APPLICATION OF THE GRADIENT/RADIANCE CHARACTERISTIC FUNCTION IN THE STUDY OF THE THERMAL TREND OF VOLCANIC AREAS

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ABSTRACT

The quantitative evaluation of some thermal parameters typical of the volcanic areas characterized by fumarolic activity has individuated a function given by the ratio between the absolute value of thermal gradient and the radiance itself, whose hystogram enables the volcanologist to monitor the situation of thermal transitory by comparing the data of two subsequent surveys. Two different situation of "anomalous" change is considered for the thermal power of conduction, the one related to the change of power with radius of the areas surrounding the fumaroles, the other connected with a temporal variation and directly dealing with the proper thermal transitory. Finally some considerations on the noticeable sensitivity of this "characteristic function" are expressed.

1. INTRODUCTION

In a previous paper (9) it was proposed to study quantitatively the volcanic areas characterized by secundary activity by monitoring the variations of four typical parameters.
They are of energetic significance, i.e. the distribution of thermal barycentres, the total emitted power and the hystograms of radiances and gradients.

The experience so far achieved indicated that the temporal changes of these quantities are little and sometimes difficult to be monitored, with special reference to the ones of baricentres and of total radiated power.

An analysis carried out on a simple geothermal model which simulates a single fumarole addressed the study to a characteristic function whose hystogram (or "physiognomy") shows the trend of the system to the stationarity or to the transitory.

2. THE POWER SUPPLYING THE RADIAL HEAT

A volcanic area characterized by secundary activity can be represented by a model constituted by a group of vertical cylinders which are heated by vapours and gases and surrounded by isotropyc material (Fig.1).
The description of the temperature versus radius is given by integrating the Fourier's equation

$$1) \qquad dQ = -K \frac{dT\,A(x)}{dx}\,dt$$

where

dQ	heat along the x direction
k	thermal conductivity coefficient
$A = 2\pi x L$	area of cylinder with length L and radius x
T	temperature
dt	time
x	radial coordinate

This equation can be written in terms of thermal conductivity power which can be constant or function of time as well as of radius. We have to consider, at least, these cases for the quantity $\frac{dQ}{dt} = P$

$$\frac{d}{dt}\frac{dQ}{dt} = \begin{cases} 0 \\ \pm \end{cases} \qquad\qquad \frac{d}{dx}\frac{dQ}{dt} = \begin{cases} 0 \\ \pm \end{cases}$$

We will hereafter consider only the simplest case of linearity

a)	$P = a$	constant
b)	$P = a \pm bx$	function of radius
c)	$P = a \pm ct$	function of time

The case a) represents the situation of stationarity. By integrating the equation 1) where $\frac{dQ}{dt} = P = a$, we obtain

$$T = T_M - H \ln \frac{x}{x_o} + \text{const}$$

where

T_M	temperature at the fumarole mouth
T	temperature along the radius
$H = \frac{P}{2\pi K L} = \frac{a}{2\pi K L}$	constant taking into account the geometry and the physical properties of the hole
x	radial coordinate
x_o	radius of the heating cylinder

The gradient is given by

$$G(x) = \frac{dT}{dx} = -\frac{H}{x}\,.$$

The relationship we would like to propose is given by ratio of the absolute value of gradient and the radiance:

$$R(x) = \frac{H}{x\left(T_M - H \ln \frac{x}{x_0}\right)}$$

The case b), which considers the conducting power as a function of radius x, gives for T(x), G(x) and R(x) respectively

$$T(x) = T - \frac{a}{2\pi\kappa L} \ln \frac{x}{x_0} \pm \frac{b}{2\pi\kappa L}(x_0 - x)$$

$$G(x) = \frac{dT}{dx} = -\frac{(a \pm bx)}{2\pi\kappa L x}$$

$$R(x) = \frac{G(x)}{T(x)} = \frac{\frac{a}{x} \pm b}{2\pi\kappa L T_M - a \ln \frac{x}{x_0} \pm b(x_0 - x)}$$

being $\frac{dQ}{dt} = a \pm bx = -\kappa A(x)\frac{dT}{dx}$

This case represents the situation of linear variation of power conductivity with radius. We consider the simplest configuration of linearity $\frac{dQ}{dt} = P = a \pm bx$ which can be used to simulate the self silling (a+bx) and the dispersion of heat (a-bx) respectively.

