POROSITY DETERMINATIONS AND THE THERMAL CONDUCTIVITY OF ROCK FRAGMENTS WITH APPLICATION TO HEAT FLOW ON CYPRUS

PAUL MORGAN*

Geology Department, Imperial College, London (Great Britain)

A standard core analysis technique has been modified to estimate porosities from measurements on rock fragments. For the range of rocks tested, chip-determined fractional porosities were within ±0.025 of the values measured on solid-core samples. This has enabled thermal conductivity measurements on rock fragments to be corrected for the effect of porosity, yielding agreement with conductivity determinations on solid core generally to better than ± 10%. The application of this is illustrated by the determination of heat flow in a 300-m borehole in western Cyprus (latitude 34°54′N, longitude 32°34′E, elevation 82 m). A decrease in temperature gradient with depth is almost completely compensated for by increasing thermal conductivity, and the best value for heat flow at this site is 23 ± 4 mW m^{-2}.

1. Introduction

In most modern theories of the evolution of the Earth's crust, geothermal mechanisms play an important, if not fundamental role. During the past decade, therefore, many terrestrial heat-flow values have been determined by the product of the local crustal temperature gradient with the local rock thermal conductivity. Temperature measurements are generally made in boreholes that have been drilled for other purposes, but, with the exception of mineral exploration holes, core samples are often not available, so that the standard methods for determining thermal conductivities (steady-state divided-bar techniques) [1–4] cannot be used.

For many boreholes the only samples available are drill cuttings and Sass et al [5] have devised a technique to use these cuttings in conductivity determinations. The rock conductivity is deduced from a steady-state measurement of the thermal conductivity of a copper and plastic cell containing water and saturated rock fragments. A simple model is then used to estimate the conductivity of the water saturated aggregate

from the conductivity of the cell, its contents, and the dimensions and conductivities of the materials of the cell. It has been demonstrated that where the conductivities of the constituents do not contrast by more than one order of magnitude, the conductivity of a mixture can be represented by the weighted geometric mean of the conductivities of its components [6]. If K_w and ϕ are the thermal conductivity and volume fraction of water in the cell respectively, and K_a is the determined conductivity of the cell contents, the weighted geometric mean conductivity of the solid components of the cell contents, K_r, is given by:

$$K_r = K_w(K_a/K_w)^{1/(1-\phi)} \tag{1}$$

Assuming the rock to be isotropic, K_r represents its conductivity if its natural porosity is zero and in this case K_r can be used as an estimate of the in-situ rock conductivity.

For porous isotropic rocks eq. 1 must be modified to account for the interstitial water present in the in situ rock. To calculate porous-rock conductivities, K_{pr}, Sass et al. [5] derive the expression:

$$K_{pr} = K_w(K_a/K_w)^{(1-\phi_0)/(1-\phi)} \tag{2}$$

where ϕ_0 denotes the natural fraction porosity of the uncrushed rock. For some boreholes, particularly oil wells, much information is available concerning the

* Present address: Department of Geological Sciences, Southern Methodist University, Dallas, Texas 75275, U.S.A.

sediment porosities. In a great many holes, however, no such data are available. In order to extend the chip-conductivity method therefore, it is necessary to either modify the techniques to determine porous-rock conductivities more directly, or to estimate the natural-rock porosities from measurements on the rock fragments.

An attempt was first made to measure porous-rock conductivities directly. The volume fraction of water in the cell is determined from the capacity of the cell, and the difference between the weights of the cell, first packed with dry rock fragments and then after saturation with water under a moderate vacuum. It was reasoned that if the rock chips were saturated and the excess water removed before packing into the cell, then the volume fraction of water then determined from the subsequent cell weights would only represent the water in the inter-chip spaces. The cell contents could then be considered as a mixture of water and saturated rock fragments, and the porous-rock conductivity determined directly using eq. 1.

This technique was attempted with several rock-fragment samples but no consistent results could be obtained, the main problems arising in the removal of the excess water from the chips. In order to overcome this difficulty, organic liquids immiscible with water were used in place of water in the mixture with the saturated rock fragments. Analine, chloroform and castor oil were tested using the appropriate liquid thermal conductivity in place of K_w in eq. 1, but again the results were unrepeatable and inconsistent. The possibilities of estimating natural-rock porosities from measurements on the rock fragments were then investigated.

