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A Galerkin–Finite Element Analysis of the Hydrothermal System at Wairakei, New Zealand

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A single-phase simulation model was applied to the hot-water hydrothermal field at Wairakei, New Zealand. A two-dimensional areal analysis was made of the production aquifer under steady state and transient flow conditions, allowing vertical flow of heat and fluid through an overlying confining bed. Calculated temperature and pressure patterns correlate well with observed patterns until approximately 1963, when increasing quantities of steam in the production aquifer invalidated the assumption of single-phase flow. For further simulation of the Wairakei reservoir the numerical model will need to be extended to incorporate phase change and three-dimensional flow. Preliminary results, however, indicate that the response of hot-water hydrothermal systems to exploitation can be simulated by using a mathematical reservoir model based on a Galerkin–finite element approach.

INTRODUCTION

Current interest in geothermal energy has stimulated increased research in the area of heat transport in porous media. Many of these investigations are concerned with understanding the physics and chemistry of both vapor-dominated and hot-water geothermal systems in order to aid exploration and help predict exploitation effects. In this study we consider only hot-water geothermal systems and present a mathematical analysis designed to compute the spatial (horizontal cross section only) and temporal distribution of pressure and temperature in saturated porous media.

The partial differential equations which describe the flow of liquid and the transport of heat in porous media are presented. Assuming the problem to be two-dimensional, we then use Galerkin's criterion to generate approximate integral equations. These equations are solved temporally by using finite difference techniques and spatially by using the method of finite elements.

The applicability of this approach to solving field problems is demonstrated by simulating over a 10-year period (1953–1962) the response to development of a hot-water hydrothermal (geothermal) system located at Wairakei, New Zealand. The Wairakei field was the first hot-water hydrothermal system to be utilized for the generation of electricity. Exploitation has caused decreased pressures and the formation of large quantities of steam in the production aquifer. The field, however, remained a hot-water system for approximately 10 years, and there are sufficient data available to formulate a numerical model.

Previous Investigations

Investigations of the simultaneous flow of fluid and heat in porous media may be classed into three groups: (1) those made in hydrological research, (2) those made in research related to the petroleum industry, and (3) those made in research associated with geothermal studies. Early work in the first group was conducted by *Philip and DeVries* [1957] and *DeVries* [1958], who developed the differential equations for heat and moisture transfer in porous media and examined the conditions of steady state heat conduction. More recently, *Stallman* [1963] and *Bredehoeft and Papadopulos* [1965] examined heat flow in porous media as a means of calculating rates of groundwater movement.

In the petroleum industry, most of the work concerned with heat and fluid flow in porous media is associated with the secondary recovery of oil by hot fluid injection and in situ combustion. Although these processes were used in the field as early as the 1930's, theoretical studies were not made until the 1950's [e.g., *Lauwerier*, 1955; *Marx and Langenheim*, 1959]. Most of the early theoretical studies considered heat flow by conduction only and ignored liquid flow. More recent studies consider both conduction and convection. Typical equations used in this work are given by *Shutler* [1970] and *Chappelear and Volek* [1968].

Most fluid flow research relating to geothermal studies involves two-dimensional models of a vertical cross section. In most cases the authors consider such problems as the source of the hydrothermal liquid and its mechanism of transport. One early modeling effort by *Einarsson* [1942] involved the use of a pipe model to study the hot springs of West Iceland. This work was extended and further developed by *Bodvarsson* [1948, 1949, 1950, 1954, 1961]. Other authors who have used pipe models include *White* [1957, 1961], *Elder* [1966], and *Donaldson* [1968*a*, 1968*b*, 1970]. Another popular vertical model consists of a saturated homogeneous porous medium heated from below. Examples of this type of model are given by *Wooding* [1957, 1958, 1959, 1960*a*, 1960*b*, 1962, 1963].

Donaldson [1962], *Elder* [1957, 1966], *McNabb* [1965], *Aziz et al.* [1968], and *Cady* [1969].

Although the above models are useful in understanding hydrothermal systems, in general they cannot be used to predict the response of a hydrothermal field to the stresses of exploitation. An early attempt to do this was made by *Whiting and Ramey* [1969], who used concepts generally applied in reservoir engineering to model the Wairakei hydrothermal field as a lumped parameter system. Another lumped parameter approach was made by *Marshall* [1966, 1970] using a vertical flow model with allowance for lateral inflow. These types of models are useful in examining the general trends of a system; however, they do not account for the spatial dependence of the solution. What is needed is a deterministic model which treats the hydrothermal field as a distributed parameter system.

Such a model is presented in this paper and consists of a digital computer program which solves the liquid flow and heat transport equations, given appropriate boundary and initial conditions. The model is currently constrained by economic considerations to two dimensions in space and one in time. The question arises as to which two spatial dimensions to consider. The vertical cross section would provide the more interesting scientific problem, since this model would allow free convection to be simulated. However, to consider the practical problem of simulating the response of the hydrothermal system to development (including well location and withdrawal distribution), it is necessary that we consider an areal model similar to those used for simulation in groundwater hydrology [e.g., *Pinder and Bredehoeft,* 1968; *Bredehoeft and Pinder,* 1973].

DESCRIPTION OF NUMERICAL MODEL

A detailed development of the governing equations is given by *Mercer et al.* [1975]. Only the major assumptions and final equations are presented here.

Assumptions

Although there are many assumptions involved in this development, only the major ones are listed below:

1. Density is assumed to be related to temperature and pressure by a first-order Taylor series. This assumption is only valid when there are small density variations about some initial density distribution ρ_0. Since temperatures, and hence densities, vary only slightly throughout the early period of exploitation at the Wairakei field, this assumption appears reasonable. The problem is to determine accurate initial densities based on initial temperatures and pressures. An examination of the variation of water density with temperature and pressure [*Meyer et al.*, 1968, p. 42] shows that density varies with pressure (1.0×10^5 to 50.0×10^5 N/m^2 at various temperatures) only about 3.0%, whereas density varies with temperature (0.0° to 250.0°C at various pressures) approximately 20.0%. Because of the larger temperature dependence, we decided to use the following empirical formula developed by I. G. Donaldson (written communication, 1972) which relates initial density to temperature alone:

$$\rho_0 = 1.00606 \times 10^3 - 2.46020 \times 10^{-1} T_0 - 2.31633 \times 10^{-3} T_0^2 \quad (1)$$

where T_0 is the initial temperature in degrees Celsius and ρ_0 has the dimensions kg/m^3. This equation is valid for liquid saturation temperatures between 100° and 280°C.

2. Although viscosity is both temperature- and pressure-dependent, it varies with temperature to a much greater extent than it does with pressure [*Meyer et al.*, 1968, p. 73]. We therefore assume that viscosity is related to temperature by the empirical equation (A. McNabb, written communication, 1967)

$$\frac{1}{\mu} = 5.38 \times 10^3 + 3.8 \times 10^3 A - 2.6 \times 10^2 A^3 \quad (2)$$

where $A = (T - 150)/100$, T is in degrees Celsius, and μ is in kg/m s. This equation is valid in the liquid saturation temperature range of 0°–300°C.

3. Heat transport is assumed to occur in both the liquid phase and the solid matrix. Further, it is assumed that thermal equilibrium between the liquid and solid matrix is achieved instantaneously.

4. Conductive heat flow in the solid phase is given by

$$h_i^s = -(1 + \phi) K_{ij}^s \, \partial T / \partial x_j \quad (3)$$

where K_{ij}^s is the thermal diffusion tensor of the solid phase and ϕ is the porosity.

5. Heat transfer in the liquid phase is somewhat more complicated, since in addition to molecular diffusion, heat transfer by mechanical dispersion may be important. The heat flux associated with these mechanisms is given by

$$h_i = -\phi K_{ij} \, \partial T / \partial x_j \quad (4)$$

where

$$K_{ij} = K_{ij}^m + K^d \psi_{ij} \quad (5)$$

K_{ij} is the hydrodynamic thermal dispersion tensor, K_{ij}^m is the mechanical thermal dispersion tensor, K^d is the molecular thermal diffusion coefficient, and ψ_{ij} is the porous medium 'tortuosity' tensor.

The hydrodynamic thermal dispersion tensor is analogous to the hydrodynamic dispersion tensor used in mass transport work and is subject to a similar analysis. Following a development analogous to that used by *Reddell and Sunada* [1970, p. 10] for the hydrodynamic dispersion tensor, we obtain the nine components of the hydrodynamic thermal dispersion tensor. The main diagonal terms have the form

$$K_{xx} = K_l \frac{v_x v_x}{v^2} + K_t \frac{v_y v_y}{v^2} + K_t \frac{v_z v_z}{v^2} + K^d \psi \quad (6)$$

where K_l and K_t are the longitudinal and transverse mechanical thermal dispersion coefficients and ψ is the tortuosity factor. The off-diagonal terms are of the form

$$K_{xy} = K_{yx} = (K_l - K_t) \frac{v_x v_y}{v^2} \quad (7)$$

6. Heat capacity c_v, vertical compressibility α, liquid compressibility β, liquid thermal volume expansion λ, and the solid density ρ_s are treated as constants. Although the above parameters are functions of temperature and pressure, their variation is relatively small. Consequently, treating these as constants does not significantly reduce the accuracy of the model.

