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E. W. PEIKERT *Northwestern University, Evanston, Ill.*[1]

Model for Three-Dimensional Mineralogical Variation in Granitic Plutons Based on the Glen Alpine Stock, Sierra Nevada, California

Abstract: Pronounced relief of the exposed surface of the Glen Alpine stock offers an opportunity for study of three-dimensional mineralogical variation in a granitic pluton. Polynomial surfaces fitted by least-squares methods to modal data from this stock indicate that, near the center of the pluton, the vertical rate of mineralogical change is large in comparison to the horizontal rate of change; whereas, near the boundaries the opposite is true. Variation of this type is most pronounced in modal potassium feldspar and color index but is present to a minor extent in modal quartz and plagioclase. These results suggest a three-dimensional model of mineralogical variation which may be appropriate for other plutons. Vertical mineralogical gradients are an important feature of this model; consequently, the model implies that the mineralogical composition of each exposed level of a pluton is unique. Thus, one must be cautious in applying observations of a two-dimensional exposed surface to the determination of the genesis of a three-dimensional pluton.

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[1] Present address: International Business Machines, Petroleum Industry Development, 2601 S. Main Street, Houston, Texas.

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INTRODUCTION

Theories of the origin of granitic plutons commonly imply structural, chemical, or mineralogical variation with depth. Until quantitative information is available concerning the nature and relative amount of vertical variation, there is no way that these theories may be tested. In the absence of such information it is impossible to establish the quantitative significance of observations made at a given erosional level within a pluton. If there is no vertical variation, observations at any erosional level are representative of a three-dimensional pluton; but if there is vertical variation, surface observations cannot be considered in this manner. Thus, the significance of any observation or set of observations is a function of whether or not there is vertical variation.

The Glen Alpine stock, Sierra Nevada, California, offers an unusual opportunity for examination of vertical variation for the relief of its exposed surface amounts to a high proportion (14 per cent) of its maximum diameter. The stock is small (1 square mile in area), but exposed so that one can adhere closely to a predetermined sampling plan designed to give objective quantitative results (*see* App. 1A). Gross mineralogical variation in this stock is comparable to that of several plutons in similar geological settings (Bald Rock batholith, Compton, 1955; Bald Mountain batholith, Taubeneck, 1957; White Creek batholith, Reesor, 1958). Except for its size, the Glen Alpine stock does not appear to be substantially different from the descriptions of these bodies. Thus the main features of mineralogical variation in the Glen Alpine stock suggest a three-dimensional model which may be applicable to other granitic plutons.

The concept of a model has served as an appropriate framework in numerous geological studies (Jahns and Burnham, 1961; Sloss, 1962; Wyllie, 1962); the geometrical model developed in the present study is but one of many possible types. Other types of models have been formulated in physical terms, for example, the clastic-wedge model in stratigraphy; an event or series of idealized events, as in the model of pegmatite genesis of Jahns and Burnham (1961); a mathematical equation, such as Stokes' Law; or merely an implied set of circumstances. In these examples, the model serves as a logical bridge between geological processes and their observed results. It functions as a framework for collection of observations and testing of hypotheses and provides a logical way in which to separate pertinent and nonpertinent information from the mass of observations that can be made concerning a given geological problem.

Models can be developed in terms of: (1) known processes such as those (dehydration and decarbonation) used by Wyllie (1962) to develop a model of metamorphism, or (2) observed responses as used in the present example to develop a model of three-dimensional mineralogical variation in granite. In the former case the processes responsible for the model are known; thus, observed responses may be tested by means of the model to see if they fit the idealized processes. In the latter approach, one can test geological processes by means of the model in order to determine whether such processes are capable of producing the observed response.

When this latter approach is used the problem is to develop a model that fits a set of observations. Because determinative and sampling errors enter into most geological observations, a model that accounts for each observation exactly may be affected by these errors to such an extent that many important features of the observations cannot be recognized. Statistical approaches are useful in developing models that focus attention on such features.

The method of fitting polynomial surfaces to observed data by the least-squares method as used by Grant (1957), Krumbein (1959), and Whitten (1959; 1961a) is useful in this regard, inasmuch as it helps to separate large-scale variability from small-scale variation for variates that have two-dimensional (map) distribution. With minor modifications the same method can be applied to observations distributed in three dimensions (Peikert, 1962; 1963). The large-scale variation separated by this method is sufficiently generalized to be included in a model, whereas the random determinative and sampling errors are included in the small-scale variation. The least-squares methods used in the present study are discussed briefly in Appendix 1 C. A more detailed discussion of the three-dimensional aspects of these methods, together with a listing and explanation of the IBM 709 program developed for the present study, has been published previously (Peikert, 1963).

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GLEN ALPINE STOCK

Many field and laboratory observations have been made on the Glen Alpine stock (jointing, distribution of inclusions, composition of plagioclase, *etc.*), but many of these do not pertain to the problem at hand, that of three-dimensional mineralogical variation. The author will give the most general field and laboratory observations to establish the geological framework of the study.

The work of many geologists (Calkins, 1930; Hamilton, 1956; Bateman, 1961; and others) has demonstrated that the Sierra Nevada batholith is a complex of many distinct bodies of granitic rock, separated from each other by contacts of varying degrees of sharpness or by elongate bodies of older metamorphic rock. The Glen Alpine stock is a small granodiorite pluton located toward the northern end of this complex (Fig. 1); it is not visibly connected to any mass of similar character, nor are similar rocks exposed for a considerable distance from its boundaries.

On the map (Fig. 2) the stock appears as an irregular ellipse measuring 6500 by 4400 feet in its principal diameters. Hornfelses of the Mount Tallac roof remnant, presumed to be Jurassic in age, surround the stock on three sides; the rocks surrounding the west side of the stock were originally conglomerates and breccias which consisted of angular fragments of chert, quartzite, limestone (or dolomite), and mafic igneous rock in a matrix of calcareous sand. The matrix has recrystallized to quartz plus a variety of calc-silicate minerals, but metamorphism has not destroyed the outlines of the coarser-grained fragments. The north and east sides of the stock are surrounded by meta-andesite flows and breccias which now exhibit hornfelsic and blastoporphyritic textures. The south side of the stock is in contact with an older quartz monzonite of unusually mafic type.

Contacts between the Glen Alpine stock and the surrounding rocks vary in character, but are generally sharp. Contact zones are commonly only a few feet in width, but areas of migmatite up to 75 feet wide are present along the eastern contact. Owing to the massive nature of the surrounding rocks the structural relationships of the stock to these rocks cannot be accurately defined.

Most of the stock is composed of granodiorite which contains biotite and subordinate hornblende; but hornblende is absent in the central part of the stock, and a quartz diorite border phase is present in a few areas. Although the mineralogical composition of the stock varies, the nature of the variation is such that boundaries between rock types cannot be accurately placed without the aid of modal analyses.

THREE-DIMENSIONAL MINERALOGICAL VARIATION

General Statement

The first step in analyzing mineralogical variation in a granitic pluton is to determine the scale at which the major portion of this variation occurs. The results of statistical tests presented in Appendix 1 B indicate that there is significant variation between sample stations for each of the four major mineralogical variates in the Glen Alpine stock. Determination of the quantitative areal variation of each of these variates should thus be an effective tool in a study of this pluton.

The position and attitude of surfaces representing, for example, 5, 10, 15, and 20 per cent potash feldspar are just as important to a petrologist studying the mineral composition of a granitic pluton as are the location and attitude of planar or linear features to a structural petrologist studying the same pluton. Unlike the structural example, it is impossible to determine directly the location and attitude of surfaces of equal mineral composition, but it is possible to estimate their "statistical" or "average" form and position by use of the least-squares method. In the present example, polynomial surfaces up to the third degree have been fitted by the least-squares method to data from 169 modes distributed throughout the Glen Alpine stock.