As no routine methods for the measurement of rock-chip porosities were known to the author, an attempt was made to adapt a core analysis technique for use with rock fragments. One of the instruments used in routine laboratory porosity determinations is the Kobe porosimeter, a single-cell Boyle's law apparatus, the performance of which has been described by Beeson [7]. Successful techniques have been developed to use this apparatus to determine rock-fragment porosities for thermal conductivity purposes.

The design and working principles of the Kobe porosimeter are briefly described below, followed by a discussion of the new techniques developed for handling chip samples. The success of the method is illustrated by a comparison of the results of porosity measurements on solid-core samples with rock fragments crushed from these cores; by a comparison of thermal conductivity measurements on solid samples with determinations on the corresponding chip samples; and finally by the determination of heat flow in a 300-m water exploration borehole in the west of the island of Cyprus.

2. The Kobe porosimeter

Fraction porosity, ϕ_0, is calculated from measurements of the bulk volume, V_b, and grain volume, V_g, of the sample using the expression:

$$\phi_0 = (V_b - V_g)/V_b \qquad (3)$$

The bulk volume is measured simply from the volume of mercury that the sample displaces at atmospheric pressure, assuming that none of the rock pores are entered by the mercury. The grain volume is determined by the volume of helium that the sample displaces, measured by a method involving the saturation of the sample with helium under pressure and an application of Boyle's law.

The version of the Kobe porosimeter used in this study is shown in Fig. 1. A mercury pump is used to inject mercury into the sample chamber, and measurement of the volume of mercury injected is made to a precision of 1 mm^3 using a vernier scale on the calibrated pump barrel. The mercury level can be accurately located at two levels, the top and bottom contacts. Gas pressure in the sample chamber can be equalized with atmospheric pressure through the outlet valve, and helium can be introduced using the gas-control valve. A two-way tap was added in series with the latter valve to allow the sample chamber to be evacuated, as discussed later. Gas pressure in the sample chamber is measured by a mercury pressure gauge connected to the mercury in the pump body. Measurements on solid-rock samples are made on cylindrical cores approximately 25 mm long and 25 mm in diameter.

Detailed experimental procedures for the use of porosimeters vary between different apparatuses [7,8] but the fundamental principles are the same and the basic stages in the use of a Kobe porosimeter are outlined in Fig. 2. The equation used to calculate the sam-

OUTLET VALVE

BOTTOM CONTAC

OUTLET TO
PRESSURE
GAUGE

Fig. 1. Diagram

ple grain vol

$V_g = (v_e - v$

(notation as
of Boyle's la
suming isoth
curate to cal
the mercury
troduces som
P_c. It is also
measuremen