7. It is assumed that flow within a confined aquifer can be considered essentially horizontal, so that only the two horizontal dimensions need be considered.

8. Finally, we assume a chemically inert single-phase (hot-water) system.

Equations

On the basis of these assumptions, the liquid flow equation and heat transport equation can be written

$$\frac{\partial}{\partial x_i} b\left(\frac{\rho k_{ii}}{\mu}\frac{\partial p}{\partial x_i}\right) = b(\rho\alpha + \phi\rho_0\beta)\frac{\partial p}{\partial t} - b\phi\rho_0\lambda\frac{\partial T}{\partial t} - rb - \frac{\rho k_{zz}}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)\bigg|_{z_2} + \frac{\rho k_{zz}}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)\bigg|_{z_1} \tag{8}$$

and

$$b[\gamma c_v + (1-\phi)\rho_s c_{vs}]\frac{\partial T}{\partial t} + b\gamma c_v v_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} b[\phi K_{ij} + (1-\phi)K_{ij}']\frac{\partial T}{\partial x_j} + b\gamma Q + b(1-\phi)\rho_s Q_s + [\phi K_{zz} + (1-\phi)K_{zz}']\frac{\partial T}{\partial z}\bigg|_{z_2} - [\phi K_{zz} + (1-\phi)K_{zz}']\frac{\partial T}{\partial z}\bigg|_{z_1} + \gamma c_v v_z(T - T')\big|_{z_2} - \gamma c_v v_z(T - T')\big|_{z_1} \tag{9}$$

where the equation parameters have been vertically averaged between the top of the aquifer z_2 and the bottom z_1 and where $b = b(x_i)$ is the saturated thickness of the aquifer. The vertical bars indicate terms which are evaluated at either the top or the base of the aquifer. In obtaining these terms it was assumed that the mass and heat flow entering the top and bottom of the aquifer are vertical and colinear with the z axis. These terms depend on observed field conditions and will be evaluated later. The sink term r in (8) represents a discharging well or wells and is defined by

$$rb = \rho q = -\rho \sum_{k=1}^{n} \hat{q}(x_k, y_k)\delta(x - x_k)(y - y_k) \tag{10}$$

where $\hat{q}$ is the discharge from the aquifer, δ is the Dirac delta function, and n is the number of sinks. The source terms Q and Q_s in the heat transport equation also represent point sources and are assumed to be zero.

The system of (8) and (9) cannot in general be solved analytically, and a numerical approach is required. The numerical scheme chosen for the spatial solution is a combination of the finite element concept and the Galerkin method of weighted residuals. Finite difference techniques are used to approximate the time derivatives. Details of this approach are given by *Mercer et al.* [1975]. General references on the application of the Galerkin-finite element method to field equations include *Zienkiewicz and Parekh* [1970] and *Pinder and Frind* [1972].

WAIRAKEI HYDROTHERMAL SYSTEM

Location and History of Development

The Wairakei field is located 8 km north of Lake Taupo and is situated on the west bank of the Waikato River (see Figure 1). At this location the river acts as a groundwater discharge area. The Wairakei region, generally considered to occupy a surface area of approximately 15 km² [*Grindley*, 1965, p. 85], extends westward from the river approximately 5 km over relatively flat valleys floored with Taupo pumice alluvium. It is bordered on the west by hills of Wairakei breccia that rise 90–150 m above the valleys and serve as a groundwater recharge area.

Centered in an active volcanic belt, Wairakei is one of many geothermal regions located between the Tongariro and White Island volcanoes. This volcanic belt appears to be a surface manifestation of a landward extension of the Pacific trench system found north of New Zealand. This hypothesis is supported by gravity, magnetic, and seismic studies which indicate that this volcanic belt is a structural depression approximately 5 km deep filled with broken block structure and penetrated by rhyolitic complexes [*Modriniak and Studt*, 1959].

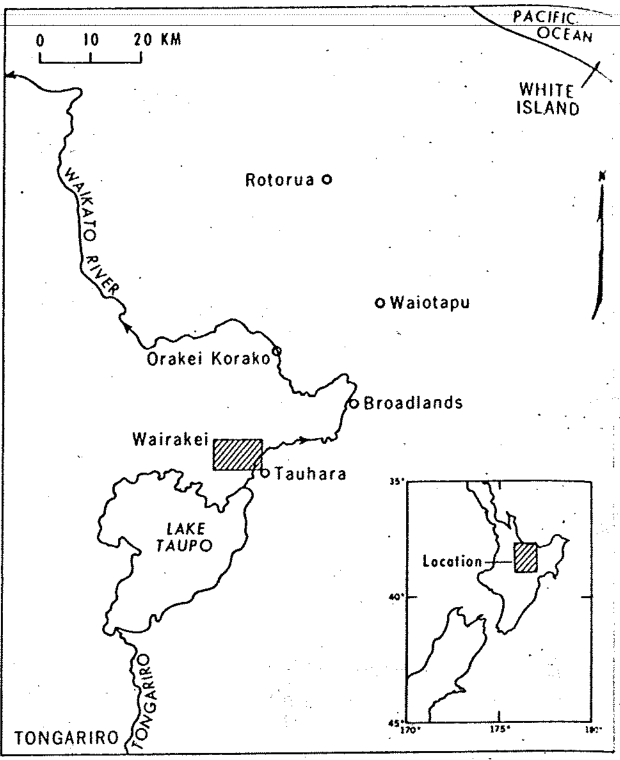

Fig. 1. Index map showing location of Wairakei, New Zealand; inset is of North Island.

Power generation began at Wairakei in 1958, and by 1968 the power stations at Wairakei were providing 192 MW, or approximately 18% of the total electrical requirements of New Zealand's North Island. Wairakei is considered to have been a hot-water system prior to exploitation [*White et al.*, 1971, p. 76] and to have remained so until approximately 1963. Utilizing steam (which flashes in the wells) to drive turbines, Wairakei became the first hot-water system to be developed for the purpose of generating electricity. The original hot-water nature of the field and the documentation of its response to development make Wairakei a logical choice for demonstrating the application of the simulation model. Once tested, this model can be used in research and development of other hot-water hydrothermal fields, such as those found in Iceland, Mexico, Japan, USSR, El Salvador, the Philippines, and the United States.

Geohydrology

The Wairakei hydrothermal field is underlain by a nearly horizontal Quaternary acidic volcanic rock sequence consisting of the following formations (in ascending order) [*Grindley*, 1965, p. 11]: Holocene pumice cover, Wairakei breccia, Huka Falls formation, Haparangi rhyolite, Waiora formation, Waiora Valley andesite, Wairakei ignimbrites, and the Ohakuri group. There are at least two aquifers in the above sequence, the Wairakei breccia and the Waiora formation. Although a deeper third aquifer may also exist in the Ohakuri group, it has been found in only one well, and its lateral extent is unknown.

The bulk of steam and hot water discharged by Wairakei wells comes from the Waiora aquifer. This unit consists of pumice breccias and vitric tuffs, ranging in thickness from

TABLE 1. Properties of the Waiora Aquifer and Huka Falls Aquitard

Property	Value	Reference
Aquifer permeability (horizontal)	$k_{xx} = k_{yy} = 1.0 \times 10^{-13}$ m^2	*Elder* [1966, p. 12]
Aquifer porosity	$\phi = 0.20$	McNabb (written communication, 1967)
Vertical compressibility of aquifer	$\alpha = 2.90 \times 10^{-10}$ m^2/N	*Jacob* [1950, p. 334]*
Heat capacity of solid phase	$c_{vs} = 0.22$ kcal/kg °C	Donaldson (oral communication, 1972)
Density of solid phase	$\rho_s = 2187$ kg/m^3	*Banwell* [1955, p. 46]*
Thermal diffusion coefficient of solid phase	$K_s = 5.2 \times 10^{-4}$ kcal/m s °C	McNabb (written communication, 1967)
Confining bed permeability (vertical)	$k' = 1.0 \times 10^{-14}$ m^2	*Wooding* [1963, p. 530]*
Confining bed porosity	$\phi' = 0.25$	McNabb (written communication, 1967)
Confining bed specific storage	$S_s = 1.0 \times 10^{-3}$/m	*Domenico and Mifflin* [1965, p. 566]*

*Estimated from given reference.

about 600 m in the western part of the production zone to more than 800 m in the eastern part. The permeability of the Waiora aquifer varies spatially depending on the amount of brecciation and is highest at fault zones and near a lower unconformable contact. Although productive wells at Wairakei are nearly always located in zones of locally high permeability associated with major faults, the reservoir as a whole responds as a porous medium defined in a continuum sense and is treated as such in this model. This assumption leads to a value of permeability which probably lies between that of the intergranular matrix and the fracture permeability. While in general the Waiora aquifer is assigned an average permeability of 1.0×10^{-13} m^2, a value of 1.0×10^{-14} m^2 was assigned to the eastern part of the production area because of fewer fault zones and generally lower calculated permeabilities. Other properties of the Waiora aquifer are given in Table 1, and associated fluid properties are given in Table 2.