Several problems arise, however, in representing polynomial surfaces. In a two-dimensional example, a variable X, measured at several points $(U,V)_i$, can be approximated by a polynomial surface. (As used here, U and V are mutually perpendicular, horizontal co-ordinates; W is the vertical co-ordinate.) Every point on this surface is defined by two geographic co-ordinates, U and V, and a magnitude X_{comp} corresponding to the least-squares estimate of X at this point. Whitten (1962a) illustrated a three-dimensional surface of this type. Contours representing equal X_{comp} values (isopleths) can be located on this surface and projected onto a two-dimensional (U,V) plane.

In a three-dimensional problem, each observation is located by three geographic coordinates (U,V, and W). Every point on the computed surface is defined by these three coordinates and a magnitude X_{comp}. Such a surface is four dimensional and cannot be fully represented on a map. However, surfaces representing equal values of X_{comp} are three dimensional, and their intersection with any two-dimensional surface (*e.g.* a U,V or V,W plane) can be located. In the present study the form and position of these so-called equal-value surfaces are illustrated by their intersection with a system of mutually perpendicular UW, VW, and UV planes. These equal-value surfaces represent the least-squares estimate of the variation of X within a three-dimensional body.

Figure 3 shows vertical cross sections through two sets of equal-value surfaces that are typical of those produced by observations confined to a topographic surface of limited local relief. As shown, the equal-value surfaces are closely spaced in the vertical or W direction, apparently suggesting a rapid rate of vertical variation in both examples. However, equal-value surfaces based on a sample population that is confined to a topographic surface always take this form because, at each U,V location, observation at only one value of W is possible. As a result, a least-squares fit can generate unreasonable values above and below the topographic surface. Unless the interpreter is aware of this limitation of the method he may interpret the rapid increase or decrease of values away from the topographic surface as an indication of a significant vertical gradient. A more detailed examination of the first partial derivatives of the function in the vertical direction reveals that such interpretation is not always justified.

If $X_{comp} = f(U,V,W)$, then $\partial X_{comp}/\partial U$, $\partial X_{comp}/\partial V$, and $\partial X_{comp}/\partial W$ represent, in the present example, mineralogical gradients in each of three mutually perpendicular directions at a given point. In this respect the distinction between Figures 3A and B is important. In both cross sections there is a surface

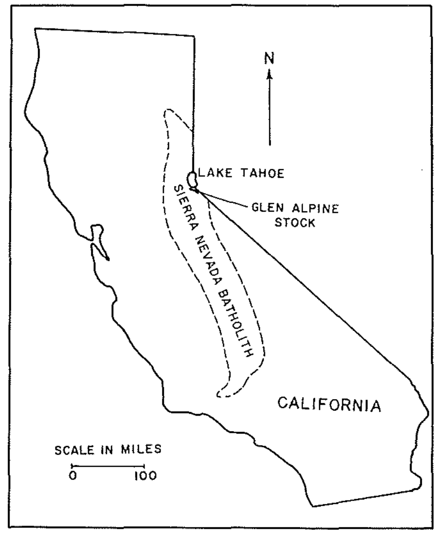

Figure 1. Location of the Glen Alpine stock. Boundaries of the Sierra Nevada batholith are approximate.

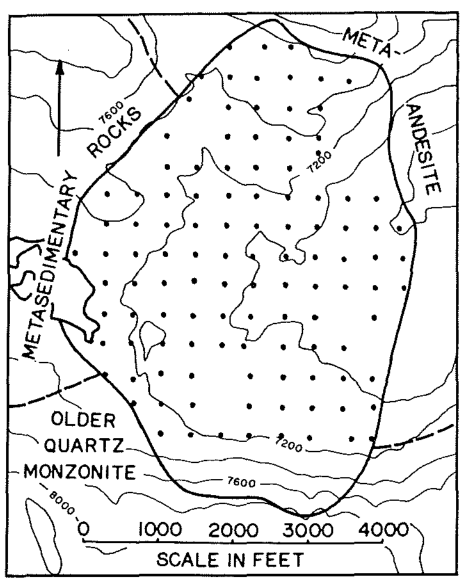

Figure 2. Glen Alpine stock and surroundings. Black dots indicate sample localities.

(dotted line) at which $\partial X_{comp}/\partial W = 0$. Toward this surface the rate of change of X_{comp} in the vertical direction approaches zero; the position of this surface in relationship to the three-dimensional distribution of data points is critical. In Figure 3A this surface nearly coincides with the topographic surface from which the sample population is available. At each point on the topographic surface $\partial X_{comp}/\partial W \cong 0$, suggesting little or no vertical variation in X_{comp}; moreover, significant horizontal gradients are suggested where the equal-value surfaces dip at a high angle as, for example, at the sample points near the edges of the diagram where $\partial X_{comp}/\partial U$ or $\partial V \gg \partial X_{comp}/\partial W$.

In Figure 3B the surface defined by $\partial X_{comp}/\partial W = 0$ is located entirely outside the realm of the data (topographic surface). At every sample point on the topographic surface $\partial X_{comp}/\partial W \neq 0$ and $\partial X_{comp}/\partial U$ or $\partial V \cong 0$, indicating that vertical gradients exceed horizontal gradients. In this example the equal-value surfaces dip at a low angle at the sample points.

Thus, sample populations that are confined to a topographic surface may produce equal-value surfaces which suggest vertical variation that is apparent (Fig. 3A) rather than real (Fig. 3B). Examination of the partial derivatives of the function at the sample points, as aided by three-dimensional fence diagrams such as Figure 4C, makes it possible to discriminate between the two extremes illustrated by Figures 3A and B. The confidence that can be placed in the interpretation increases with the vertical range of the sample points within a given area.

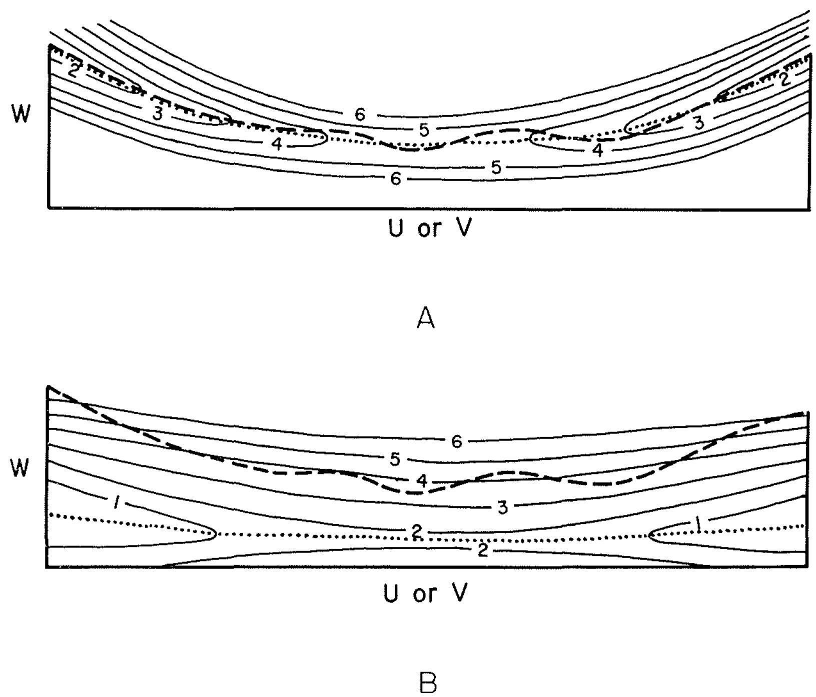

Figure 3. Idealized cross sections showing variation as interpreted by equal-value surfaces. Dashed line represents topography; dotted line indicates surface for which $\partial X_{comp}/\partial W = 0$. A, Primarily horizontal variation; B, Primarily vertical variation

Potassium Feldspar

Mean and standard deviation over all the modes for each of the four major mineralogical variates are given in Table 1. Of these variates, potassium feldspar shows greatest over-all variability ($\sigma = 5.13$). Figure 4A shows its variation as approximated by a third-degree polynomial surface based on U,V co-ordinates only. The isopleths show relatively high percentages in the central and southern part of the stock with decreasing percentages toward the margins. In this illustration elevation differences among sample stations are not considered in the computation. Many different three-dimensional interpretations of these isopleths are possible because the dip of the equal-value surfaces is not determined. Nonetheless, this is the type of information on which most studies of granitic plutons are based.

