_S (\partial\sigma_{ij}{}^\beta/\partial\beta)_{u_i}\, u_i\, dS_j \tag{1}$$

As stated above and expressed mathematically in (1), we must solve the elasticity problem illustrated in Figure 1b. As the first step in this calculation, I calculated the body forces in an infinite elastic medium due to rotating a magnetic dipole. So far, I have considered only the case where the infinite medium contains randomly-oriented anisotropic grains with principal susceptibilities (ψ_{11}, ψ_{22}, ψ_{33}) but the analysis can be easily extended to media with preferred orientation of magnetic minerals, should that be warranted. I find, for example, that the only body force in an isotropic medium is a force $\bar{F}$ per unit volume having strength

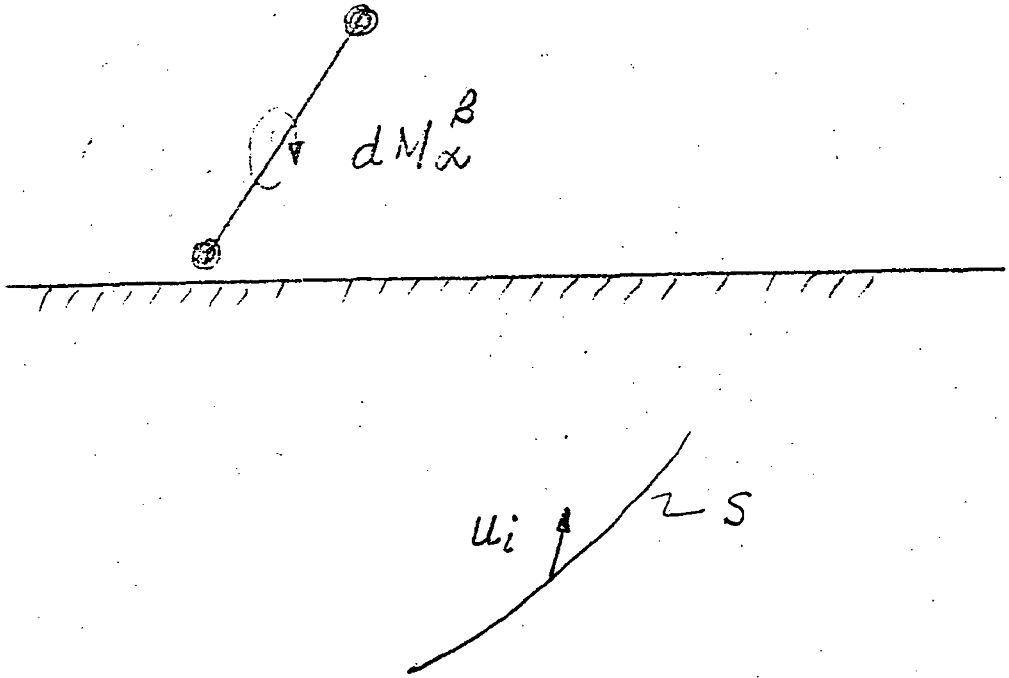

Fig. 1(a): A dislocation on the surface S_β in the half-space causes a change $\Delta M_\alpha{}^\beta$ in the moment on a dipole along β about an axis α.