The Waiora aquifer is overlain by lacustrine shales of the Huka Falls formation which range in thickness from 30 to 300 m and act as confining beds. In some locations the shales are interbedded with breccia. These confining beds are also treated as a porous medium defined in a continuum sense. Properties of the Huka Falls formation pertinent to this analysis are shown in Table 1.

The Wairakei ignimbrite, a welded rock of low primary permeability, underlies the Waiora aquifer. The base of the Waiora aquifer is not well defined because of secondary permeability afforded by fracture zones and the irregular surface of the unconformity between the ignimbrites and the aquifer. The problem of determining the base of the aquifer is further complicated in the south and southwest, where rhyolite overlies the ignimbrite and progressively cuts out the Waiora aquifer from below. The rhyolite also displays secondary permeability in its upper layers [*Studt*, 1958, p. 708].

The stratified volcanic sequence at Wairakei is complexly faulted and draped over a basement horst. Most of the major fault zones are well-defined through drilling and strike in a southwest-northeast direction [*Bolton*, 1970]. It is generally thought that such fault zones in the underlying ignimbrites are the source of the hot water in the Waiora aquifer.

Heat and Mass Flows and Temperatures at Wairakei

The natural heat flow at Wairakei has been measured several times beginning in 1951. *Fisher* [1964] gives a summary of results obtained by various authors over the period from 1951–1959, and his tabulations are shown in Table 3. As may be seen, the values range from 82,000 kcal/s [*Ellis and Wilson*, 1955] obtained by using chemical methods to 163,000 kcal/s [*Thompson et al.*, 1961] measured by using physical methods. *Elder* [1966, p. 68, 1965, p. 224] interpreting data from *Ellis and Wilson* [1955] and *Thompson et al.* [1961] estimates that the preexploitation heat flow at Wairakei was 252,000 kcal/s.

One common conclusion of all the heat flow surveys is that the majority of the natural heat flow is due to convection and is thus associated with a natural mass discharge. Although the total mass discharge was not measured, it can be estimated

TABLE 2. Properties of the Liquid

Property	Value	Reference
Compressibility coefficient of the liquid phase	$\beta = 4.78 \times 10^{-10}$ m^2/N	*Jacob* [1950, p. 334]*
Coefficient of thermal volume expansion for the liquid phase	$\lambda = 5.0 \times 10^{-4}$/°C	*Harlow and Pracht* [1972, p. 7044]
Heat capacity of the liquid phase	$c_v = 1.0$ kcal/kg °C	*Weber et al.* [1959, p. 206]
Thermal diffusion coefficient (thermal conductivity of the liquid phase)	$K^d = 1.553 \times 10^{-4}$ kcal/s m °C	*Elder* [1966, p. 26]

*Estimated from given reference.

TABLE 3. Natural Heat Flow From Wairakei [After *Fisher*, 1963]

Reference	Measurement Period	Reference Temperature, °C	Conduction Through the Soil, kcal/s	Convection, kcal/s	Total, kcal/s
Banwell [1955]	1951-1952	12	5,100	127,700	132,800
Healy [1956]	1951-1952	12	5,000	125,000	130,000
Gregg [1958]	1951-1952	0	5,100	144,900	150,000
Benseman [1959]	1951-1952(?)	12(?)	14,300(?)	128,700(?)	143,000
Ellis and Wilson [1955]	1954	12	···	···	82,000
Thompson et al. [1961]	1958-1959	15	9,000	154,000	163,000
Fisher [1964]	1951-1952	12	2,800	99,700	102,500
Fisher [1964]	1958	12	2,800	98,200	101,000

from the measured heat flow. *Fisher* [1964, p. 183], for example, using a mean enthalpy of 245 kcal/kg and omitting the heat flow due to conduction, obtains a mass discharge of 440 kg/s for the 1951-1952 period. On the other hand, if 144,900 kcal/s (excludes conductive heat flow) [*Gregg*, 1958] is used and the mean enthalpy is estimated to be 200 kcal/kg, a mass discharge of approximately 710 kg/s is obtained for the 1951-1952 period. *Elder* [1966, p. 68, 1965, p. 224] suggests that the mass discharge for Wairakei could have been as high as 900 kg/s (300 kg/s steam and 600 kg/s water).

Temperatures at Wairakei increase rapidly with depth down to the top of the Waiora aquifer, where the temperature in the hotter regions is about 200°C. Through the aquifer the temperature gradient is reduced, and in some locations temperatures reach approximately 250°C at 460-m depth. In the lower part of the aquifer the gradient is either very small or becomes negative (below 600 m).

WAIRAKEI MODEL

Consistent wih the above description of the Wairakei system, our conceptual model assumes the existence of a principal producing reservoir, the Waiora formation, overlain by the less permeable Huka Falls formation and underlain by the relatively impermeable Wairakei ignimbrites. We assume that flow in the Waiora formation is essentially horizontal and can be simulated using a two-dimensional areal model. We further assume that the Huka Falls formation not only transmits significant quantities of fluid vertically but also transmits heat by both conduction and convection. In contrast, the Wairakei ignimbrites are assumed to be impermeable relative to the Waiora formation, and heat is transmitted vertically through the ignimbrites by conduction alone. This latter assumption is in conflict with a generally held opinion that the hot water in the Waiora aquifer is principally due to convection through fault zones in the underlying ignimbrites. Tritium measurements, however, indicate a circulation time of some water in the Waiora aquifer of less than 50 years [*Hulston*, 1961]. This short residence time suggests a source for the water in the Waiora aquifer other than the underlying ignimbrites. It is likely that water is entering the Waiora aquifer from both above and below; however, present data on the ignimbrites are so limited that we feel treating the ignimbrites as impermeable is a reasonable approximation. Although this assumption is somewhat restrictive in the steady state model, it is less restrictive in the transient model since, once exploitation becomes significant, it is anticipated that flow in the aquifer will be dominated by inflow from above.

Boundary Conditions

Although the zone with the high density of wells is the main production area, the Wairakei field is considered to be more extensive (see Figure 2, which is an enlargement of the shaded area labeled Wairakei in Figure 1). It is approximately the area bounded by the Waikato River north of Huka Falls in the southeast, State Highway 1 in the northeast, and Poihipi Road in the west [*Bolton*, 1970]. Most of the wells located outside the main production area in Figure 2 are known as the 200 series wells and were used primarily to help define the boundaries of the Wairakei field. The boundaries used in this study are shown in Figure 3 and are similar to Bolton's, with the exception of the western boundary, which has been extended beyond Poihipi Road. This extension is necessary to account for the pressure decrease in wells located near Poihipi Road. It is interesting to note that these boundaries coincide approximately with those defined by ground subsidence, gravity surveys [*Hunt*, 1970], and electrical resistivity surveys (G. E. K. Thompson, written communication, 1972).

One set of boundary conditions for pressure and one for temperature are required to solve the flow equation and the heat transport equation over the region in Figure 3. The boundary conditions for pressure are considered to be of the Neumann type, in which the outward normal derivative of pressure on the boundary is zero (i.e., the boundary is considered impermeable). The boundary conditions for temperature are considered to be of the Dirichlet type, in which the temperature is a known function of the space variables around the perimeter of the system. Thus conductive heat flow may occur at the boundary, but convective heat flow may not, since the normal component of velocity is zero at the boundary as prescribed by the pressure boundary conditions.

These boundary conditions are consistent with the physical boundary conditions observed at the Wairakei field. As was pointed out in the geohydrology section, the southern boundary of the Wairoa aquifer is cut off by rhyolite, which may be considered impermeable. The northern and eastern boundaries are also considered impermeable, since the pressure response in wells near these boundaries appears to be independent of the production field [*Bolton*, 1970]. Finally, the western boundary is extended far enough from the production field that the transient solution is not significantly affected by it, and it too is treated as impermeable. Inasmuch as temperatures throughout the Wairakei field have changed only slightly with time [*Grindley*, 1965, Figure 29C], using estimated boundary temperatures as Dirichlet boundary conditions very closely represents the field situation.