The case c) takes into account the temporal variation of the supplying power with time which is the actual case of transitory. In the first approximation of being the variation linear with time we obtain

$$\frac{dQ}{dt} = P = a \pm ct = \kappa A(x)\frac{dT}{dx}$$

$$T(x) = T_M - \frac{a \pm ct}{2\pi\kappa L} \ln \frac{x}{x_0} + \text{const}$$

$$G(x) = -\frac{a \pm ct}{2\pi\kappa L x}$$

$$R(x) = \frac{G(x)}{T(x)} = \frac{\frac{a \pm ct}{2\pi\kappa L}}{x\left(T_M \ \frac{a \pm ct}{2\pi\kappa L} \ln \frac{x}{x_0}\right)}$$

The physiognomies of R(x) defined as the area interested by the same value of the function, are given for the three cases by:

$$\alpha)\quad \frac{dA}{dR} = \frac{2\pi x^3 \left(T_M - H \ln \frac{x}{x_o}\right)}{\left(\frac{a}{2\pi k L}\right)^2 + \left(\frac{a}{2\pi k L}\right)^2 \ln \frac{x}{x_o} - \frac{a}{2\pi k L} T_M}$$

$$\beta)\quad \frac{dA}{dR} = \frac{-\frac{a}{x^2}\left[2\pi k L T_M - a \ln \frac{x}{x_o} \pm b(x_o - x)\right] + \left(\frac{a}{x} \pm b\right)^2}{\left[2\pi k L T_M - a \ln \frac{x}{x_o} \pm b(x_o - x)\right]^2}$$

$$\gamma)\quad \frac{dA}{dR} = \frac{2\pi x^3 \left(T_M - \frac{a \pm bt}{2\pi k L} \ln \frac{x}{x_o}\right)}{\left(\frac{a \pm bt}{2\pi k L}\right)^2 + \left(\frac{a \pm bt}{2\pi k L}\right)^2 \ln \frac{x}{x_o} + \left(\frac{a \pm bt}{2\pi k L}\right) T_M}$$

It can be shown that the curve of equation α) divides the plane into two regions, the one characterized by a positive increment of P with radius or with time, the other by the negative one. In particular the upper part of the plane divided by the curve is the one of the decreasing thermal transitory while the lower zone is occuped by the parametric curves of the increasing transitory (Fig.2).

This fact is of great importance. Namely, although it is practically impossible to determine by geological or by geophysical way the stationarity condition for an actual volcanic area, the comparison of two subsequent infrared surveys can indicate the general thermal trend of the system.

The considerations we have so far performed are valid to describe the thermal behaviour of a single source. We have to continue on our discussion analysing what happens for a system of heating sources for the situations considered before.

The actual distribution of fumaroles in an area characterized by fumarolic activity, can be sketched with a system of grouped heating sources which induces at the surface the distribution observed by infrared scanning.
Considering a sketch with n sources of coordinates x_i, Y_i in the x, y plane, the value of surficial temperature can be obtained by equaling the conducted heat to the emitted one

$$\text{conducted power} = \sum_i^n \frac{-k\, dx\, L\,(T_i - T)}{\sqrt{(x-x_i)^2 + (y-y_i)^2}} \cos\vartheta_i + \sum_i^n \frac{-k\, dy\, L\,(T_i - T)}{\sqrt{(x-x_i)^2 + (y-y_i)^2}} \operatorname{sen}\vartheta_i$$

$$\text{radiated power} = \pi \sigma \varepsilon\, dx\, dy\, T^4$$

obtaining at the end the equation:

$$T^4 \pi\sigma\varepsilon + \sum_i^n \frac{k L\,(T_i - T)}{(x_i - x)^2 + (y_i - y)^2} \quad (x_i - x) + (y_i - y) = 0$$

The solution T(x,y) which cannot be analitically achieved serves to plott the curves of T, G and R physiognomies.
A semiqualitative analysis can be useful to better understand shape and behaviour of the T, G and R physiognomies for a model like this. Consider we have n sources disposed on a regular network. The physiognomies referred to the group of "hot spots" look very like to the corresponding ones of just a single source, being simply a sum of the n contributions untill the circular area of influence surrounding each source has a radius so large to be overlapped one to another, generating "interacti

on". This last fact appears as a relative maximum in the physiognomy of T pointing out the temperature value of the privileged percentage area. As far as an uniform distribution of sources is concerned the interaction happens for the lower radian ce values. On the other hand if a general distribution is considered the relative maximum will occur for an intermediate value of the radiance. This last case is the one we more frequently encounter in the actual volcanic areas.