CAS CONTROL VALVE

TWO WAY TAP

OUTLET VALVE

HELIUM

VACUUM

REMOVABLE CAP

TOP CONTACT

CAP LOCKING RING

SEAL

POROSIMETER BODY

BOTTOM CONTACT

MERCURY

CALIBRATED PUMP BODY

OUTLET TO
PRESSURE
GAUGE

SCREW-TYPE LOW PRESSURE MERCURY PUMP

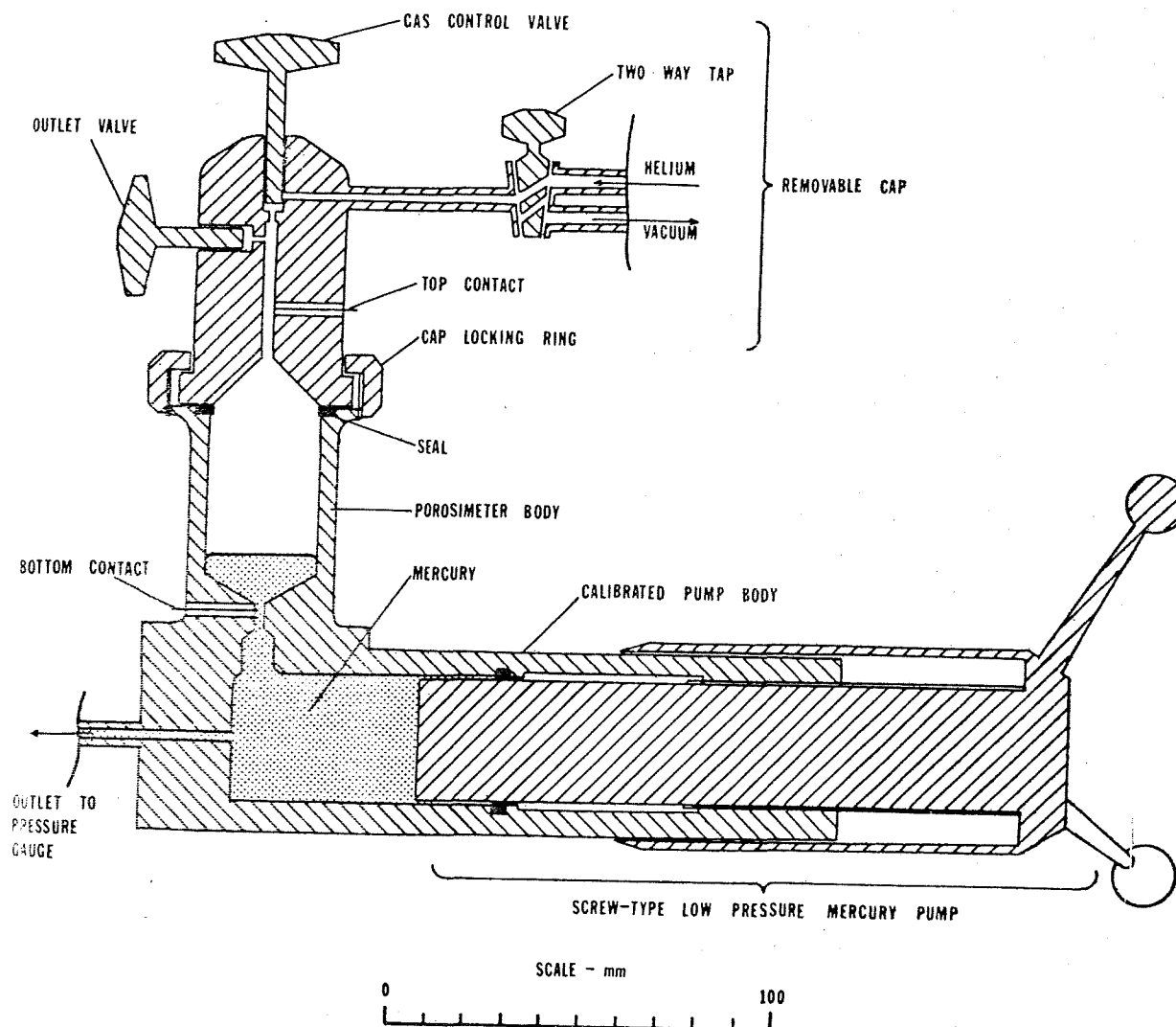SCALE – mm

0   100

Fig. 1. Diagrammatic cross-section of the Kobe porosimeter.

ple grain volume:

$$V_g = (v_e - v_s)/(1 - P_a/P_c) \qquad (4)$$

(notation as in Fig. 2) is derived from an application of Boyle's law to the gas in the sample chamber assuming isothermal conditions. In practice it is inaccurate to calculate V_g from eq. 4, as measurement of the mercury pressure and not the helium pressure introduces some uncertainties in the absolute value of P_c. It is also necessary at the start of each session of measurements to reject the first two or three deter-

minations as temperature differences through the apparatus introduce errors. Calibration runs are therefore made using a sample of known volume to determine a value for $(1 - P_a/P_c)$, and to ensure that the apparatus is in thermal equilibrium. A convenient method of effecting this is to pump a known volume of mercury, V_m, into the cell before the compression and again measure volume change, v_m, required to compress the helium to P_c. The measurement is repeated until consistent results are obtained, and used to calibrate the instrument by substituting the relevant

BULK VOLUME DETERMINATION
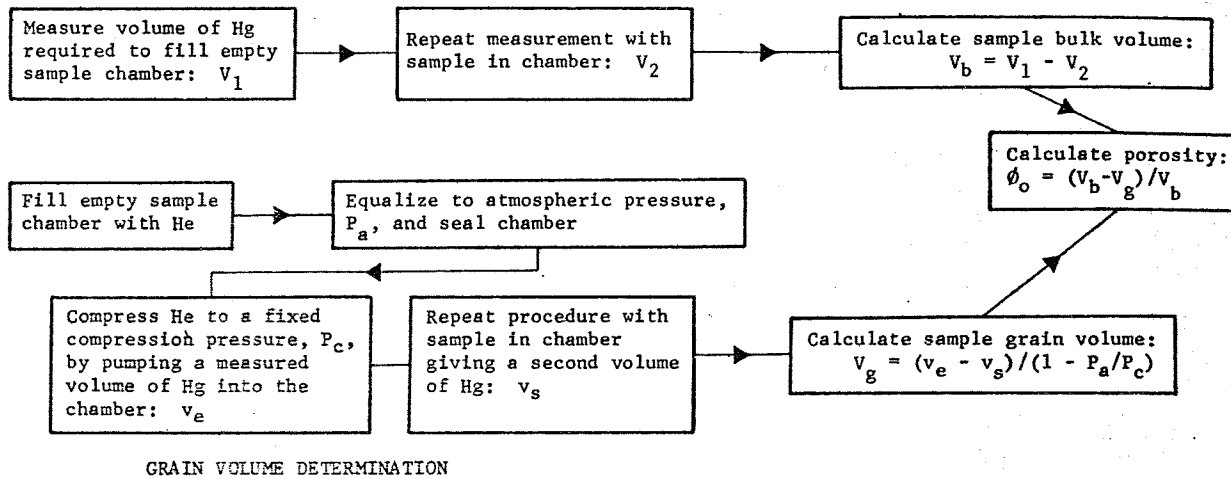
Fig. 2. Flow chart showing the basic stages in porosity determination using the Kobe porosimeter.