Vertical Flow at the Top and Base of the Aquifer

To obtain (8) and (9), the three-dimensional flow equation and heat transport equation were integrated over the aquifer thickness. The resulting two-dimensional areal equations in-

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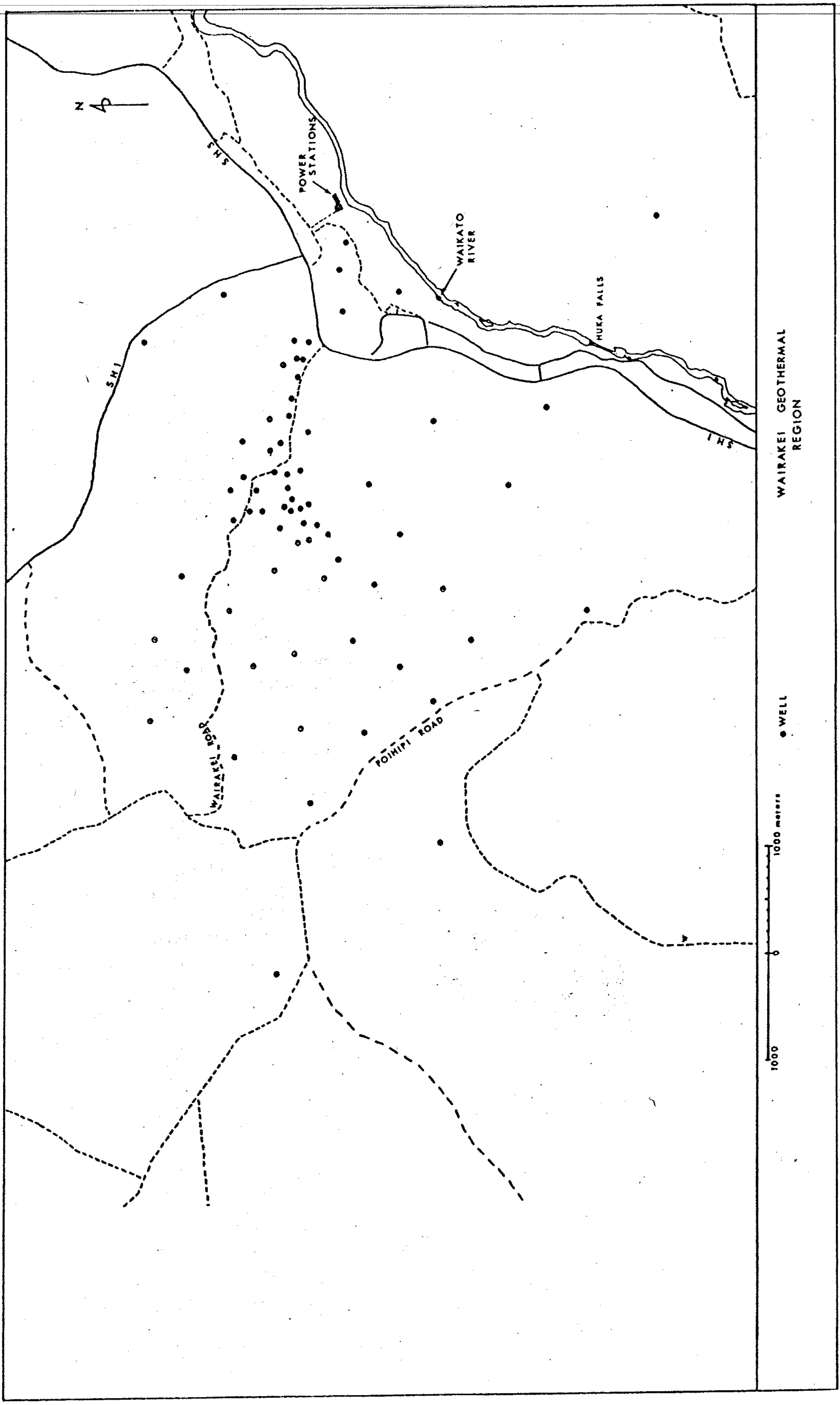

Fig. 2. Map of Wairakei geothermal region showing main production area and local features.

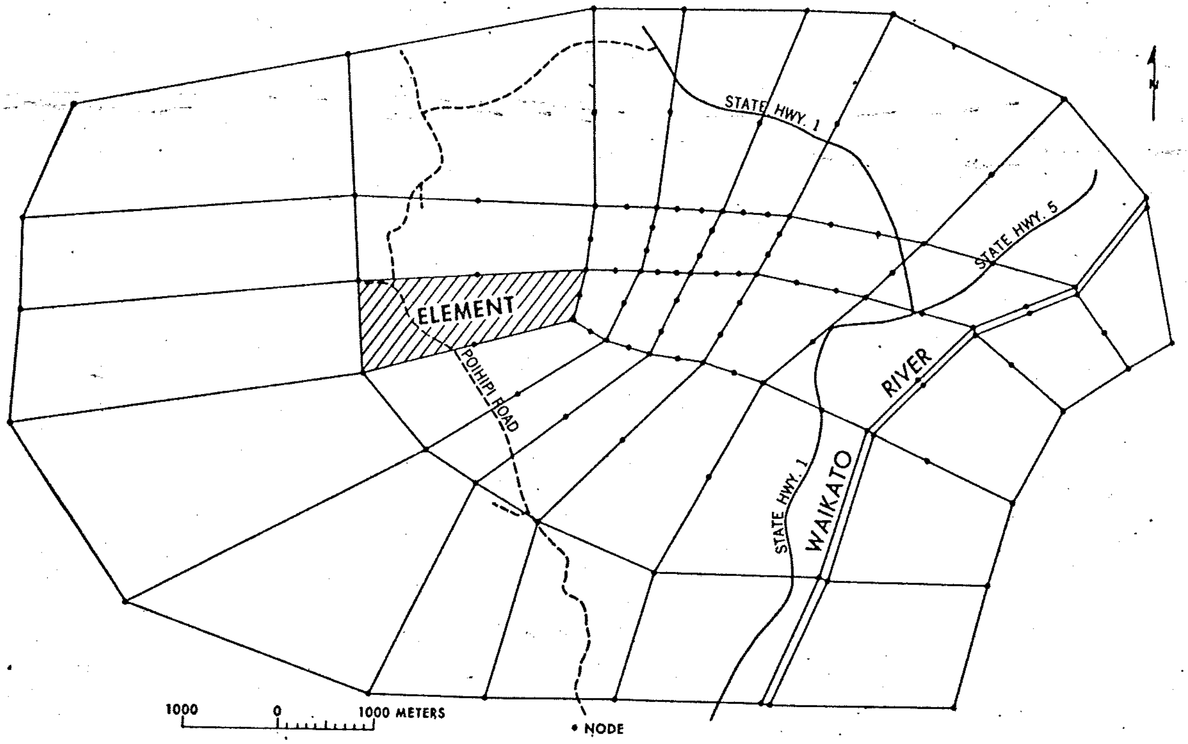

Fig. 3. Wairakei element configuration for Galerkin–finite element analysis.

clude terms which account for vertical mass and heat flows at the top and base of the aquifer (these are sometimes referred to as leakage terms). These terms need to be evaluated in order to model the Wairakei system.

The conceptual model of Wairakei consists of a layered system as shown in the idealized geologic cross section in Figure 4. Mass and heat are allowed to flow vertically through the confining beds (i.e., the Huka Falls formation). The direction of flow is determined by the potential and thermal gradients across the confining beds and may vary spatially and temporally. To determine the direction and magnitude of these flows, the pressure and temperature distributions at the top of the Huka Falls formation are required. To obtain the pressure distribution, the Wairakei breccia and the Holocene pumice are treated as a single aquifer ranging in thickness from 520 m in the west to 160 m in the east, where they are cut by the Waikato River. It is assumed that recharge (rainfall) is sufficient in this upper aquifer to maintain a water table near land surface. Head values in this upper aquifer can be estimated by using a topographic map of the Wairakei region, and pressures at the top of the Huka Falls formation can be calculated. The temperature distribution for the top of the Huka Falls formation was obtained from data of *Grindley* [1965, Table 1, p. 109] and is presented in Figure 5.

Using these assumptions, we can evaluate the vertical flow terms in (8) and (9) for the top of the aquifer. For the flow equation,

$$\frac{\rho k_{zz}}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)\bigg|_{z_2} = \rho q' \tag{11}$$

where q' represents vertical liquid flow through the Huka Falls formation. For the steady state case, q' becomes

$$q' = (k'/\mu b')(p' - p + \rho g b') \tag{12}$$

where k' is the permeability and b' is the thickness of the Huka Falls formation, p' is the pressure distribution above the Huka Falls formation, and p is the pressure in the Wairoa formation. It is assumed that transient flow in the Huka Falls formation is due to a stepwise change in pressure in the Waiora aquifer and for the transient case q' may be approximated by [*Bredehoeft and Pinder*, 1970]

$$\begin{aligned} q' = {} & (p_0 - p)\frac{K'}{\rho g b'(\pi K' t'/2b'^2 S_s)^{1/2}} \\ & \cdot\left\{1 + 2\sum_{n=1}^{\infty} \exp\left[-\frac{n^2}{(K' t'/2b'^2 S_s)}\right]\right\} \\ & + \frac{K'}{\rho g b'}(p' - p_0) + K' \end{aligned} \tag{13}$$

where $K' = k'\rho g/\mu$ is the hydraulic conductivity and $S_s = \rho g(\alpha + \phi'\lambda)$ is the specific storage of the Huka Falls formation and

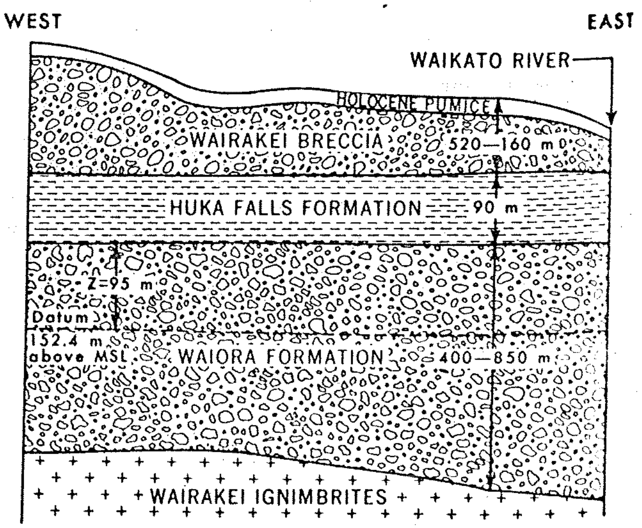

Fig. 4. Idealized geologic cross section of Wairakei, New Zealand.