If a transitory takes place, which involves the whole area, the characteristic curves moves in the plane rotating around the origin (Fig.3). On the contrary if just a zone within the whole area is going toward a transitory, we will observe a shift only in a part of the R physiognomy curve (Fig.4).

As a summing up, two are, at least, the aspects to be studied in respect with a given area which shows a change in its thermal stability: the presence and posi tion of a relative maximum in the T physiognomy and the rotation with time of the physiognomy of gradiens to radiance ratio.
Shape and subsequent position of the R physiognomy describes the trent of the whole area or of a part of it.
On the sensitivity of functions "physiognomies" of T and R in respect with the para meters maximum radiance at the fumaroles mouth N_m and thermal conductivity power P we can write:

1 - Sensitivity of $ph(N)=\frac{dA}{dN}$ (physiognomy of N) in respect with N_m

$$\frac{\frac{d\,ph(N)}{dN_m}}{ph(N)} = -\frac{2}{H} = -\frac{4\pi K L}{a}$$

2 - Sensitivity of $ph(N)=\frac{dA}{dN}$ in respect with P

$$\frac{\frac{d\,ph(N)}{dP}}{ph(N)} = -\frac{1}{H} - \frac{2}{H^2} = -\left(\frac{2\pi K L}{P} + \frac{8\pi^2 K^2 L^2}{P^2}\right)$$

3 - Sensitivity of $ph(R)=\frac{dA}{dR}$ (physiognomy of gradient/radiance) in respect with N_m:

$$\frac{\frac{d\,ph(R)}{dN_m}}{ph(R)} = \frac{8\pi^2 x^6\left(N_m - H\ln\frac{x}{x_0}\right)^3\left[H^2\ln\frac{x}{x_0}\left(H^2 - HN_m\right)\right] + H2\pi x^2\left(N_m - H\ln\frac{x}{x_0}\right)^2}{\left[H^2\ln\frac{x}{x_0} + \left(H^2 - HN_m\right)\right]}$$

4 - Sensitivity of $ph(R)=\frac{dA}{dR}$ in respect with P

$$\frac{\frac{d\,ph(R)}{dP}}{ph(R)} = \frac{2\ln\frac{x}{x_0}\left[H^2\ln\frac{x}{x_0} + H^2 - HN_m\right] - \left(N_m - H\ln\frac{x}{x_0}\right)\left(2H\ln\frac{x}{x_0} + 2H - N_m\right)}{\left[H^2\ln\frac{x}{x_0} + \left(H^2 - HN_m\right)\right]\left(N_m - H\ln\frac{x}{x_0}\right)}$$

We can observe in both cases of dependence on N_m or on P, the higer sensitivity of the function ph(R), physiognomy of ratio between gradient and radiance.

3. CONCLUSIONS

The thermal infrared data gathered over volcanic areas in subsequent surveys can be of great help in monitoring and distinguishing the trend of the whole zone or of part of it. The repetition rate of the thermal measurement would range from one to some weeks depending on the type of volcanic phenomenology. A seasonal repe tition would anyway be a very powerful control expecially for monitoring the very small variations by means of the $\frac{G}{N}$ hystogramm. This last function shows the advanta ge of having a noticeable sensitivity in respect with the radiance of the hot spots and of thermal conducting power, enabling the volcanologist to detect spatial or tem poral variations even at very low level and close to the one of background.

