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Theory of Heat Extraction From Fractured Hot Dry Rock

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A theory of heat extraction from fractured hot dry rock is presented, based on an infinite series of parallel vertical fractures of uniform aperture. Fractures are uniformly spaced and drain heat from blocks of homogeneous and isotropic impermeable rock. Cold water enters at the bottom of each fracture, and solutions are given in terms of dimensionless parameters from which the exiting water temperatures at the top of the fractures can be determined. An example of the application of the theory demonstrates how a multiply fractured system provides a more efficient mechanism for heat extraction than a single fracture in hot dry rock.

Utilization of geothermal energy is currently limited to a small number of naturally occurring geothermal steam and hot water reservoirs. The growing interest in this new source of energy has also stimulated attempts to develop a method of extracting thermal energy from the numerous regions of the earth's crust containing deposits of hot dry rock, which may constitute a resource much larger than that permeated by groundwater. The basis of this concept is to develop an adequate fracture surface to be used for heat transfer purposes. Because of the low thermal conductivity of rock a very large heat transfer area must be provided; otherwise, meaningful amounts of energy cannot be extracted at practical rates.

One proposal [*American Oil Shale Corporation and U.S. Atomic Energy Commission,* 1971] describes a method for recovering heat through a closed loop cycle of surface water from dry geothermal sites previously fractured by a suitable array of sequentially fired, fully contained nuclear explosives. Results from a preliminary analysis forecast operation of a 200-MW power plant for 30 years [*Burnham and Stewart,* 1973]. Soviet workers have also been considering the use of in situ explosions to create a highly fractured rock system so that circulating water could extract geothermal heat [*Diadkin et al.,* 1973].

Another technique, developed by *Robinson et al.* [1971] of the Los Alamos Scientific Laboratory, requires drilling two parallel deep boreholes, the second of which is directed so as to intersect a vertically oriented crack produced by hydraulic fracturing in the first hole. Water circulating down one well, through the crack, and up the other would carry off heat from the hot rock to the surface. A supporting theoretical analysis was presented by *Harlow and Pracht* [1972], which indicates that many tens of megawatts of thermal power could be supplied for several decades, provided that the initial fracture zone could be extended through the effects of thermal stress cracking in the adjacent hot rocks. A field experiment conducted in a test well drilled to a depth of 780 m at one edge of a volcanic caldera in the Jemez Mountains of northern New Mexico has shown that granite could indeed be fractured hydraulically and was impermeable enough in this region to hold water tightly [*Hammond,* 1973]. The propagation of the fracture from thermal stress effects, however, has not yet been demonstrated.

A third concept that could greatly increase the economic life of hot rock geothermal systems is that proposed by *Raleigh et al.* [1974]. Observing that in many regions the stresses in subsurface rock are reasonably constant over large areas, they suggest that geothermal wells be drilled at an angle in a direction perpendicular to the expected orientation of fractures and that a series of parallel, vertical cracks be created from a single well (Figure 1). *Strukbar et al.* [1974] have discussed some of the problems of creating multiple vertical fractures from an inclined well bore.

The purpose of this paper is to present the results of a mathematical analysis of this third method.

MATHEMATICAL MODEL

The following discussion will be based on a linear model involving an infinite series of parallel, equidistant, vertical fractures of uniform thickness, separated by blocks of homogeneous and isotropic, impermeable rock, the width of the individual fracture being assumed to be negligible in comparison with the spacing between the fractures.

Owing to the spatial periodicity of the temperature field it is possible to replace the infinite system by a finite one consisting of a single vertical fracture of thickness $2b$ between two matrix blocks with an insulating outer boundary at a distance from the midplane of the fracture equal to half the fracture spacing. As is illustrated in Figure 2, a rectilinear coordinate system is placed such that the $x = 0$ plane coincides with the midplane of the fracture. Water is injected at $z = 0$ and is flowing upward in the fracture.