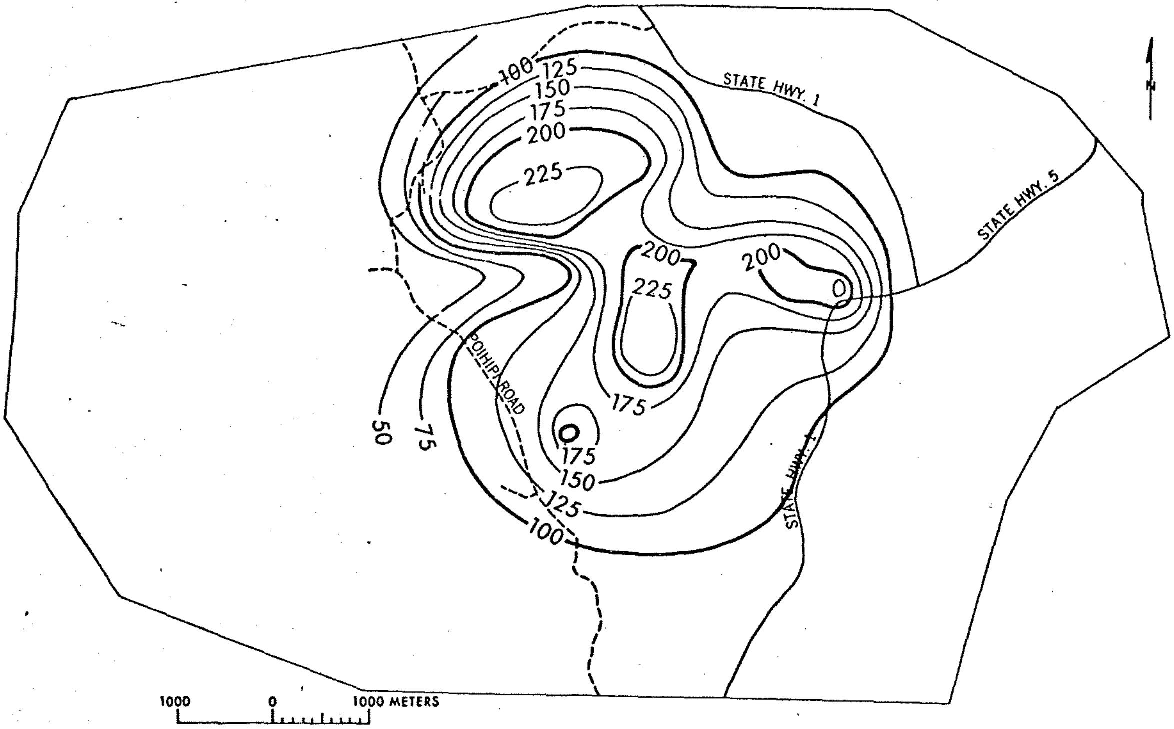

Fig. 5. Temperature distribution in degrees Celsius above the Huka Falls formation; contour interval is 25°C.

p_0 is the initial pressure in the Waiora aquifer. As was indicated in Table 1, the Huka Falls formation is considered to have an average vertical permeability of 1.0×10^{-14} m^2 except under the Waikato River, where it is estimated to be 5.0×10^{-14} m^2. Although this increase in permeability under the Waikato River was required to obtain a satisfactory pressure solution, the increase may be justified physically, since in the Wairakei area the Waikato River trends in the same direction as the regional faults and may be fracture-controlled.

For the heat transport equation (9) there are two vertical flow terms which must be evaluated at the top of the aquifer, one for conduction and one for convection. These terms are

$$[\phi K_{zz} + (1 - \phi) K_{zz}{}^{s}] \frac{\partial T}{\partial z}\bigg|_{z_2} = [\phi' K_{zz} + (1 - \phi') K_{zz}{}^{s}] \frac{T' - T}{b'} \tag{14}$$

$$\gamma c_v v_z (T - T')\big|_{z_2} = \rho q' c_v (T - T') \tag{15}$$

where K_l' and K_s' are the vertical components of the liquid thermal dispersion and solid thermal diffusion coefficient for the Huka Falls formation, T' is the temperature distribution above the Huka Falls formation, and T is the temperature distribution in the Waiora aquifer. As can be seen, convective heat flow depends on q' as defined earlier. For conductive heat flow a Fourier-type equation is used for both steady state and transient conditions. An approach similar to that used for pressure could have been used for calculating temperatures under transient conditions, but for the Wairakei system, temperature changes are small, and the simpler approximation is considered adequate.

Vertical flow at the base of the aquifer (through the Wairakei ignimbrites) is not well understood. As was stated earlier, it is assumed that the Wairakei ignimbrites are impermeable relative to the Waiora formation. Consequently, liquid flow and convective heat flow are assumed not to occur at the base of the aquifer. Since only the heat flow at the surface of the Wairakei area is known, a spatially distributed (conductive) heat function is used in the steady state model to reproduce the observed temperature distribution. Applying a trial and error approach, we adjust the distributed heat function until the calculated temperature distribution matches the observed temperature distribution and the calculated heat flow leaving the top of the model reproduces that measured at the Wairakei field. The vertical flow terms in (8) and (9) for the bottom of the Waiora aquifer are

$$\frac{\rho k_{zz}}{\mu_l} \left(\frac{\partial p}{\partial z} + \rho g \right) \bigg|_{z_1} = 0 \tag{16}$$

$$[\phi K_{zz} + (1 - \phi) K_{zz}{}^{s}] \frac{\partial T}{\partial z}\bigg|_{z_1} = -Q' \tag{17}$$

$$\gamma c_v v_z (T - T')\big|_{z_1} = 0 \tag{18}$$

where Q' is an areally distributed (conductive) heat source.

Initial Conditions

The Wairakei hydrothermal system is considered to have been at steady state prior to exploitation. The first step in modeling the Wairakei system is therefore the reproduction of the observed virgin or steady state conditions. These results will be used as the initial conditions for the transient model of exploitation. Inasmuch as wells drilled in the early 1950's had little discharge, temperature and pressure measurements made in these original wells are considered representative of steady state conditions. Figure 6, taken from *Studt* [1958, p. 712], is a 1955 potentiometric surface of the main production area. The early wells were shallow, and an upper datum of 152.4 m above sea level was chosen, since more data are available for this level. Nevertheless, the observed potentiometric surface is limited in extent, and few data are left on which to calibrate a simulation model.

Studt's head values were calculated from wellhead pressures

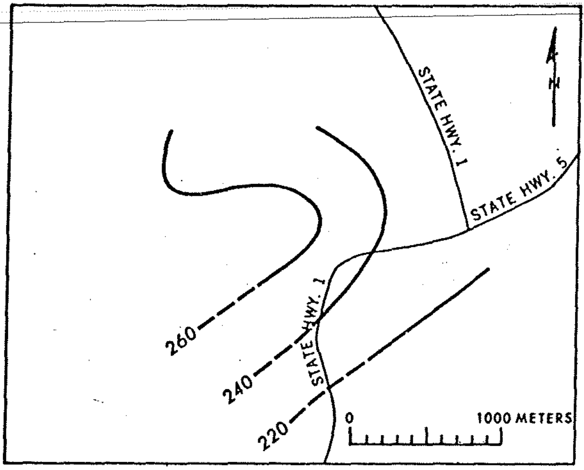

Fig. 6. Wairakei measured steady state potentiometric surface, 1955 [after *Studt*, 1958]. Datum is 152.4 m above sea level; contour interval is 20 m.

and downhole temperature measurements made at 30.48-m intervals. By assuming that each density value calculated from the recorded temperatures could be considered constant over an interval of 15.24 m above and below where the temperature was measured, he was able to estimate pressures from which he calculated equivalent or 'cold water' head values.

The observed steady state temperature distribution is shown in Figure 7. This surface is more extensive than the observed steady state potentiometric surface because it is based not only on measurements made in the early 1950's but also on recent temperature measurements. These recent values were utilized in preparing Figure 7 because temperatures have changed only slightly since exploitation began and the recent measurements still represent near steady state conditions. It should be pointed out that although temperatures vary with depth in the Waiora aquifer, vertically averaged temperature values must be used because the problem is treated as two-dimensional in the areal plane. These average temperatures are based on data collected from wells which penetrate the top and bottom of the Waiora aquifer [*Grindley*, 1965, pp. 123, 109].