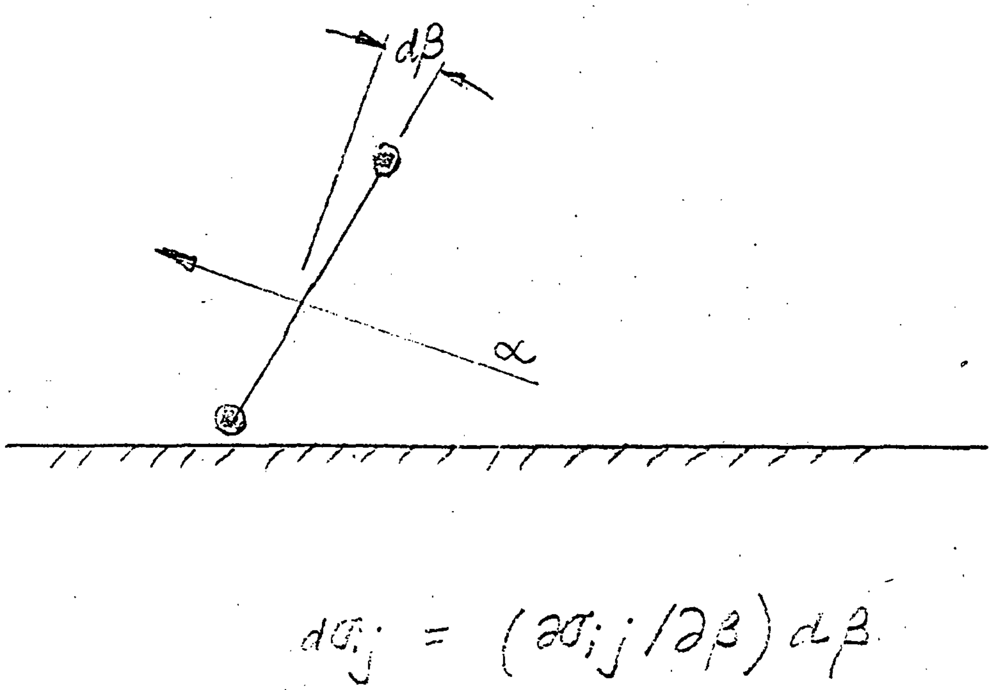

Fig. 1(b): A rotation about an axis α of a magnetic dipole oriented parallel to β causes a change $d\sigma_{ij}$ in the stresses in a uniformly magnetic elastic half-space.

$$\bar{F} = \frac{1}{6}\mu_o \psi_{ii} \nabla(H^2) \tag{2}$$

where μ_o is the permeability of a vacuum and H is the vector field due to the field of the dipole H* and the earth's field H°; that is

$$\bar{H} = \bar{H}^\circ + \bar{H}^* \tag{3}$$

Assuming that the earth's field is uniform on a local scale and much larger than that due to the dipole, I find from (2) and (3) that

$$\bar{F} = \frac{1}{3}\mu_o \psi_{ii} \nabla(\bar{H}^* \quad \bar{H}^\circ) \tag{4}$$

Carrying out the operations in (4), one finds that the components of the body-force field can be represented by analytic expressions. The next step is to calculate the stress field in the infinite space due to the force field in (4). We are interested in the stresses induced in a half-space rather than an infinite medium, however. To find the stresses in a half-space, the last step is to create a free surface by removing the stresses acting on the plane in the infinite space representing the free surface. I have started, but not completed, these two final steps.

In addition to the analytical work described above, which was carried out at M.I.T., I spent one month at the Office of Earthquake Studies in Menlo Park, California, working with M.J.S. Johnston on problems associated with field measurements.

Gravity Change Due to Body Forces

Rice and I [Walsh and Rice, 1979] derived the Green's functions relating a dislocation at depth to the gravity change at the surface. During the past year, I derived the Green's functions needed to calculate the change in gravity at the surface of an elastic half-space due to a change in body forces. The technique I used was practically the same as in the previous derivations, except that here I had to calculate the displacements of $d\delta_i$ (instead of the change in stress) due to moving a point mass m located on the surface a small distance dc. Once this elasticity problem has been solved, Betti's reciprocal theorem is used to show that the change Δg in gravity is

$$\Delta g = (1/m \int_S \sigma_{ij} \; (\partial\delta_i/\partial c) \; dS_j \qquad (5)$$

where S is the surface over which the differential body forces σ_{ij} dS_j are distributed.

I derived the displacement fields $(\partial\delta_i/\partial c)$ needed in (5) to calculate the changes in both the vertical and horizontal components of gravity. I calculated the change in gravity and the change in vertical that would be expected for a volcanic eruption where the change in body force represents the vertical displacement of magma from the reservoir to the surface. I also have calculated the effect on gravity of horizontal translation of material - the erosion of a mountain and filling of a valley, for example. This work is now being written up for publication.

Gravity change with non-uniform density

The density of the earth increases with depth, and clearly this must affect to some extent the changes in gravity that are measured. The Green's functions that Rice and I derived, as discussed above, were calculated using a homogeneous, elastic half-space as the model for the earth. To estimate the error in our calculation which is introduced by assuming uniform density, I derived the Green's functions for the case where the elastic properties of the half-space are uniform but the density of a surface layer is lower than that of the rest of the medium.

I have carried this derivation through to completion, but I have not checked the calculations. The approach, as in the other analyses above, involves Betti's reciprocal theorem, but the subsequent calculations are simpler for the case where the layer is much thinner than the depth of the event. The algebra involved is not insignificant, however, and this must yet be checked.

Piezomagnetism

I have begun an analysis of the piezomagnetic effect in rocks, using what appears to be a new approach. Consider magnetic grains dispersed in a matrix of non-magnetic material which is assumed to be homogeneous, elastic, and isotropic. The crystals are anisotropic with known magnetostrictive and elastic properties. When a magnetic field is turned on, the shape and size of each magnetic crystal changes because of the magnetostrictive effect in the free crystal and the elastic constraint provided by the matrix. The distortion of a crystal can be calculated following Eshelby's [1957] technique, and the change in shape of the sample as a whole is found using our ever-helpful reciprocal theorem. Once the effective magnetostrictive effect for the sample is known, the piezomagnetic properties can be found from the reciprocal relationship between the two.

I have begun this calculation by considering the simplest possible case, where the magnetic crystals are spherical with isotropic elastic properties and with polar symmetry in their magnetostrictive properties. I have gone through the steps that I outlined above and the procedure works. The next step is to do the calculation for a realistic case where both the elastic and magnetic properties are anisotropic.