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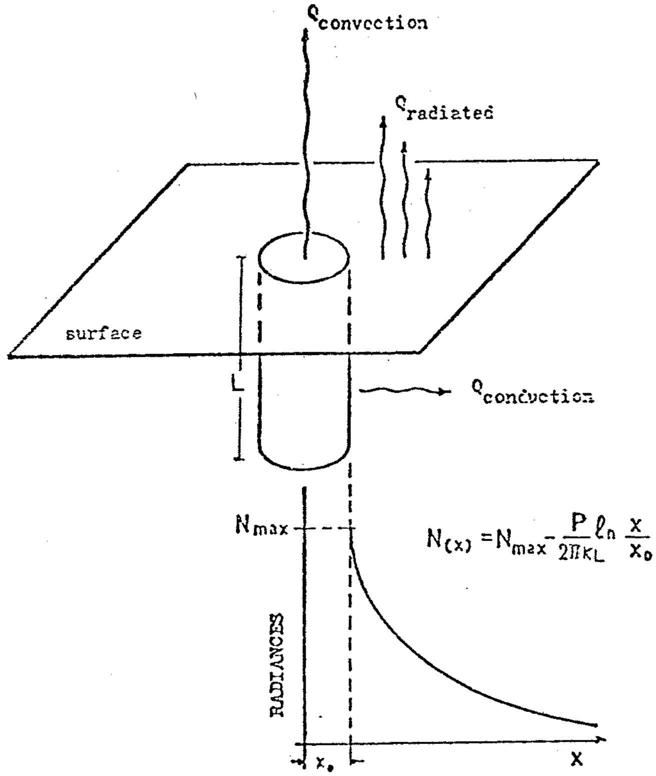

Fig. 1 - Simple geothermal model showing a vertical heating cylinder surrounded by an uniform medium, and the radiance distribution versus radius, the thermal conducting power P being constant.

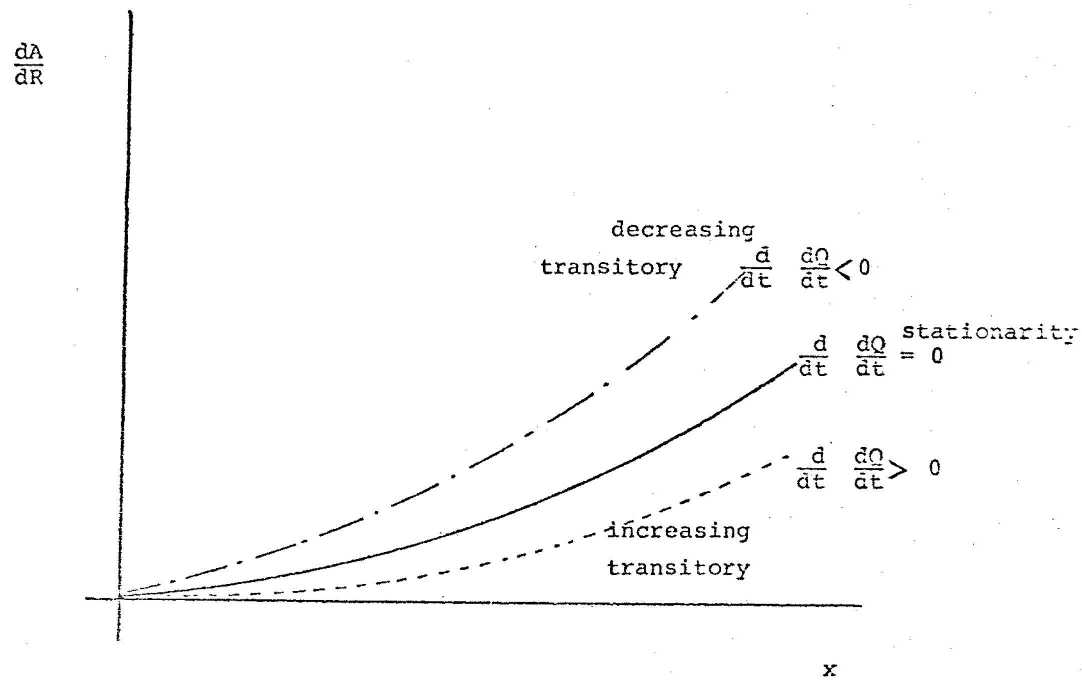

Fig.2 - The curve of the function $\frac{dA}{dR}$ divides the plane into two zones, the one characterized by increasing, the other by the decreasing thermal transitory.