To determine which of the many three-dimensional interpretations is correct the author included in the computation the elevation, or W co-ordinate, of the sample stations. The isopleths in Figure 4B represent the intersections of three-dimensional equal-value surfaces with the topography. A greater percentage of the total variability is accounted for by these isopleths (35.7 per cent versus 29.8 per cent); thus, Figure 4B provides a better approximation of the potassium feldspar percentage at the exposed surface of the stock than does Figure 4A. In Figure 4B the potassium feldspar maximum is farther north, and a subsidiary maximum is located in the southern part of the stock. The differences between Figures 4A and B result from vertical relief that amounts to only 14 per cent of the maximum diameter of the stock. If no vertical component of potassium feldspar variation were present, the two figures would be identical.

Figure 4C shows the equal-value surfaces of potassium feldspar percentage in orthographic projection. In all the fence diagrams (Figs. 4C, 5C, 6C, and 7C) the patterns are used to emphasize unreasonable values that occur outside the realm of the data and to focus attention on the behavior of the equal-value surfaces in the vicinity of the sample points. Except for the extreme northern and southern ends of the stock, potassium feldspar variation conforms to example B, Figure 3. At each data point in the central portion of the stock $\partial X_{comp}/\partial W < 0$; consequently, a systematic decrease in modal potassium feldspar with elevation is indicated. The equal-value surfaces have shallow dips where they intersect the topography and are closely spaced in the vertical direction, an indication that the vertical component of variation is relatively large. Near the northern and eastern margins of the stock the dip of the equal-value surfaces increases slightly, as is indicative of horizontal gradients. Excluding the southern end of the stock from which no data are available, one may state that modal potassium feldspar decreases upward and outward from the center of the stock.

An example from the exposed surface of the stock illustrates the importance of vertical variation. If the 5 per cent potassium feldspar equal-value surface is taken as the granodiorite-quartz diorite boundary, a quartz diorite border is present at the topographically high northern end, but not at the topographically low eastern boundary of the stock. At both contacts the stock is surrounded by similar meta-andesites. Elevation within the stock appears to be the most significant geological difference between the two localities. At the eastern margin the quartz diorite may have been present at some higher elevation than that exhibited by the present topographic surface. In view of the similarity of the surrounding rocks at both localities, it is difficult to attribute the absence of the quartz diorite at the eastern margin solely to lateral variation.

Table 1. Mean and Standard Deviation of Major Mineralogical Variables Based on 169 Modes

	Mean per cent	Standard deviation (σ)
Quartz	26.4	3.51
Plagioclase	45.6	4.59
Potassium feldspar	12.1	5.13
Color index	15.9	3.45

Color Index

Figure 5A, based on U,V co-ordinates only, shows that in 2 dimensions color-index variation is the reverse of potassium feldspar variation, with low percentages in the center of the stock and high percentages toward the margins. When the effect of W is included in the computation (Fig. 5B), a larger percentage of the total variability is accounted for by the third-degree surface (50.0 per cent versus 40.2 per cent), and the major color-index minimum shifts to the north. The three-dimensional, equal-value surfaces (Fig. 5C) are also similar to example B, Figure 3, in the central and eastern portions of the stock. In these areas $\partial X_{comp}/\partial W > 0$ at each sample point, predicting a systematic increase in color index with elevation. Where sample control is available near the margins of the stock, the equal-value

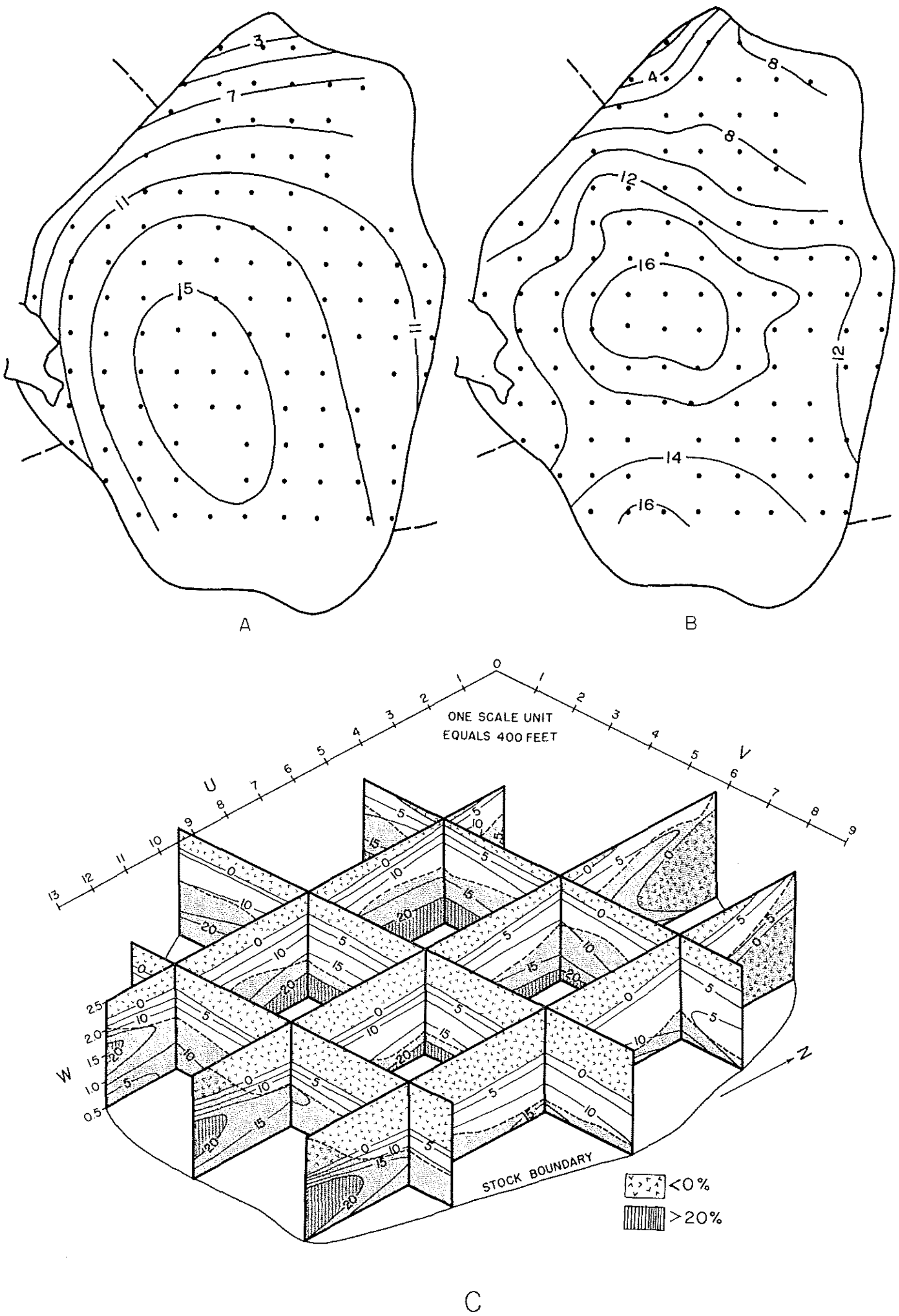

Figure 4. Variation of modal potassium feldspar. A, Degree-three least-squares surface based on U,V co-ordinates; 29.8 per cent sum-of-squares reduction. Dots indicate sample localities. B, Intersection of degree-three equal-value surfaces based on U,V,W co-ordinates with topographic surface; 35.7 per cent sum of squares reduction. Dots indicate sample localities. C, Equal-value surfaces of B in orthographic projection. Dashed line represents topographic surface.