The following simplifying assumptions are made:

1. The product of the density and heat capacity for both the water and the formation, and the formation thermal conductivity are constant. The linear volumetric and mass flow rate of the water is constant in the fracture.
2. The water temperature $T_W(z, t)$ is uniform in any cross-

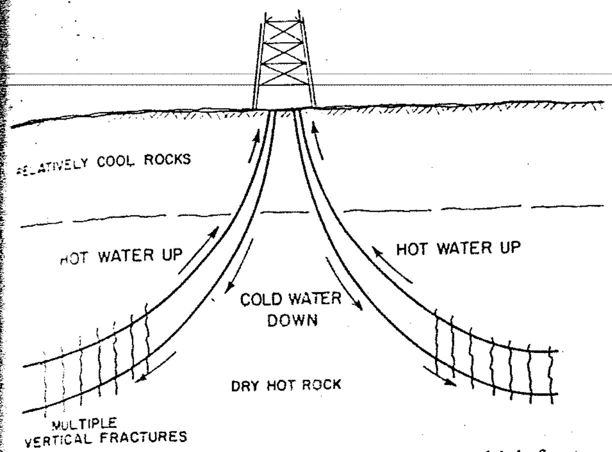

Fig. 1. Schematic diagram of heat extraction from multiple fractures in hot dry rock.

section of the fracture and is equal for all z to the formation temperature at $x = b$.

3. There is no heat transfer by radiation within the fracture or by conduction in the vertical direction within the fracture or the formation. All heat transport is by horizontal conduction in the rock and by forced convection along the z axis within the fracture.

4. Initially, both the water in the fracture and the formation are at the same temperature. This temperature is not uniform along the fracture but depends on z, if one takes the geothermal gradient into account. At a given z value the initial temperature is given by the initial rock temperature at the point of injection T_{R0} minus the product of distance z above the injection point and the geothermal gradient ω, which is assumed to be constant. At time $t = 0$, water is injected at constant injection temperature T_{W0}. No heat flux is assumed across the boundary at $x = x_E$.

The differential equation governing the water temperature $T_W(z, t)$ is obtained by writing a heat balance on an element of fracture volume $(2b \cdot dz \cdot 1)$ between elevation z and elevation $z + dz$ above the injection point. Because of the symmetry with respect to the fracture midplane this can be written as

$$b\rho_W c_W \left[\frac{\partial T_W(z, t)}{\partial t} + v\,\frac{\partial T_W(z, t)}{\partial z}\right] = K_R \left.\frac{\partial T_R(x, z, t)}{\partial x}\right|_{x=b} \quad (1)$$

where v is the water velocity, ρ_W and c_W are the water density and specific heat, and K_R is the rock thermal conductivity; $T_R(x, z, t)$ is the rock temperature, which is governed by the heat conduction equation

$$\frac{\partial^2 T_R(x, z, t)}{\partial x^2} = \frac{\rho_R c_R}{K_R}\,\frac{\partial T_R(x, z, t)}{\partial t} \quad (2)$$

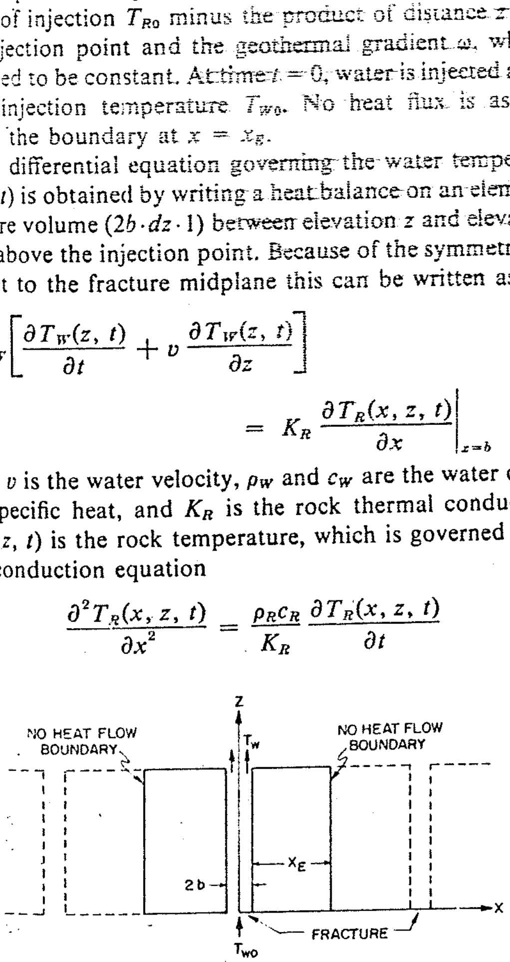

Fig. 2. Mathematical model for fractured hot dry rock.