GRAIN VOLUME DETERMINATION

expression containing these values for $(1 - P_a/P_c)$ in eq. 4, i.e.:

$$V_g = V_m(v_e - v_s)/(v_e - v_m) \qquad (5)$$

Grain volumes are thereby calculated without the need for values of P_a and P_c, and before actual sample measurements are started the thermal equilibrium of the system is tested.

3. Porosity measurements on rock fragments

For porosity measurements on rock fragments, approximately 12.5 cm^3 of the chips were loosely packed into a stainless steel cage (Fig. 3a). The bulk and grain volumes of the fragments are determined by subtracting the appropriate cage volume from the measured combined cage and chip volumes. Certain modifications to the basic techniques are necessary, however, to obtain consistent results.

In the bulk volume determinations it is important that the mercury completely surrounds all the chips and is not excluded from any regions by trapped pockets of air. To overcome this problem a two-way tap was fitted to the instrument, in series with the gas-control valve, to allow a moderate vacuum (0.7–1.4 kN m^{-2}) to be applied to the sample while the mercury is pumped into the cell. The system is then re-

turned to atmospheric pressure for the actual measurement. This has the undesired effect, however, of increasing the penetration of the mercury into the pores of the sample. By consideration of the pressure drop across a spherical mercury–air interface [9], it was estimated that, without the use of the vacuum
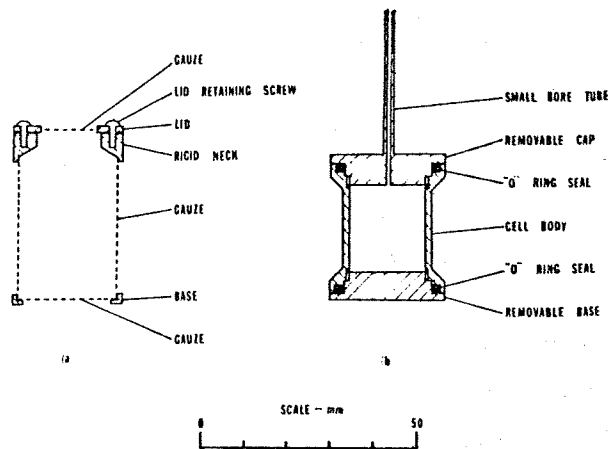
Fig. 3. a. Diagrammatic cross-section of porosity cage. The 26-mesh stainless-steel gauze is held into the stainless-steel body parts by epoxy resin. Horizontal cross-section circular. b. Diagrammatic cross-section of porosity cell. The cell was designed and constructed in stainless steel by the Exploration and Production Research Division of the British Petroleum Co. Ltd. Horizontal cross-section circular.

on solid-core samples with measurements on rock fragments crushed from these cores. These results are shown in Table 1 and it can be seen that there is agreement to within ±0.025, which is within the experimental repeatability of the apparatus.

4. Application to thermal conductivity measurements

The effect of natural porosity on whole-rock thermal conductivities is demonstrated in Fig. 4. It can be seen that the reduction in conductivity is most marked for rocks with a high geometric mean conductivity of the solid constituents, but is still very significant even with relatively low-conductivity rocks. Porosity is therefore an important parameter in the determination of thermal conductivities from measurements on rock fragments using eq. 2.