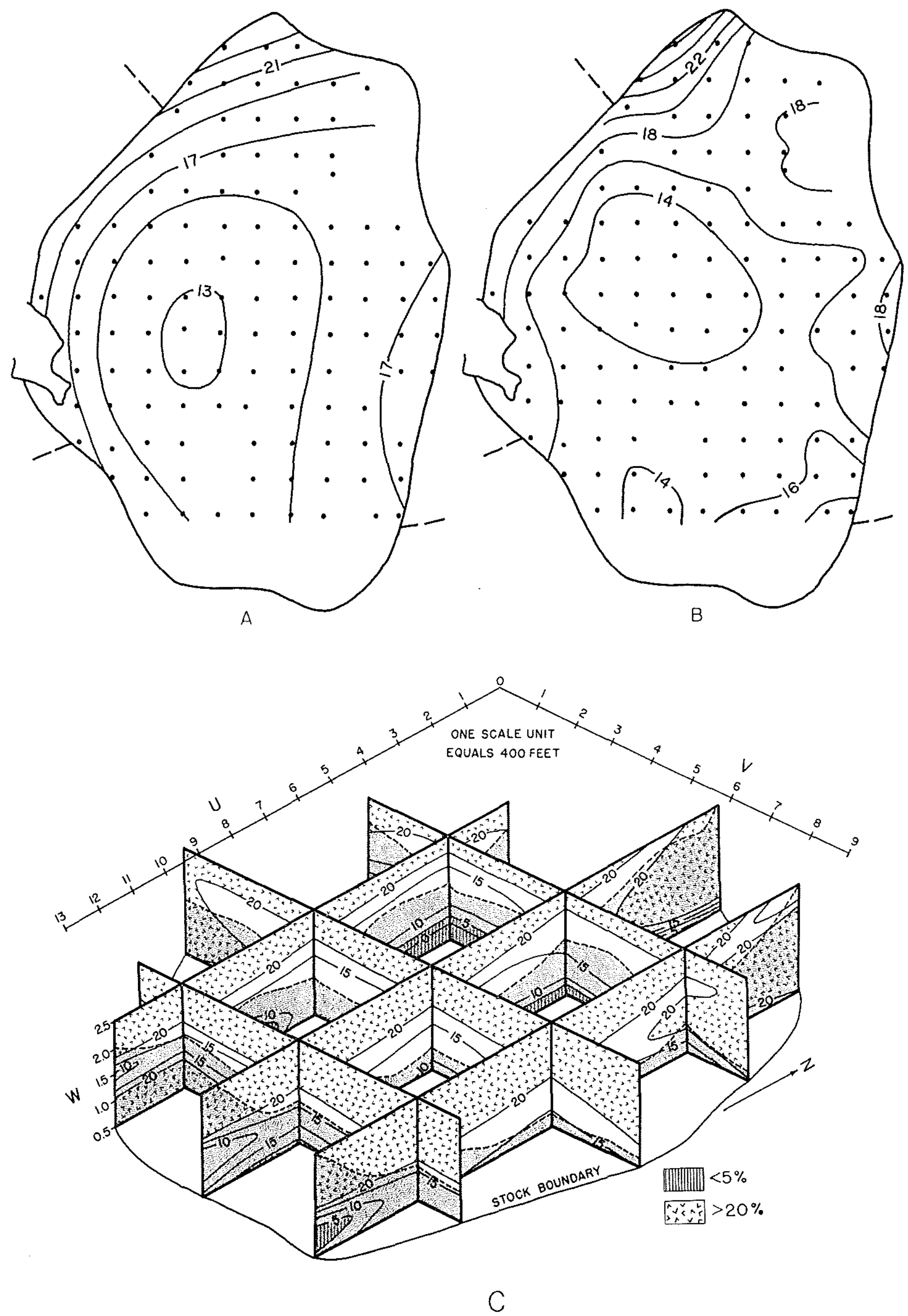

Figure 5. Color index variation. A, Degree-three least-squares surface based on U,V co-ordinates; 40.2 per cent sum-of-squares reduction. Dots indicate sample localities. B, Intersection of degree-three equal-value surfaces based on U,V,W co-ordinates with topographic surface; 50.0 per cent sum-of-squares reduction. Dots indicate sample localities. C, Equal-value surfaces of B in orthographic projection. Dashed line represents topographic surface.

surfaces conform more closely to example A, Figure 3, suggesting the importance of horizontal variation in these areas. Even in three dimensions, variation in color index is essentially the inverse of potassium feldspar variation.

Quartz

The two-dimensional variation in modal quartz (Figure 6A) shows that the highest percentages are located along a northeasterly trend. Modal quartz decreases in all directions away from this high. When one considers the effect of W (Fig. 6B) this high becomes four separate maxima, and there is a particularly large area of uniform quartz percentage (26–28 per cent) in the center of the stock. Figure 6B is a better approximation of the variation of modal quartz at the exposed surface of the stock because it is associated with a significantly higher sum of squares reduction than is the corresponding U,V surface (Fig. 6A).

The three-dimensional illustration of variation in quartz percentage (Fig. 6C) can be divided into several areas in which the variation has different three-dimensional characteristics. In the central and western parts of the area $\partial X_{\text{comp}}/\partial W > 0$ at the sample points, predicting an increase in quartz percentage with elevation. Variation in the northwestern part of the area appears to be primarily horizontal, with quartz percentages decreasing toward the margins of the stock. At the eastern boundary $\partial X_{\text{comp}}/\partial W < 0$, an indication that modal quartz decreases with elevation in this area. An example of the information gained by study of the three-dimensional aspects of mineralogical variation may be cited from this diagram. Although the quartz percentages at the eastern and western margins are similar, the equal-value surfaces predict that these percentages increase with elevation at the western margin, but decrease with elevation at the eastern margin. This is of interest because the metasedimentary rocks on the west could have supplied quartz at some higher elevation if contamination were greater at this level; whereas under the same conditions the meta-andesites on the east side would not have supplied additional quartz to the pluton.

Plagioclase

The two-dimensional variation in modal plagioclase (Fig. 7A) shows a circular high in the south-central part of the stock. Although the isopleths become irregular in form when the effect of W is considered (Fig. 7B), the pattern of variation does not change substantially except for the development of a low in the northeastern quadrant. This is not surprising in view of the small increase in the sum-of-squares reduction associated with the addition of W in the computations (26.4 per cent versus 23.3 per cent). As with quartz, modal plagioclase is nearly uniform in a large area in the central and southern part of the stock. Figure 7C shows the equal-value surfaces in orthographic projection. In the center and at the eastern margin of the stock $\partial X_{\text{comp}}/\partial W > 0$ predicts an increase in plagioclase percentage with elevation. Variation in the remainder of the stock is an excellent actual example of the hypothetical case shown in Figure 3A, indicating that much of the variation shown in Figures 7A and B is horizontal.

Correlation of Variables

One can predict the general aspects of the correlation matrix (Table 2) from the U,V,W isopleth patterns. The matrix shows that (1) modal potassium feldspar has a negative correlation with modal plagioclase and color index and a positive one with modal quartz, (2) modal plagioclase has a negative correlation with modal quartz and a positive correlation with color index, and (3) modal quartz has a negative correlation with color index. The magnitude of the correlation coefficients, however, cannot be predicted accurately from examination of the isopleth patterns. As pointed out by Chayes (1962), determination of the geological and statistical significance of these correlations must await development of statistical tests for closed-table variables.

Considerable redundance may exist in the three-dimensional mineral-variation patterns. For example, variation of color index could be controlled entirely by potassium feldspar variation in the closed-number system. This is not an important factor at this stage of the study where description of the geometry of mineral variation is the primary concern, but it will become important when the origin of this variation is considered.