The temperatures must also satisfy the following conditions:

$$T_R(x, z, t) = T_W(z, t) = T_{R0} - \omega z \qquad t < z/v \quad (3)$$

$$T_R(x, 0, t) = T_W(0, t) = T_{R0} \qquad t < 0$$
$$T_R(x, 0, t) = T_W(0, t) = T_{W0} \qquad t \geq 0 \quad (4)$$

$$T_W(z, t) = T_R(b, z, t) \qquad \forall\, z, t \quad (5)$$

$$\left.\frac{\partial T_R(x, z, t)}{\partial x}\right|_{x=x_E} = 0 \quad (6)$$

The simultaneous solution of (1) and (2) subject to conditions (3)–(6) is derived in Appendix A. The result for the outlet water temperature at a distance z from the injection point can be expressed in a general form as a function $T_{WD}(\beta, X_{ED}, t_D')$ of dimensionless parameters such that

$$T_{WD} = [T_{R0} - T_W(z, t)]/(T_{R0} - T_{W0}) \quad (7)$$

$$\beta = \omega z/(T_{R0} - T_{W0}) \quad (8)$$

$$X_{ED} = (\rho_W c_W/K_R)(Q/z)x_E \quad (9)$$

$$t_D' = [(\rho_W c_W)^2/K_R \rho_R c_R](Q/z)^2 t' \quad (10)$$

where Q is the volumetric flow rate per fracture per unit thickness of the system in the y direction and $t' = t - (z/v)$; z/v is the time lag between the departure of the water from the injection point and the arrival at point z. Because this time lag is very small in comparison with the lengths of time involved in our problem, t' is practically identical to t.

The dimensionless parameter β was found to affect the time variation of the outlet water dimensionless temperature at small values of dimensionless time. This can occur when there is a high geothermal gradient or a large fracture length. The influence of geothermal gradient in the single fracture case is shown in Figure 7. For simplicity in the multiple fracture case the geothermal gradient has been neglected, T_{R0} now being taken as the average rock temperature over the zone of interest. The results for this case are shown in Figure 3, where T_{WD} is plotted as a function of t_D' for various values of the dimensionless half-fracture spacing X_{ED}.

The ratio of the amount of heat extracted by the water flowing through the fracture to the initial total heat available in the rock was also computed and is shown in Figure 4.

Although the model presented in this paper is strictly valid only for an infinite number of fractures, it can also be used as a good approximation when the number of fractures is finite but large enough; this should be the case in most practical applications.

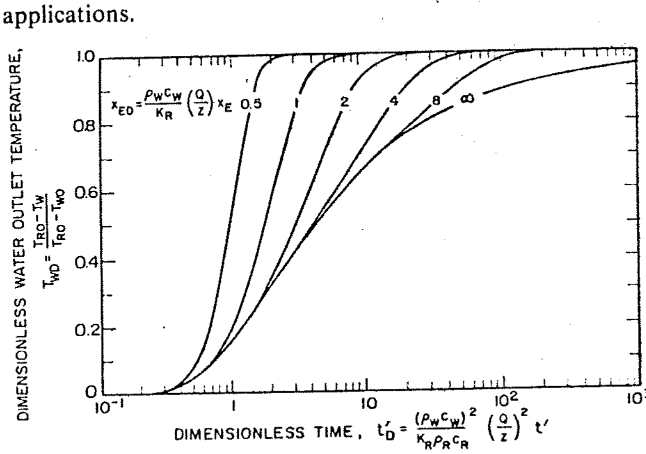

Fig. 3. Dimensionless water outlet temperature versus dimensionless time showing effect of fracture spacing.