Experimental studies have shown that rock fragment fractional porosities can be determined using modified techniques with the Kobe porosimeter to within ±0.025. The fractional error in the calculated conductivity, $\partial K_{pr}/K_{pr}$, due to an uncertainty ϕ_0 in porosity, is given by:

$$\partial K_{pr}/K_{pr} = \phi_0 (\ln K_w - \ln K_r) \qquad (6)$$

and the error limits for a ±0.025 uncertainty in volume fraction porosity are shown in Fig. 3. It can be seen that this error is higher for more conductive rocks, rising from 3.0 to 5.7% as K_r increases from 2 to 6 W m^{-1} °C^{-1}. Sass et al. [5] give an accuracy limit of ±10% for the basic chip conductivity measuring method, and this limit must be extended to ±16% for higher-conductivity material where porosity measurements are included in the determination. If care is taken to remove all systematic errors from the measurements, however, it should be possible to improve this accuracy by repeating measurements on the same sample.

A comparison of conductivity determinations from solid-core and rock-fragment samples for most of the rock types listed in Table 1 is shown in Fig. 5 and tabulated in Table 2. Solid-core conductivities were measured in a brass divided-bar apparatus (as described by Beck [3] but with no guard-ring fitted to the bar), and chip conductivities were determined using
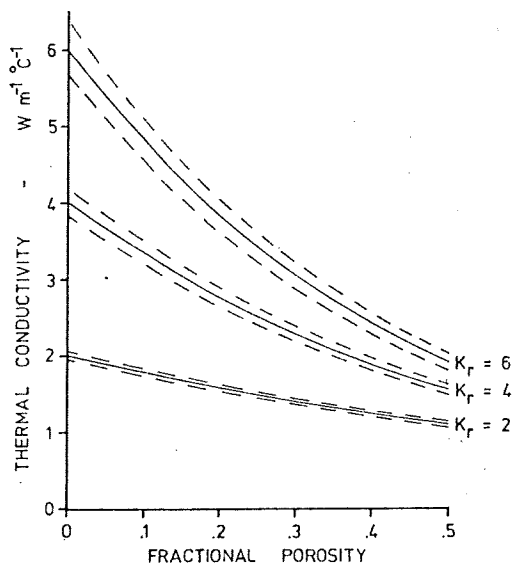

Fig. 4. The effect of porosity on thermal conductivity for different values of K_r. Dashed curves show the conductivity errors for a ±0.025 error in fractional porosity.
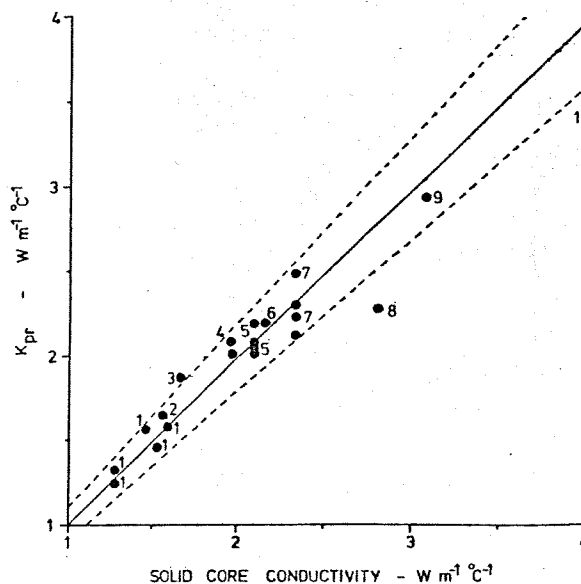

Fig. 5. Comparison between solid- and chip-determined conductivities. Dotted lines show the ±10% error limit for K_{pr}. Numbers by points refer to sample lithology as listed in Table 2.

TABLE 2

Conductivity data used in Fig. 5; K_r, K_{pr}, K_d: Non-porous chip, porous-chip and solid-core conductivities respectively in W m^{-1} °C^{-1}

Key no.	Sample description	K_r	ϕ_0	K_{pr}	K_d	Dif
1	Basaltic pillow lava	1.34	0.101	1.24	1.28	−3.3
		1.47	0.124	1.31	1.28	+1.6
		1.82	0.150	1.56	1.46	+6.9
		1.86	0.216	1.46	1.53	−4.7
		1.93	0.172	1.58	1.59	−0.8
		4.03	0.089	3.41	3.98	−14.5
2	Basalt	1.96	0.146	1.65	1.56	+6.2
3	Miocene chalk	2.80	0.267	1.87	1.67	+12.1
		3.37	0.302	2.01	1.97	+1.9
4	Miocene sandy chalk	2.29	0.072	2.08	1.96	+6.2
5	Miocene to Pliocene marl	2.32	0.105	2.01	2.09	−3.6
		2.62	0.164	2.05	2.09	−1.0
		2.52	0.136	2.08	2.09	−0.4
		2.56	0.109	2.19	2.09	+5.2
	Miocene fine grained sandstone	2.24	0.051	2.10	2.12	−1.2
6	Miocene marl	2.71	0.143	2.19	2.15	+1.6
7	Miocene and Pliocene sandy marl	2.48	0.111	2.12	2.33	−9.1
		2.69	0.127	2.23	2.33	−4.5
		2.60	0.095	2.30	2.33	−1.4
		2.93	0.151	2.31	2.33	−1.1
		3.08	0.132	2.48	2.33	+6.3
8	Altered dunite	2.35	0.023	2.28	2.80	−18.5
9	Mesozoic limestone	3.04	0.025	2.92	3.05	−4.2