THREE-DIMENSIONAL MINERAL-VARIATION MODEL

Variation patterns of the individual minerals in the Glen Alpine stock have several features in common; these can be summarized in a model that expresses the most important large-scale features of three-dimensional mineral variation. Variation patterns based on U,V co-

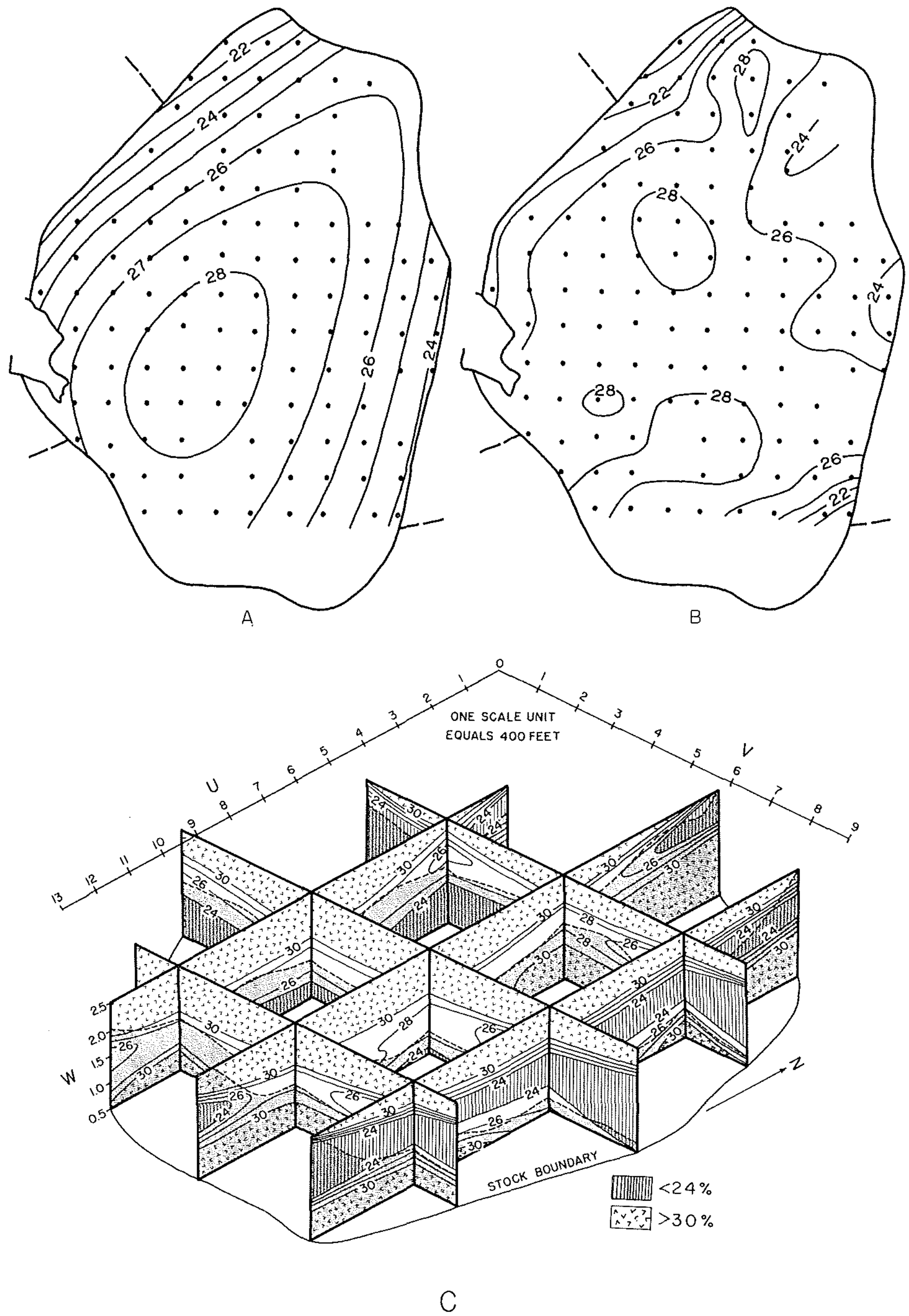

Figure 6. Variation of modal quartz. A, Degree-three least-squares surface based on U,V co-ordinates; 20.0 per cent sum-of-squares reduction. Dots indicate sample localities. B, Intersection of degree-three equal-value surfaces based on U,V,W co-ordinates with topographic surface; 34.5 per cent sum-of-squares reduction. C, Equal-value surfaces of B in orthographic projection. Dashed line represents topographic surface.

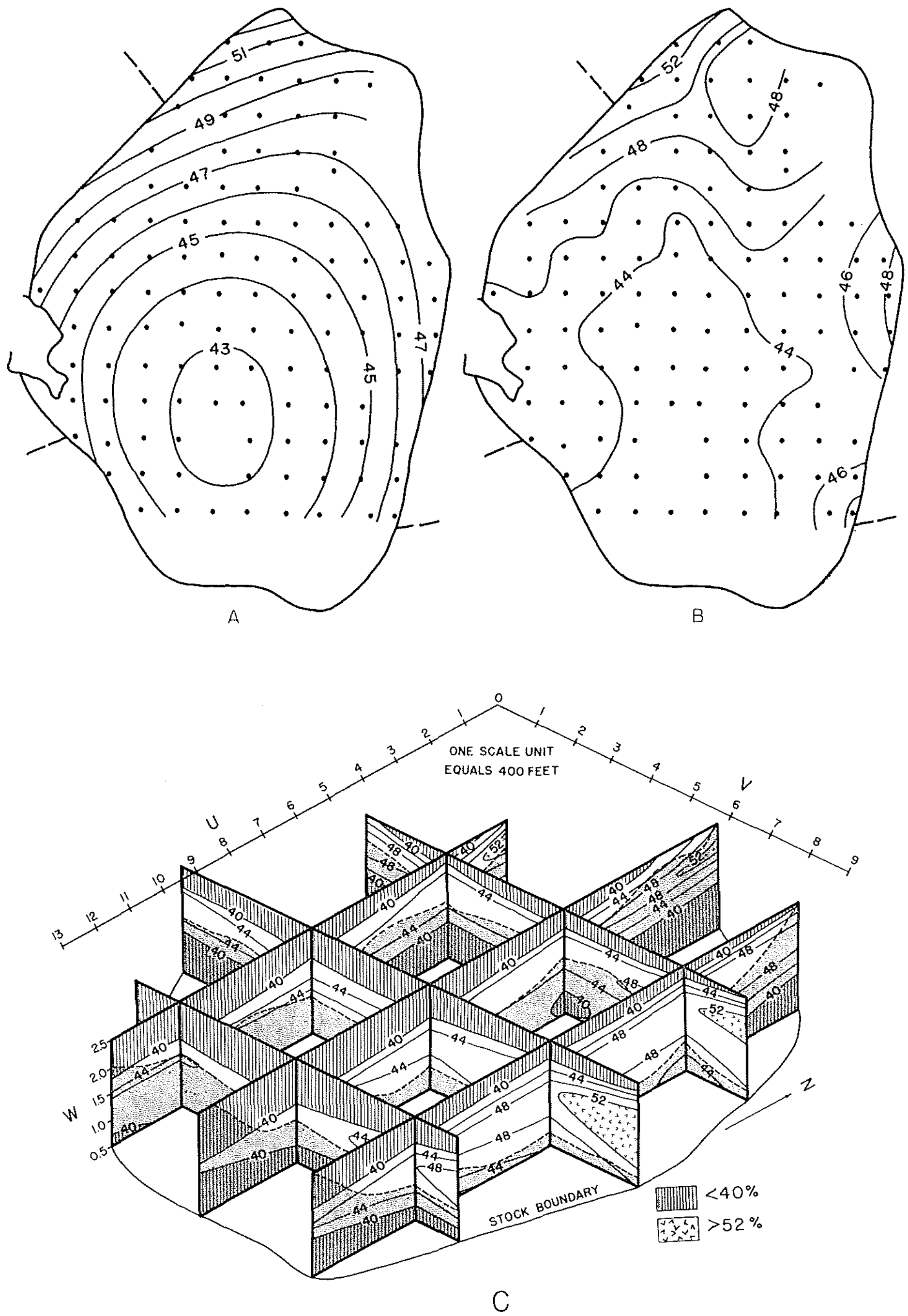

Figure 7. Variation of modal plagioclase. A, Degree-three least-squares surface based on U,V co-ordinates; 23.3 per cent sum-of-squares reduction. Dots indicate sample localities. B, Intersection of degree-three equal-value surfaces based on U,V,W co-ordinates with topographic surface; 26.4 per cent sum-of-squares reduction. Dots indicate sample localities. C, Equal-value surfaces of B in orthographic projection. Dashed line represents topographic surface.