The finite element configuration selected for modeling the Wairakei system is illustrated in Figure 3 and consists of 41 elements and 109 nodes. The flexibility of the irregular isoparametric element is demonstrated in the definition of the Waikato River, where increased vertical flow is permitted through the stream bed. The advantage of using mixed elements is apparent through an examination of elements in the main production area as contrasted with those located away from the main field. It is important to have higher-order elements and their associated higher-order basis functions in and around the main production area in order to approximate better the more pronounced changes which will occur there during exploitation. On the other hand, since very little change is anticipated farther from the main production area, linear elements should be adequate.

The steady state model consists of solving the equations of flow and heat transport, omitting the time-dependent terms. A solution is obtained by iterating between the temperature and pressure equations and using updated densities and viscosities. Although temperature and pressure are obtained for each node, corresponding head values can also be calculated, since the average densities are constant in the vertical direction. The simulated potentiometric surface with datum 152.4 m above sea level is shown in Figure 8, where the insert is the main production area and is shown in Figure 9. To facilitate comparing the observed and calculated surfaces, Figure 9 has the same border as the observed potentiometric surface in Figure 6. The calculated temperature distribution is shown in Figure 10 and is based on a dispersivity coefficient of 10 m in both the X and the Y direction.

Comparison of the calculated potentiometric surface in

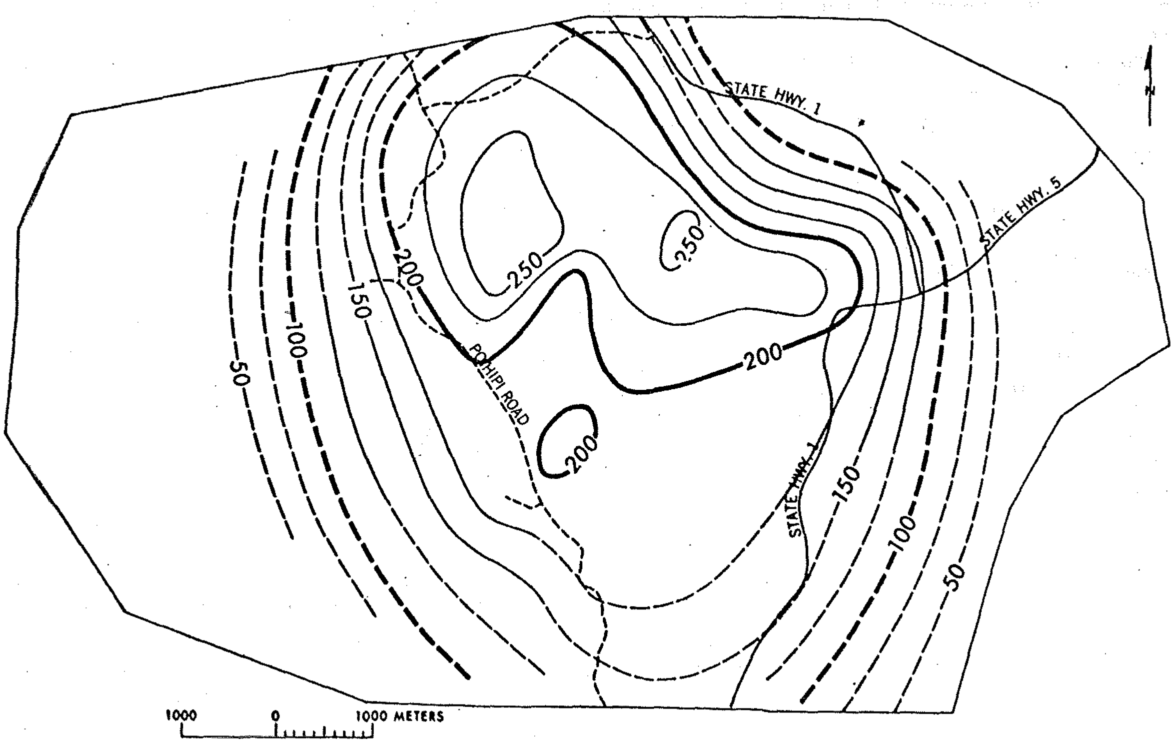

Fig. 7. Measured Wairakei steady state (vertically averaged) temperature distribution in degrees Celsius; contour interval is 25°C.

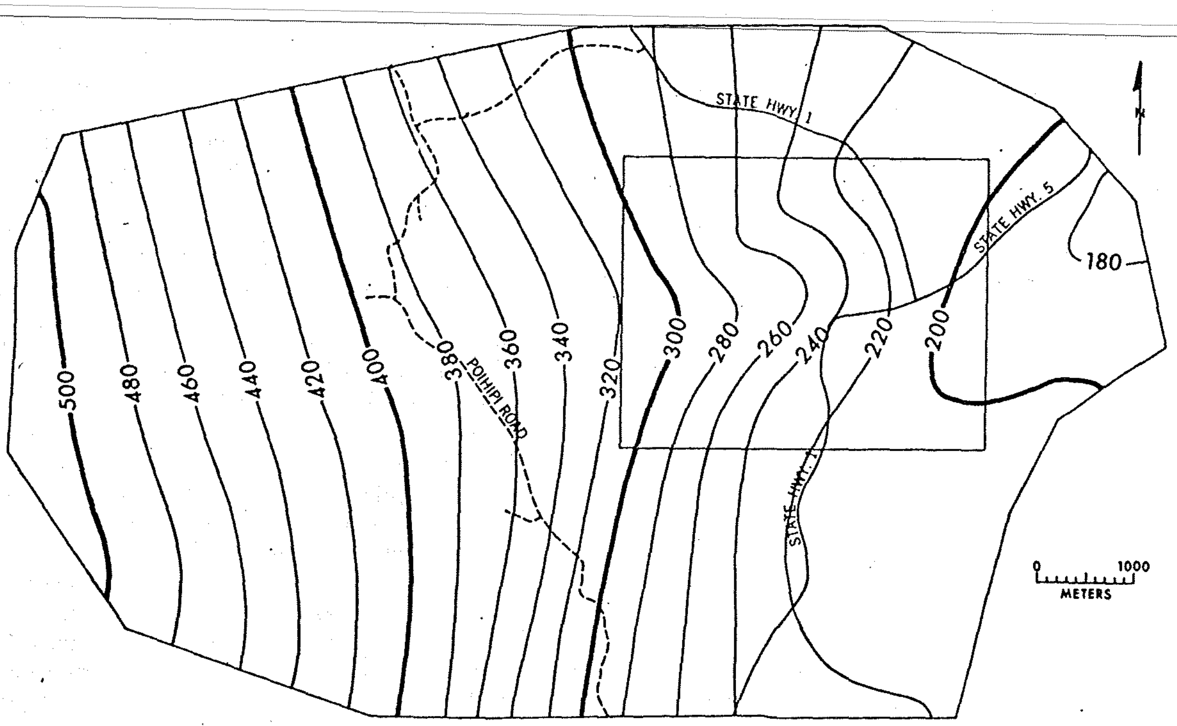

Fig. 8. Wairakei calculated steady state potentiometric surface. Datum is 152.4 m above sea level; contour interval is 20 m.

Figure 9 with the measured surface in Figure 6 reveals that both surfaces show a flow pattern which is basically from west to east. The observed potentiometric surface shows a distinct 'ridge' and 'valley' which is less pronounced in the calculated surface. This could possibly be the effect of 3 years of discharge from the field or the result of treating the Wairakei field as a porous medium and neglecting fault zones. The existence of a fault or an implied fault in the axes of both the valley and the ridge supports this latter hypothesis. Whatever the case, the surfaces still compare well, and the general trends are the same.

The computed temperature distribution in Figure 10 compares favorably with the observed temperature distribution in Figure 7. The most difficult task in obtaining a satisfactory temperature solution was adjusting the unknown heat source at the base of the aquifer. Another difficulty involved the selection of the element configuration. As was indicated earlier, the element configuration used in this problem was designed for the pressure solution, and consequently high nodal density and higher-order elements do not necessarily coincide with regions having large temperature gradients.

To test the accuracy of the model, mass and heat balances were made on the steady state results, and the 'mass out' compared to the 'mass in' of the model had a 0.023% error, while the heat difference had a 0.003% error. Since the boundaries of the model are impermeable, the balances were made on the vertical flow of fluid and heat entering and leaving the aquifer.

The heat flow calculated in the model was 214,700 kcal/s by convection and 18,380 kcal/s by conduction. As can be seen, convective heat flow is approximately an order of magnitude larger than conductive flow, which is consistent with what is observed at Wairakei [*Fisher*, 1964, p. 173]. The total calculated heat flow is 233,000 kcal/s (using a reference temperature of 0°C), which is higher than most of the observed values discussed earlier. Examination of Figures 2 and 3 suggests that the larger calculated value may be due to the larger area considered in the model. Further, using a constant heat capacity of 1.0 kcal/kg °C (Table 2) may have caused the calculated value to be high. The mass discharge calculated in the model amounted to 1,133 kg/s, which is higher than the estimated values. The estimated values, however, are generally based on a single mean enthalpy and consequently a single mean temperature, whereas the model considers the wide range of temperatures shown in Figure 10. If the assumed mean enthalpy is high, then the observed mass flux will be underestimated. Using an average enthalpy instead of an areally distributed one could account in part for the difference in mass flows. The larger area used in the model may also have contributed to the larger calculated mass flux.