Where the samples were very friable making single solid-core determinations unreliable, the mean of the solid determinations is given. ϕ_0: Fractional porosity. Dif: Differences determined by Dif = $(K_{pr} - K_d)/K_d \cdot 100$.

the same apparatus [5], corrected for the effect of porosity as described above. The chip samples were crushed from the actual cores on which the solid-core conductivities were measured to minimize variations due to compositional heterogeneity. Most of the solid and chip conductivities agree to within the ±10% error limit for K_{pr} and, as the precision of a single solid determination was generally ±3%, these are well within the predicted limits of experimental error. Only two results are significantly outside the ±10% error limit: the high-conductivity basaltic pillow-lava chips were difficult to permeate with helium during the grain-

volume measurement, and the porosity determination is thought unreliable; the altered dunite rock fragments were possibly hydrated during saturation for the conductivity measurement, as slight swelling occurred and some of the sample was lost from the conductivity cell, making the conductivity determination inaccurate. The results then show the success of the incorporation of the porosity measurements into the chip conductivity technique.

Horai and Simmons [10] have adopted an alternate technique for measuring the thermal conductivity of rock fragments. The chips are crushed into powder, saturated with water, and the conductivity of the resulting mixture is measured by the needle-probe method [11]. As with the steady-state technique of Sass et al. [5], however, some estimate of the rock porosity is required to calculate the porous-rock conductivity. Determinations of porosities using the Kobe porosimeter as outlined above would allow this calculation to be made. King and Simmons [12] determined porosities for this purpose from density measurements on dry and water-saturated specimens of the uncrushed rock. As the size of the rock fragments decreases, however, this method of porosity determination becomes impractical as it becomes increasingly difficult to remove the excess water from the saturated chips. The Kobe porosimeter technique does not suffer from this problem and is therefore more generally applicable than the wet- and dry-density method.

Choice of the relationship for determining the thermal conductivity of the solid component of the fluid–solid mixture is made purely on an empirical basis. Sass et al. [5] have chosen the weighted geometric mean relationship (eq. 1) which has been adopted for the present study. With the needle-probe method both the geometric mean (e.g. see [12]), and the average of the upper and lower bounds given by Maxwell's relations for non-interacting spheres of one conductivity in a matrix of another (e.g. see [10,13]) have been used. The precision of the experimental data is not sufficiently high to indicate the more representative of these relationships. In a study of the thermal conductivity of vesicular basalt, Robertson and Peck [14] have concluded that the thermal conductivity of the fluid–solid rock system is best represented by a mean of parallel and series models or a quadratic relationship. Whichever relationship is chosen, however, the measurement of natural-rock porosities using the Kobe

260
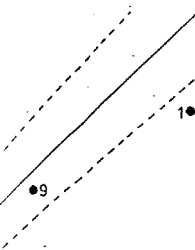
porosimeter will increase the precision of determinations of porous-rock conductivities.

5. Heat flow in Cyprus

The application of rock-fragment porosity determinations to terrestrial heat-flow determinations is demonstrated by the results from a 300-m water exploration borehole on Cyprus. The borehole penetrated Miocene marls, chalks, chert and gypsum of the Pakhna formation, but no solid-core samples were recovered. Drill cutting samples were collected, however, and the rock conductivities were estimated from measurements on these samples by the techniques outlined above. Measured porosities were in the range 0.18–0.47, and the effects of these on the calculated conductivities are shown in Fig. 6. Least-squares temperature gradients were calculated from the temperature data within each depth range represented by a conductivity sample and Fig. 6 shows some inverse correlation between the gradient and conductivity data. Component heat flow values were calculated using conductivity values both

corrected and uncorrected for porosity. It can be seen that there is less scatter in the former results.