ordinates show that the mineralogical composition of the stock is relatively uniform in its center, but varies rapidly toward its margins. In the center of the stock variation is primarily vertical, giving rise to equal-value surfaces that have shallow dips. Near the margins these surfaces dip more steeply, the steepest dips being at the highest elevations. Vertical mineralogical variation becomes relatively less important radially outward from the center of the stock. Figure 8 is a cross section through a generalized three-dimensional model in which mineralogical variation has these properties. For convenience the author sketched the model on the basis of a uniform gradient over the center of the pluton. Figure 8 represents the model geometrically, but the model could also be expressed as a generalized mathematical equation relating mineral composition to position within a three-dimensional pluton. Such an equation would be a further step in quantification of the original observations.

PETROLOGIC IMPLICATIONS OF THE MODEL

An example has already been given (Peikert, 1962) of a situation in which shallowly dipping, equal-value surfaces could cause misinterpretation of the direction of mineralogical gradients. The following discussion considers only a few of the many implications of a model, such as Figure 8, in which vertical variation is important. Although this model has been developed on the basis of variation in mineral percentages, it may be applicable to other parameters such as chemical composition, properties of individual minerals, textures, and structural data.

Quantitative measurements of the composition and compositional variation of granitic plutons are just beginning to appear in the geological literature (*e.g.* Whitten, 1961a; 1961b). These measurements represent a great advance, but the information has been limited to the exposed surfaces of the plutons which are essentially two dimensional. If the model (Fig. 8) is of general importance, only in the most fortuitous circumstances is the composition of a given exposed surface representative of a three-dimensional pluton. If the mineral composition of a granitic pluton varies significantly in a vertical direction, it is obvious that few hypotheses of the origin of this variation can be supported or eliminated on the basis of observations from a two-dimensional exposed surface of a pluton.

Table 2. Correlation Matrix

	Quartz	Plagioclase	Potassium feldspar	Color index
Quartz	1.00	−0.55	0.18	−0.55
Plagioclase	..	1.00	−0.78	0.38
Potassium feldspar	..	..	1.00	−0.64
Color index	..	..	..	1.00

A specific example helps to illustrate this point. Figure 9A shows variation at the exposed surface of a granitic pluton that is to be accounted for by a genetic hypothesis. If three-dimensional variation takes place according to model C (Fig. 9), it may be possible to eliminate, for example, assimilation as unlikely because of the large quantities of material that would have to be assimilated; but if variation takes place according to model B, (Fig. 9), the quantity of assimilated material would be so much less that this restriction might not apply.

The author has assumed that the Glen Alpine stock has not been tilted since its initial crystallization. This assumption seems justified in the present example (regional tilting of the Sierra Nevada block is a few degrees, at most, in this area) but may not be justified in plutons of active orogenic areas or in much older plutons. Tilting of such plutons could cause confusion regarding the significance of mineralogical gradients. Figure 10 shows how this may arise. A pluton develops zoning according to the

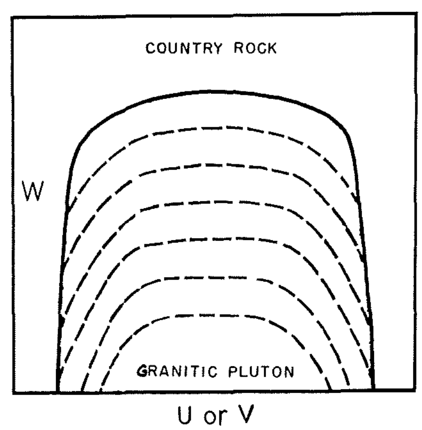

Figure 8. Generalized cross section through mineral-variation model based on the Glen Alpine stock. Dashed lines represent trace of surfaces of equal mineral composition.

model and is then tilted and eroded. If there is no evidence of tilting or the amount of tilting, or if one does not recognize the possibility of original vertical variation, the observed mineralogical gradients may be difficult to explain. Recognition of these facts, however, may lead to the correct interpretation.

The mineral-variation model may also help to explain other observed phenomena. Thus far, all discussion has been restricted to plutons in which mineral variation is gradational. What of the zoned plutons whose individual zones are bounded by sharp textural and mineralogical discontinuities? The mineral-variation model, shown in Figure 8, offers one possible and simple explanation of such bodies which does not resort to the development of separate magmas that some petrologists find objectionable. The essentials of this explanation are: (1) development of mineral variation along the lines of the model, and (2) differential vertical movement within the partially or completely consolidated mass. If vertical mineral-

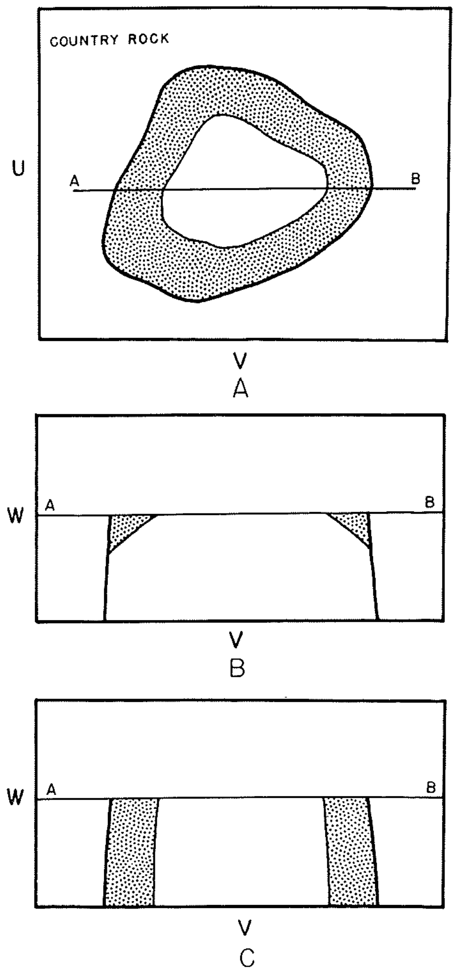

Figure 9. Alternative three-dimensional interpretations of map (U,V) variation within a granitic pluton. Stippled area represents border zone of composition different from core. A, Observed map variation; B and C, Alternative three-dimensional interpretations of variation along section A–B

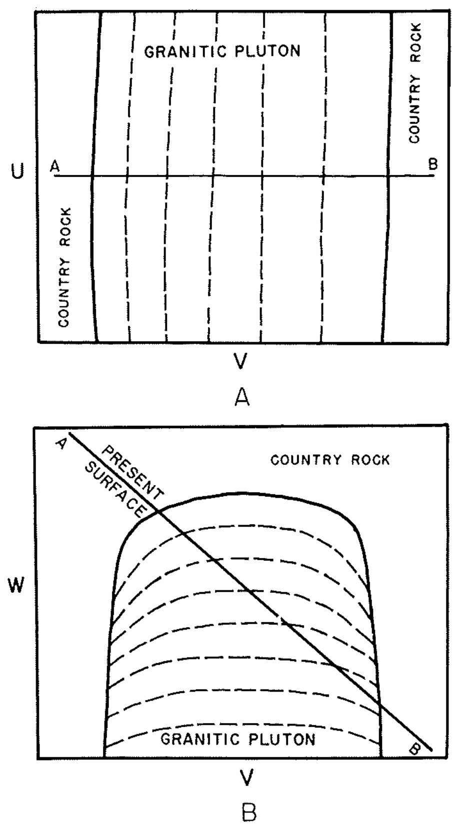

Figure 10. Possible explanation of an observed mineralogical gradient in a granitic pluton. Dashed lines represent trace of surfaces of equal mineral composition. A, Observed map (U,V) variation; B, Cross section through pluton suggesting possible three-dimensional explanation of observed variation

ogical gradients are steep, only a limited amount of differential vertical movement would be necessary to produce sharp mineralogical discontinuities at a given erosional level. Figure 11 illustrates development of a zoned pluton in this manner.