A sensitivity analysis was performed on selected parameters in the steady state model. One of the parameters varied was the permeability of the Huka Falls confining bed. Reducing only the permeability of the confining bed produced a decrease in

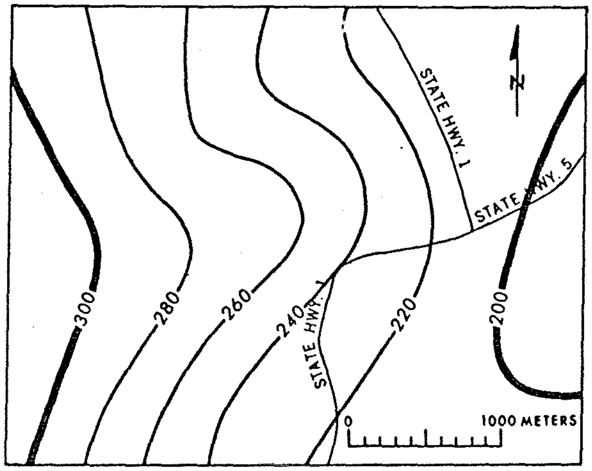

Fig. 9. Wairakei calculated steady state potentiometric surface of the production area. Datum is 152.4 m above sea level; contour interval is 20 m.

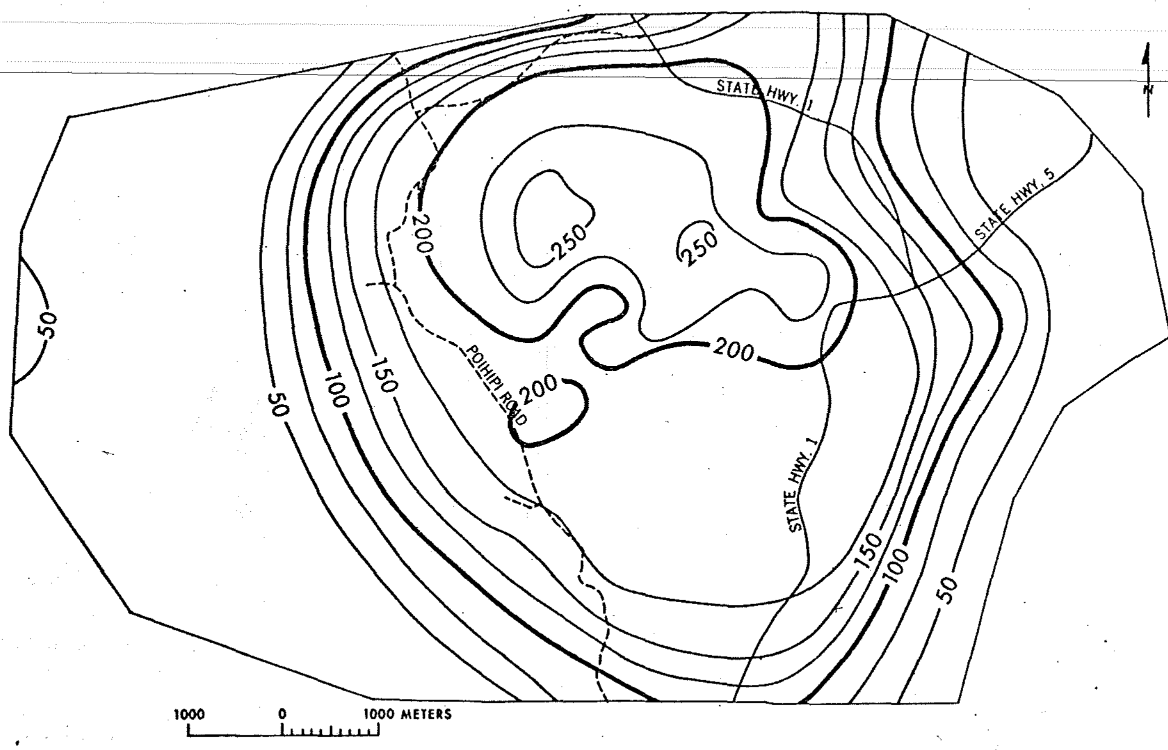

Fig. 10. Calculated Wairakei steady state temperature distribution in degrees Celsius; contour interval is 25°C.

mass discharge through the confining bed and an increase in the temperatures in the aquifer, while increasing permeability of the confining bed produced the opposite effect. It appears that for this particular application the confining bed permeability has a considerable effect on the temperature distribution.

The dispersivity coefficient was also varied in an attempt to examine the sensitivity of the temperature solution to this parameter. When this coefficient was increased from 10 to 100 m, the temperature changes were insignificant. When the dispersivity coefficient was increased to 1000 m, temperature changes were usually less than 10°C. Thus it appears that in contrast to point source problems where the dispersivity coefficient plays an important role, for this areal simulation the dispersivity influences the temperature distribution only slightly.

Transient Simulation

By using the steady state temperature and pressure solutions as initial conditions, well discharge rates were incorporated into the model and the transient effects of exploitation simulated. Discharge rates were averaged for each well over 30-day intervals. Since the locations of the nodes in the model do not correspond exactly with the locations of the wells in the field, the temporally averaged discharge rates were distributed to adjacent nodes using linear interpolation. For example, a well located in an element with 10 nodes would have its discharge distributed to those 10 nodes, the nearest node having the largest fraction of the discharge. The discharge for the entire field may be obtained by summing the discharge for each well and is presented in Figure 11. As may be seen, little discharge took place before power generation began in 1958. By estimating the enthalpy of the mass discharged, the approximate heat output may be calculated and is also presented in Figure 11.

Since the simulation is over an extended period of time, a time step of 1 month was utilized. The 1-month time step in conjunction with a backward difference scheme was found to provide a satisfactory solution.

Measured potentiometric surfaces of the main production area are shown in Figures 12 and 13 for 1958 and 1962, respectively. The 1958 surface [after *Studt,* 1958, p. 709] was constructed in the same fashion as Figure 6. The 1962 surface was contoured by using data from *Grindley* [1965, p. 124, Table 20]. Grindley gives downhole pressure values at sea level datum for several wells. These pressure values were converted to head values at a datum of 152.4 m above sea level by using

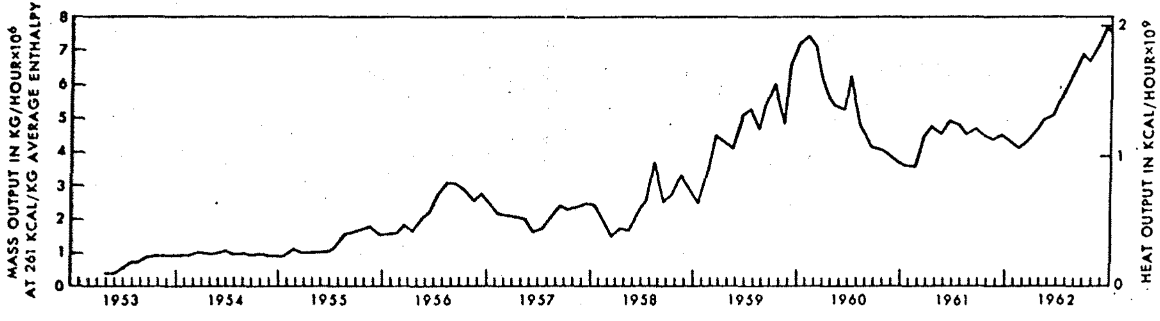

Fig. 11. Mass and heat discharged through Wairakei wells, 1953–1962; 200 series wells excluded [after *Grindley,* 1965].

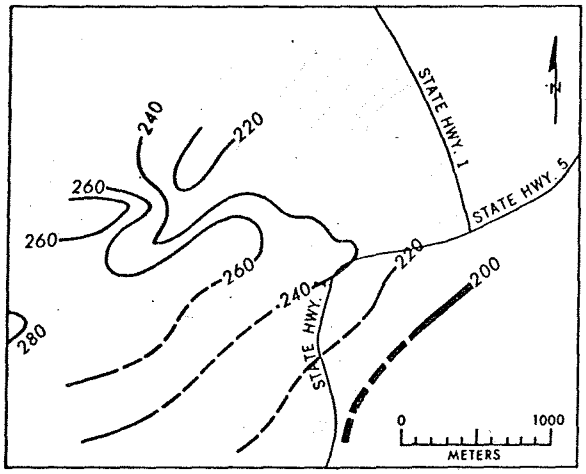

Fig. 12. Wairakei measured potentiometric surface, 1958 [after *Studt*, 1958]. Datum is 152.4 m above sea level; contour interval is 20 m.