Except for the upper 40 m of the hole, temperature logs made on 10 June 1970 and 14 May 1971 agreed to within ±0.02 °C and the data are considered to be free from the effects of short-term transient disturbances. The heat flow for the interval 38–304 m was calculated to be 19.4 ± 0.1 mW m^{-2} using a least-square fit to the temperature and thermal resistance data [15]. Correction for the steady-state influence of local topography [16] increases this value to 21.0 mW m^{-2} and corrections for the topographic evolution of the area increase this by a further 5–10% (based on data from Henson et al. [17] and Vaumas [18]). These corrections cannot explain the decrease in heat flow with depth shown in Fig. 6 and this could be the result of Quaternary climatic changes, horizontal variations in thermal conductivity, or regional water flow. Lack of information prevents the reliable estimation of these effects, and the best value for the heat flow is estimated to be 23 ± 4 mW m^{-2}.

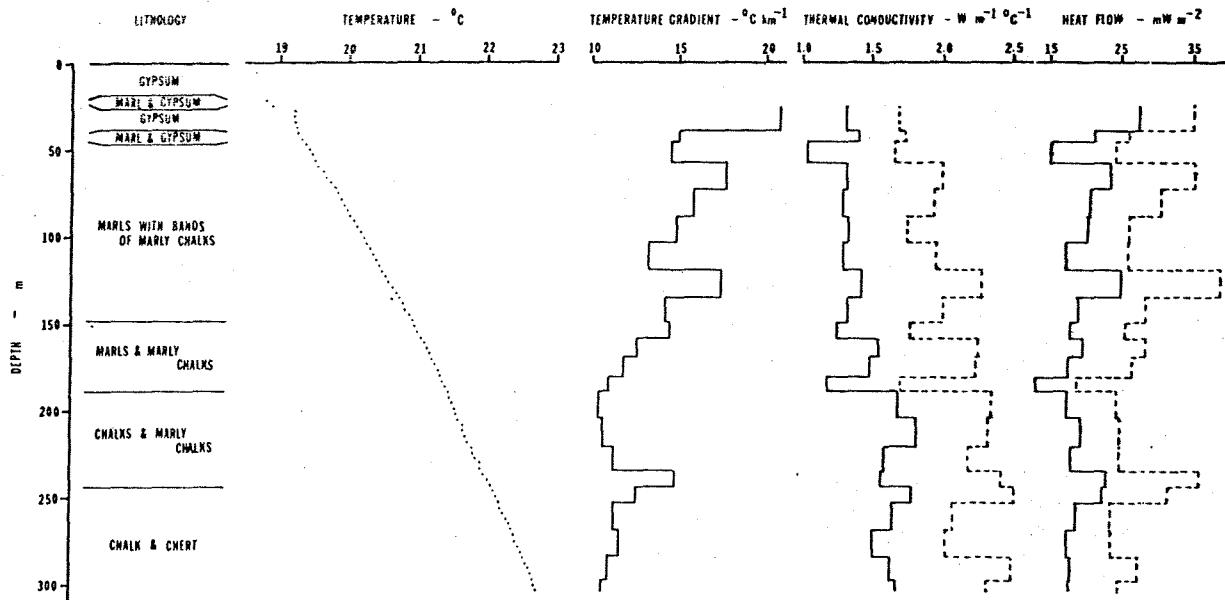Additional heat-flow data from Cyprus confirm this result [8], and marine heat-flow determinations

Fig. 6. Heat-flow results from Cyprus government water borehole PB35 (latitude 34°50′N, longitude 32°34′E, elevation 82 m). Drilling started 7 October 1969, completed 25 November 1969. Temperature logs made 10 June 1970 and 14 May 1971. Dashed lines show conductivities and calculated heat-flow values uncorrected for the effect of porosity.

in the eastern Mediterranean have yielded similar low values [19,20]. Discussions of these results are given by Erickson [19], Ryan et al. [20] and Morgan [8].

6. Conclusions

The results shown in Table I indicate that with a modification of the basic techniques the Kobe porosimeter can be used to estimate natural-rock porosities from measurements on rock fragments. Fractional porosities have been determined to within ±0.025 which is quite adequate for the correction of thermal-conductivity measurements on chip samples for the effects of natural-rock porosity. Porous isotropic rock thermal conductivities determined from measurements on rock fragments agree to within ±10% of solid-core determinations (Fig. 5). Random errors of this magnitude are acceptable for terrestrial heat-flow determinations, as variations in conductivity due to heterogeneity between hand samples are of the same order [5]. Data from a Cyprus borehole (Fig. 6) illustrate the application of the chip porosity and conductivity technique.