CONCLUSIONS

Much information is available about two-dimensional mineralogical variation in granitic plutons from maps showing distribution of rock types and, more recently, from isopleth maps (Whitten, 1959; 1961a; 1961b; Dawson and Whitten, 1962) which illustrate quantitative mineralogical variation. Two-dimensional maps of this type are subject to many different three-dimensional interpretations. Two extremes of such interpretation are represented by case (1), in which the surfaces of equal mineral composition are visualized as dipping at high angles, and case (2), in which these surfaces are presumed to dip at low angles. Interpretation by case (1) would imply that mineralogical variation is primarily horizontal. This would mean that there is little variation among the different vertical levels of a pluton and, more important, that representative samples from a given exposed surface can be used to approximate the composition of a three-dimensional pluton. Interpretation by case (2) implies that mineralogical variation is primarily vertical and that each vertical level of a granitic pluton has a unique mineralogical composition. In this case representative samples from an exposed surface could not be used to approximate the quantitative mineralogical or chemical composition of a pluton.

The relief of the exposed surface of the Glen Alpine stock permits examination of its mineralogical variation in three dimensions; the least-squares interpretation of this variation supports case (2). In the center of the stock the surfaces representing equal percentages of a given mineral appear to have shallow dips ($\partial X_{\text{comp}}/\partial W > \partial W_{\text{comp}}/\partial U$ or ∂V), *i.e.*, variation is primarily vertical. The dip of these surfaces increases radially outward from the center of the stock as the importance of horizontal variation increases ($\partial X_{\text{comp}}/\partial W < \partial X_{\text{comp}}/\partial U$ or ∂V).

There are several reasons why this geometrical model of mineral variation based on the Glen Alpine stock merits careful consideration in studies of other plutons. Many granitic plutons have geological settings similar to that of the Glen Alpine stock in that they are individual units in large batholithic complexes. Mineralogical variation in the Glen Alpine stock is similar to that of several plutons described in the geological literature; the major difference is in the distance over which this variation takes place. Variation according to the model could be produced by numerous different, possibly interacting, petrogenetic processes that may be of widespread geological importance. Several observed phenomena may yield a simpler interpretation in light of this model.

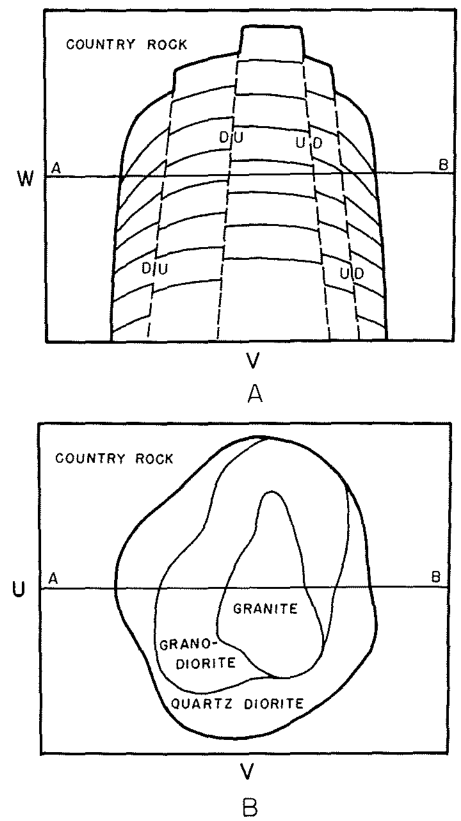

Figure 11. Possible development of zoned pluton in which individual zones have sharp boundaries. A, Differential vertical movement takes place in pluton in which mineralogical variation has the geometry of the model in Figure 8, V,W cross section; B, Result observed on U,V map.

This study emphasizes the need for three-dimensional observations and three-dimensional thought regarding granitic plutons. Only additional quantitative information concerning three-dimensional variation in other plutons will determine whether the model based on the Glen Alpine stock is common or the exception to the rule.

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APPENDIX 1: STATISTICAL CONSIDERATIONS

A. SAMPLE PLAN: A square grid, based on cells 400 feet square, was used to obtain an even distribution of samples from the exposed surface of the stock. Outcrops are so common that, in all, samples were collected from 85 per cent (113) of the grid intersections within the stock. Only the southern extremity and part of the eastern edge of the stock are not represented by samples. The unsampled area is small in comparison to the area over which samples are uniformly distributed. Duplicate specimens were collected at odd-numbered localities to estimate the amount of variation between closely spaced samples (Peikert, 1963).

Ideally, equal representation of all vertical levels would be a desired part of the sample plan; however, where vertical control is dictated entirely by topography, equal representation could only be obtained by drilling. Figure 12 shows the vertical distribution of samples. The distribution is nearly uniform between 7000 and 7250 feet, but representation diminishes toward higher and lower elevations. If vertical variation is important, such a distribution will tend to reduce the over-all variance of the measurements; if vertical variation is not statistically significant, results will not differ from those produced from a uniform vertical distribution of samples.

B. SCALE OF MINERALOGICAL VARIABILITY: A preliminary step in studying mineral variation in a granitic pluton is to determine the scale at which the most significant variation occurs. This gives an idea of the way that a pluton may be usefully studied. For example, if variation within an outcrop is more significant than variation on a larger scale, mapping of individual outcrops might prove useful in studying the pluton; if variation between widely spaced specimens is most significant, patterns of areal variability should be considered in a petrogenetic study.

The statistical significance of mineralogical variability at a given scale may be estimated in several ways. In the present example a components-of-variance model such as that given by Dixon and Massey (1957, Table 10:27, p. 175) is used. Testing by this method estimates and compares the variances associated with different scales of observation. Unless the variance at a given scale is a very small percentage of the variance at a smaller scale, it is statistically significant. Numerical tests may be applied but are not necessary to support the major points of the following discussion.

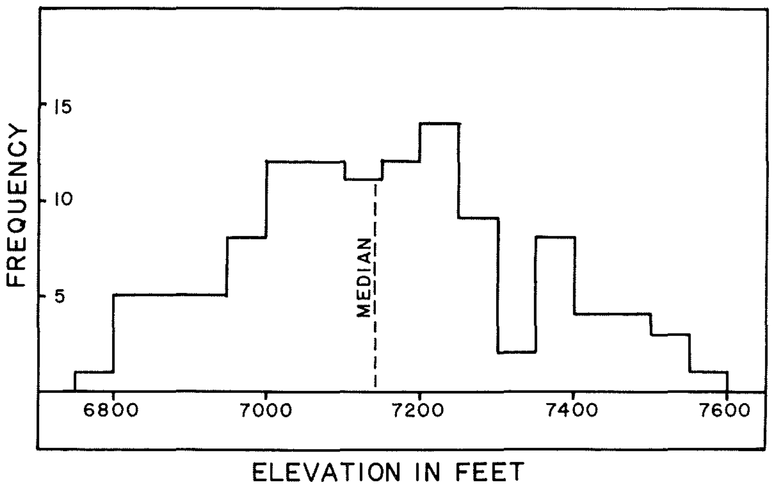

Figure 12. Histogram showing vertical distribution of sample stations in the Glen Alpine stock

The sample design used is a two-level nest; two different sets of samples were used to test over-all variation against determinative error and over-all variation against variation at a sample locality. All modes in the Glen Alpine stock were determined by a count of 1500 points; a 0.3-mm interval between points and a 1-mm interval between traverses were used. The area covered is approximately 450 mm^2 for each specimen.