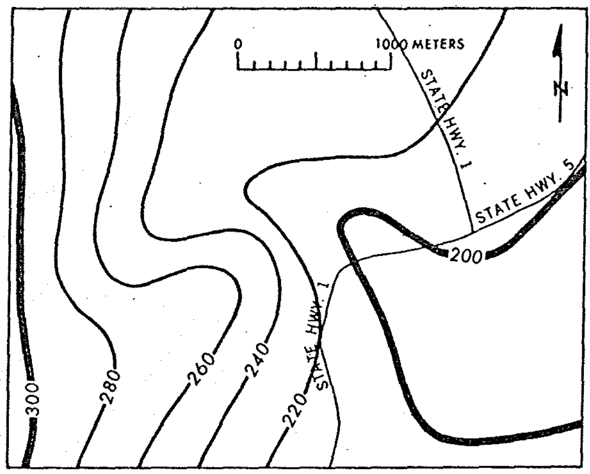

Fig. 14. Wairakei calculated 1958 potentiometric surface of the production area. Datum is 152.4 m above sea level; contour interval is 20 m.

density values computed from the average temperature data in Figure 7. As is indicated by the dashed lines, there is a considerable amount of uncertainty in the 1962 potentiometric surface. As exploitation continues, pressures decrease sufficiently to cause the formation of steam in the aquifer. This leads to inaccurate calculation of densities, and water levels in the wells can no longer be accurately measured. Thus potentiometric surfaces for periods after 1958 are difficult to calculate, and no contoured surfaces could be found in the published literature. Since the observed temperature distributions for 1958 and 1962 are approximately the same as the one given in Figure 7, no new temperature surfaces are presented.

The computed potentiometric surfaces of the main production area for August 1958 and December 1962 are presented in Figures 14 and 15, respectively. Computed temperatures showed a slight decrease throughout the field, a maximum decrease of approximately 4°C in December 1962 coinciding with the maximum drawdown. Since the change in the calculated temperature distribution is very small, once again no new surfaces are presented.

The major changes in the potentiometric surface occur in the vicinity of the main production area and consist of a general decline in head due to the withdrawal of water and heat. A comparison of computed and observed surfaces for this area can be made for 1958 by using Figures 12 and 14 and for 1962 by using Figures 13 and 15. The maximum drawdown has increased from approximately 40 m in 1958 to approximately 80 m by 1962. In general, flow is still from west to east with exploitation effects causing a local flow toward the production area. Possible causes for differences between the observed and computed surfaces include the way in which the discharge data was distributed to the nodes, steam forming in the aquifer causing errors in either the model (which is single phase) or the observed data, error in the original steady state conditions, error in not giving fault zones special attention, and error in treating a three-dimensional problem as two-dimensional.

The single-phase model was not adequate to reproduce historical data after 1962 because of the considerable quantity of steam that had formed in the Waiora formation [*Grindley*, 1965; *Bolton*, 1970, 1973]. Results of simulations from 1953 to 1962, however, indicate that hot-water hydrothermal systems can be modeled and evaluated. In the case of the Wairakei model the mass and heat outputs due to exploitation were known. This information was used to calibrate the simulation model by comparing observed and computed temperature and potentiometric surfaces. To apply this basic model to forecast

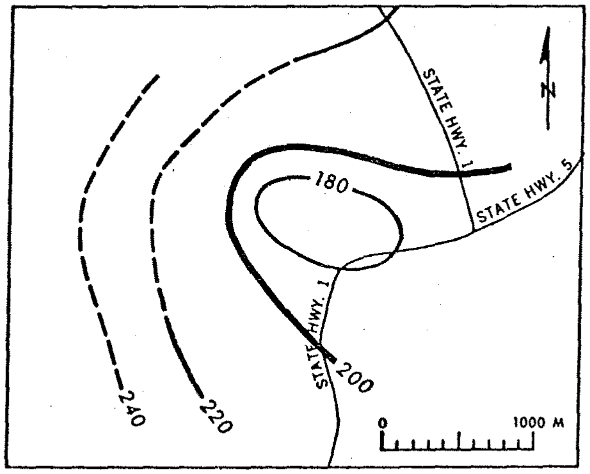

Fig. 13. Wairakei measured potentiometric surface, 1962 [data from *Grindley*, 1965]. Datum is 152.4 m above sea level; contour interval is 20 m.

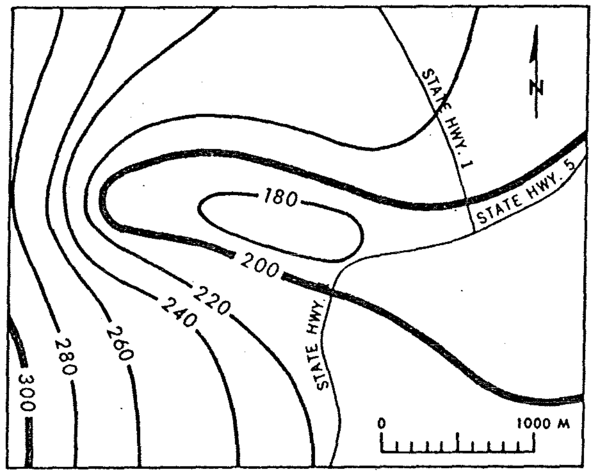

Fig. 15. Wairakei calculated 1962 potentiometric surface of the production area. Datum is 152.4 m above sea level; contour interval is 20 m.

the performance of other hot-water systems, mass outputs will need to be assumed, perhaps using economic considerations. Such a model will indicate, within the accuracy of the description of the geothermal system, the amount and rate at which mass may be withdrawn and if the enthalpy of the mass is known, will permit the heat output to be estimated.

CONCLUSIONS

A two-dimensional (areal) model has been developed in which the groundwater flow equation and the heat transport equation are solved by using finite difference approximations for the time derivatives and Galerkin–finite element approximations for the space derivatives. The approach is flexible and efficient, and once developed, the basic program can be reused, possibly with some modification, for a variety of groundwater problems involving heat transport.

The study of the Wairakei hydrothermal system described in this paper illustrates the potential of this approach for simulating both the predevelopment conditions (assumed steady state) of a hot-water geothermal system and its response over a development period (assuming that steam within the aquifer plays a relatively minor role in the field behavior). It has been shown that the initial patterns of, and changes in, temperature and pressure in such a system can be reproduced using basic field and well information. Thus the amount of heat energy available in a field and the rate at which it can be removed can be estimated using numerical modeling techniques. Since most hydrothermal systems are hot-water dominated [*White,* 1970], such a predictive tool could be very helpful in the economic evaluation and management of such systems.

The limitations of the present model preclude a full assessment of the Wairakei field, since it can no longer be considered a hot-water system after 1962. A more complete investigation of geothermal systems using a three-dimensional numerical model incorporating the behavior of steam-water interactions is now underway.

NOTATION

b,	saturated thickness of the aquifer, L.
b',	saturated thickness of the confining bed, L.
c_v,	heat capacity at constant volume per unit mass, $L^2t^{-2}T^{-1}$.
g,	gravitational constant, Lt^{-2}.
h_i,	heat flux associated with diffusion and/or dispersion, Mt^{-3}.
K_{ij},	hydrodynamic thermal dispersion tensor (K_{zz}, vertical component), $MLt^{-3}T^{-1}$.
K_{ij}^m,	mechanical thermal dispersion tensor, $MLt^{-3}T^{-1}$.
K^d,	molecular thermal diffusion coefficient, $MLt^{-3}T^{-1}$.
K_{ij}^s,	thermal diffusion tensor of the solid phase (K_{zz}^s, vertical component), $MLt^{-3}T^{-1}$.
K_t, K_l,	transverse and longitudinal mechanical thermal dispersion coefficients, $MLt^{-3}T^{-1}$.
K',	hydraulic conductivity of the confining bed, Lt^{-1}.
k_{ij},	local intrinsic permeability tensor (k_{zz}, vertical component), L^2.
k',	permeability of the confining bed, L^2.
p,	pressure, $ML^{-1}t^{-2}$.
p',	pressure distribution above the confining bed, $ML^{-1}t^{-2}$.
Q,	heat source term per unit mass, L^2t^{-3}.
Q',	areally distributed heat source, Mt^{-3}.
q,	strength of a point mass source function, Lt^{-1}.
q',	vertical mass flow term, Lt^{-1}.
r,	time rate of supply of mass per unit volume, $ML^{-3}t^{-1}$.
S_s,	specific storage of confining bed, L^{-1}.
T,	temperature, T.
T',	temperature distribution at the top (or bottom) of the aquifer (in practice the temperature distribution above the confining bed is used), T.
t,	time, t.
t',	length of pumping period, t.
v_i,	velocity (v_z, vertical component), Lt^{-1}.
α,	vertical compressibility, Lt^2M^{-1}.
β,	compressibility coefficient of the liquid phase, $LM^{-1}t^2$.
γ,	mass density on a bulk volume basis where $\gamma = \rho\phi$, ML^{-3}.
λ,	coefficient of thermal volume expansion for the liquid phase, T^{-1}.
μ,	viscosity, $ML^{-1}t^{-1}$.
ρ,	average density, ML^{-3}.
ϕ,	porosity, dimensionless.
ψ_{ij},	porous medium tortuosity tensor, dimensionless.
ψ,	tortuosity factor, dimensionless.
$(\ )^s$,	refers to the solid phase.
$(\ )_z$,	refers to the z direction (vertical).
$(\ )'$,	indicates properties associated with confining beds.
$(\ )_0$,	refers to a reference or initial quantity.

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