Useful results can also be obtained with anisotropic rocks, although independent information on the magnitude of the anisotropy and the orientation of the principal conductivity axes in situ is also required [5]. If porosity measurements are to be made on high-permeable rock fragments, control tests should also be made on similar solid-core samples to check the bulk volume determinations. No inconsistencies were noted in the results from the range of rocks tested in this study, but calculations suggest that, with the use of the vacuum in the bulk-volume measurements, mercury will enter the rock pores connected by channels greater than 0.006 mm diameter. In this case, trapped pockets of air may be removed by the use of a mechanical vibrator on the system, with or without the aid of a reduced vacuum, or by very loose packing of the chips and using a very slow rate of advance of the mercury into the chamber.

The development of the chip conductivity technique and its extension to porous rocks where no borehole-porosity information is available should allow terrestrial heat-flow measurements to be made in many existing boreholes which were previously unsuitable. It is hoped that this will enable geothermal data to be collected relatively inexpensively from

new areas, and make a valuable contribution to the understanding of the thermal mechanisms within the Earth.

References

1 F. Birch, Heat flow in the Front Range, Colorado, Bull. Geol. Soc. Am. 61 (1950) 567.
2 A.E. Beck, A steady state method for the rapid measurement of the thermal conductivity of rock, J. Sci. Instrum. 34 (1957) 186.
3 A.E. Beck, Techniques of measuring heat flow on land, In: W.H.K. Lee (Editor), Terrestrial Heat Flow, Geophys Monogr. Ser. No. 8 (1965) 24 pp.
4 A.M. Jessop, The effect of environment on divided bar measurements, Tectonophysics 10 (1970) 39.
5 J.H. Sass, A.H. Lachenbruch and R.J. Munroe, Thermal conductivity of rocks from measurements on fragments and its application to heat flow determinations, J. Geophys. Res. 76 (1971) 3391.
6 W. Woodside and J.H. Messmer, Thermal conductivity of porous media, J. Appl. Phys. 32 (1961) 1988.
7 C.M. Beeson, The Kobe porosimeter and the oilwell research porosimeter, Am. Inst. Min. Metall. Pet. Eng., Trans. 189 (1950) 313.
8 P. Morgan, Terrestrial heat flow studies in Cyprus and Kenya, Ph. D. Thesis, University of London (1973).
9 H.L. Ritter and L.C. Drake, Pore size distribution in porous materials, Ind. Eng. Chem., Anal. Ed. 17 (1945) 782.
10 K. Horai and G. Simmons, Thermal conductivity of rock-forming minerals, Earth Planet. Sci. Lett. 6 (1969) 359.
11 R. von Herzen and A.E. Maxwell, The measurement of thermal conductivity of deep-sea sediments by a needle-probe method, J. Geophys. Res. 64 (1959) 1557.
12 W. King and G. Simmons, Heat flow near Orlando, Florida

and Uvalde, Texas determined from well cuttings, Geothermics 1 (1972) 133.

13 K. Horai, Thermal conductivity of rock-forming minerals, J. Geophys. Res. 76 (1971) 1278.

14 E.C. Robertson and D.L. Peck, Thermal conductivity of vesicular basalt from Hawaii, J. Geophys. Res. 79 (1974) 4875.

15 E.C. Bullard, Heat flow in South Africa, Proc. R. Soc. London, Ser. A 173 (1939) 474.

16 E.C. Bullard, The disturbance of the temperature gradient in the Earth's crust by inequalities of height, Mon. Not. R. Astron. Soc., Geophys. Suppl. 4 (1940) 360.

17 F.R.S. Henson, R.V. Browne and J. McGinty, A synopsis of the stratigraphy and geological history of Cyprus, Q. J. Geol. Soc. London 105 (1949) 1.

18 E. de Vaumas, Further contributions to the geomorphology of Cyprus, In: Annual Report for the Year 1960, Geol. Surv. Dep., Nicosia, (1961) 24 pp.

19 A.J. Erickson, The measurement and interpretation of heat flow in the Mediterranean and Black Sea, Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge, Mass. (1970).

20 W.B.F. Ryan, D.J. Stanley, J.B. Hersey, D.A. Fahlquist and T.D. Allan, The tectonics and geology of the Mediterranean Sea, In: A.E. Maxwell (Editor), The sea, 4 II, Wiley-Interscience, New York, N.Y. (1970) 387 pp.