With the aid of a random number table, the author randomly selected 10 thin sections for replicate determination. The grid of the second determination did not exactly coincide with the first, so the factor σ_d includes both operator error and variation within the individual section. The complete components-of-variance model is shown for quartz (Table 3), but only results are shown for other variates (Table 4). For each mineralogical

TABLE 3. COMPONENTS-OF-VARIANCE MODEL FOR QUARTZ: BETWEEN-DETERMINATION VARIANCE

	Sum of squares	*df*	Mean square
Between sections (Estimate of $\sigma_d + 2\sigma_o$)	91.55	9	10.17
Between determinations (Estimate of σ_d)	10.02	10	1.00
Total	101.57	19	..

σ_d = variance of replicate determinations.
σ_o = variance of randomly selected sections.

TABLE 5. COMPONENTS-OF-VARIANCE MODEL FOR QUARTZ: LOCAL VERSUS AREAL VARIABILITY

	Sum of squares	*df*	Mean square
Between localities (Estimate of $\sigma_l + 2\sigma_a$)	1201.7	55	21.85
Within localities (Estimate of σ_l)	305.5	56	5.45
Total	1507.2	111	..

σ_l = variance within sample localities.
σ_a = variance between localities.

variate the between-section variance is several times the between-determination variance.

Duplicate samples (up to 30 feet apart) from 56 localities were used to estimate variation between as opposed to variation within sample localities. The complete calculation is shown for quartz (Table 5) with results reported directly for other variates (Table 6). The variance within localities (σ_l) contains variance of all smaller scales that cannot be separated without a complete heararchical design which would require a prohibitive number of modes. In Table 6 the between-locality variance (σ_a) is not only significant with respect to the within-locality variance (σ_l) but is actually larger for every variate. This means that there is significant areal mineralogical variation in the stock, over and above the possible error incurred in assigning a single thin-section mode to represent a given sample locality.

C. LEAST-SQUARES METHODS: To determine the position of equal-value surfaces directly would require samples from numerous elevations at each grid point, presumably from drilling. Where vertical control is dictated by topography, as in the present example, it is only possible to estimate the position of these surfaces.

This estimate could be made in several ways. The method of least squares, used in the present study, is relatively simple and has yielded significant results in many geological studies (Whitten 1959; 1961a; 1961b; Allen and Krumbein, 1962; and others). In this method observations are approximated by polynomial surfaces of varying degree using conventional least-squares techniques. The reader is referred to Grant (1957) and Krumbein (1959) for discussion of the theory and mathematical details of the method and to Peikert (1963) for a discussion of the IBM 709 program used in the present study.

The least-squares or expected value of a dependent variable (X_{comp}) is defined as a function of geographic co-ordinates U,V, and W in a power series polynomial of the type:

$$X_{comp} = a_0 + a_1U + a_2V + a_3W + a_4U^2 + a_5UV + a_6UW + a_7V^2 + a_8VW + a_9W^2 + a_{10}U^3 + a_{11}U^2V \cdots .$$

The polynomial coefficients a_0, a_1, a_2, . . . are evaluated according to the least-squares principle so that $\Sigma\ (X_{obs} - X_{comp})^2$ is a minimum where X_{obs} is a single observation at point $(U,V,W)_i$. The term $(X_{obs} - X_{comp})$, called the deviation (X_{dev}), will be referred to in further discussion.

In the present study observational data have been successively approximated by linear (first-degree), quadratic (second-degree), and cubic (third-degree) polynomial surfaces. The per cent reduction in the total sum of squares associated with each least-squares surface is shown in Table 7. If the least-squares surfaces were to be used as predicting devices, sum-of-squares reductions of these magnitudes would not be satisfactory. However, in the present study the least-squares method is used only to approximate the form and position of equal-value surfaces within the stock. If one considers the number of observations, the sum-of-squares reductions appear to be sufficiently high to justify the model constructed from the equal-value surfaces.

One of the major problems in applying this

TABLE 4. RESULTS FOR OTHER VARIATES OF ANALYSIS USED IN TABLE 3

	σ_d	σ_o
Quartz	1.00	4.59
Plagioclase	1.11	8.56
Potassium feldspar	0.94	8.05
Color index	1.19	5.32

TABLE 6. RESULTS FOR OTHER VARIATES OF ANALYSIS USED IN TABLE 5

	σ_l	σ_a
Quartz	5.45	8.20
Plagioclase	6.55	14.96
Potassium feldspar	7.03	18.98
Color index	5.65	5.96

TABLE 7. PER CENT REDUCTION IN TOTAL SUMS OF SQUARES DUE TO LEAST-SQUARES POLYNOMIAL SURFACES BASED ON U,V,W CO-ORDINATES

	Degree 1	Degree 2	Degree 3
Quartz	6.2	21.9	34.5
Plagioclase	15.6	23.3	26.4
Potassium feldspar	21.6	32.5	35.7
Color index	17.2	38.9	50.0
Hornblende	11.3	37.2	51.2

method is the determination of how many polynomial terms are necessary to express the regional variation in a set of observations. If observations are on an orthogonal grid it is convenient to compute a Z^2 array (Grant, 1957) which shows the percentage of the total sum of squares accounted for by each polynomial coefficient. Grant illustrated the procedure for separating this array into a portion representing regional variation (trend) and a portion representing only random local variation (residual).

Mandelbaum (1963) has shown that it is also possible to determine the sum-of-squares reduction associated with the addition of each successive term in a polynomial expression. This applies to observations that are irregularly spaced for which the polynomial coefficients are not independent, as in the orthogonal example. Mandelbaum defined the polynomial that best separates the regional from the local variation as that for which ${}^{ss}X_{\text{dev}}/(N - P)$ is a minimum, where ${}^{ss}X_{\text{dev}}$ is the sum of squares of the deviations, N is the number of observations, and P is the number of polynomial terms included in the regional trend. The term $(N - P)$ is the degrees of freedom of the deviations.

TABLE 8. MANDELBAUM'S TEST
Values given are ${}^{ss}X_{\text{dev}}/(N - P)$

	Degree 1	Degree 2	Degree 3
Quartz	11.750	10.158	9.085
Plagioclase	18.122	17.079	17.496
Potassium feldspar	21.052	18.800	19.118
Color index	10.081	7.710	6.740

The program used in the present study computes only the sums of squares associated with the complete first-, second-, and third-degree polynomial surfaces, corresponding to $P = 4$, 10, and 20. If Mandelbaum's test is applied to the information given in Table 7, one can see (Table 8) that the degree-three surfaces are the best approximation of the large-scale variation of modal quartz and color index, but that these degree-three surfaces contain more terms than are necessary to express the regional variation of modal plagioclase and potassium feldspar. For comparative purposes, the degree-three surfaces are used throughout to illustrate mineral variation.

Whitten (1959; 1962b) has shown that the areal distribution of X_{dev} values can be of geological significance. This situation arises when the deviations contain systematic variation in addition to random variation. Additional study may show that the deviations from the degree-three surfaces used in the present study are of geological importance. Nonetheless, these deviations do not affect the large-scale variation on which the present considerations are based and may result from entirely different geological processes.