The zones covered by the various configurations $\frac{d}{dt}\frac{dQ}{dt} < 0$ or $\frac{d}{dt}\frac{dQ}{dt} > 0$ are also involved by the situations of being $\frac{d}{dx}\frac{dQ}{dt} < 0$ and $\frac{d}{dx}\frac{dQ}{dt} > 0$ respectively.

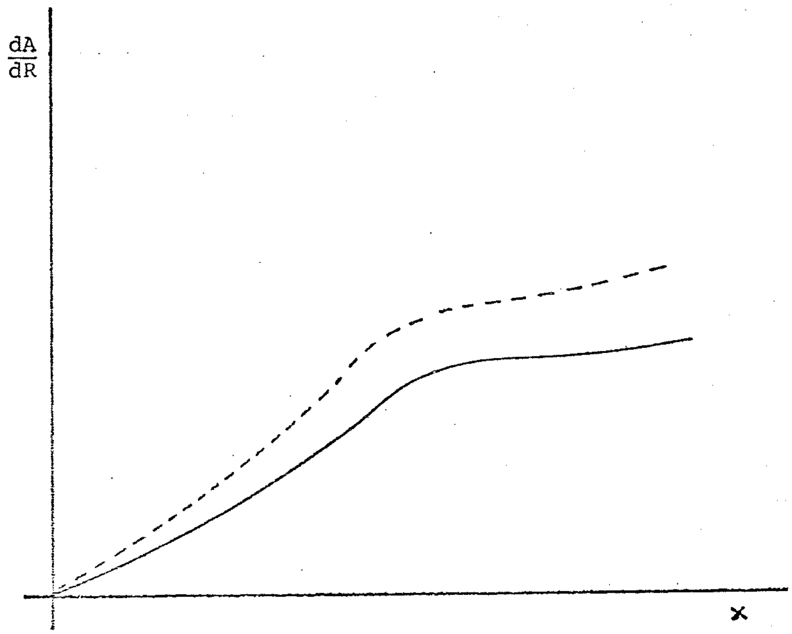

Fig. 3 - Case of thermal decreasing transitory involving the whole area (dotted line) compared with the situation of stationarity which shows the presence of interaction.

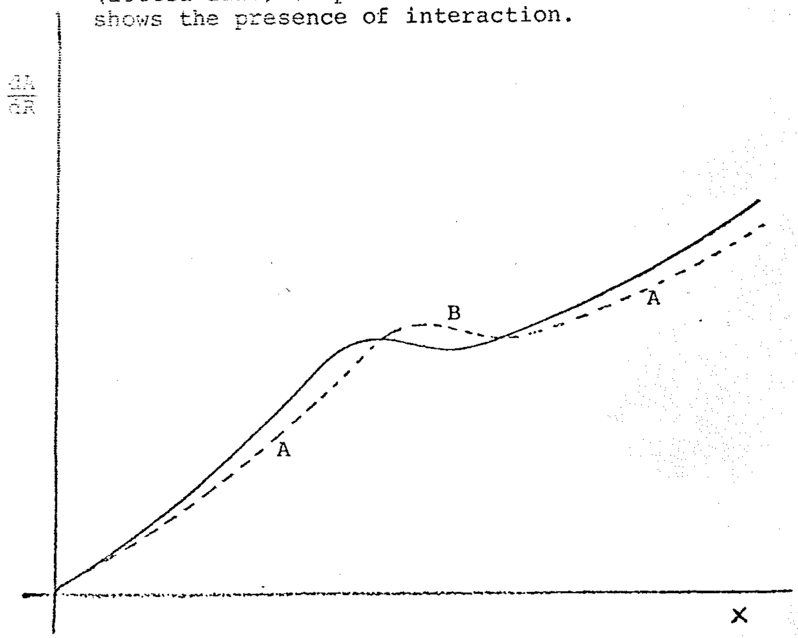

Fig.4 - Case of transitory involving just a part of the investigated area. The level interested by interaction shows a "shift" while two zones with increasing A and decreasing transitory B appear simultaneously.