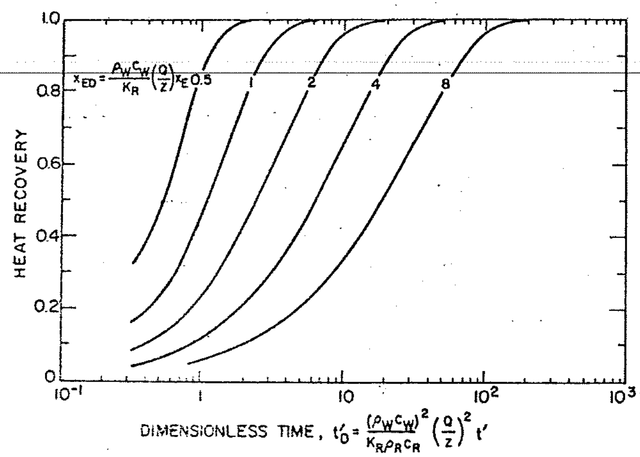

Fig. 4. Fractional heat recovery versus dimensionless time.

EXAMPLE OF CALCULATION

In designing a geothermal power plant that uses this concept a number of parameters are imposed, and others have to be adjusted. In the system under consideration the rock and water properties are given, and the total volumetric flow rate, minimum usable outlet temperature, and useful reservoir life are fixed by technical and economic considerations. What has to be chosen is the number of fractures, the dimensions of the individual fractures, and the spacing between the fractures. These parameters can be determined from Figure 3. An example of this type of calculation is described below.

In order to allow comparison with the system proposed by the Los Alamos group the data given by *Harlow and Pracht* [1972] were used in the computations. They assumed a single fracture with a height of 1 km and a length (y direction) of 1 km and used a volumetric flow rate of 1.45×10^5 cm³/s. They adopted a rock temperature of 300°C and an inlet water temperature of 65°C. Their remaining material properties were $K_R = 6.2 \times 10^{-3}$ cal/cm s °C, $\rho_R = 2.65$ g/cm³, $c_R = 0.25$ cal/g °C, $\rho_W = 1.0$ g/cm³, and $c_W = 1.0$ cal/g °C. Resulting temperatures are shown in Figure 5. With a single fracture the water outlet temperature drops very quickly after only a few years. This should be the case in Harlow and Pracht's model if thermal fracture propagation turns out to be negligible.

On the other hand, if the same total volumetric flow rate is now divided equally between 10 vertical fractures, the much reduced flow rate per fracture results in a very substantial increase in the water outlet temperatures. This is shown on Figure 5 for fracture spacings that vary from 40 to 160 m. One can anticipate that this will be the case from the theoretical results on Figure 3. Since water outlet temperatures will not begin to fall below T_{R0} until $t_D' > 0.2$, reducing Q to 1/10 of its value means that the actual time required to reach $t_D' = 0.2$ will be 100 times longer if all the flow passes through 10 fractures than if it passes through only one fracture. Thereafter if 10 fractures are used, the water outlet temperature will drop at a much lower rate that is dependent on fracture spacing.

The number of fractures in our example was determined by the requirement of a 297°C water outlet temperature ($T_{WD} = 0.013$) after 20 years. It can be seen from Figure 3 that this temperature corresponds to $t_D' = 0.325$ and $X_{ED} > 0.5$, which for $t' = 20$ years yields $Q = 0.145$ cm³/s. Since this number by definition is also equal to the total volumetric flow rate (1.45×10^5 cm³/s) divided by the number of fractures N and by the fracture length (1 km), the number of fractures follows immediately. The result is $N = 10$.

The spacing between the fractures is determined by the value of the dimensionless spacing parameter X_{ED}, which for this example would be >0.5. It is obvious from Figure 3 that temperature will drop at a faster rate for smaller X_{ED}. On the other hand, one can see from Figure 4 that the fraction of energy recovered from the rock will be higher. For example, if one were to increase the time period to 100 years ($t_D = 1.6$), it can be seen from Figures 3 and 4 that there would be no advantage in taking X_{ED} greater than 2 (which corresponds to a fracture spacing of 160 m), because a lower heat recovery would result and the drilling costs would be increased with no improvement in the outlet temperatures.

Obviously, the best choice for the fracture spacing is that which would yield the highest electrical power outlet for the longest period of time. The electrical power output has been computed from the water in situ exergy by assuming 75% flash efficiency and 65% mechanical efficiency [*Bodvarsson and Eggers*, 1972]. The results are shown in Figure 6. It can be seen that electrical power output is highest for 10 fractures with a 160-m spacing, and this spacing is capable of producing many times the power output of a single fracture. If 20 fractures were used instead of 10, the maximum output of about 19 MW could be maintained for a century.

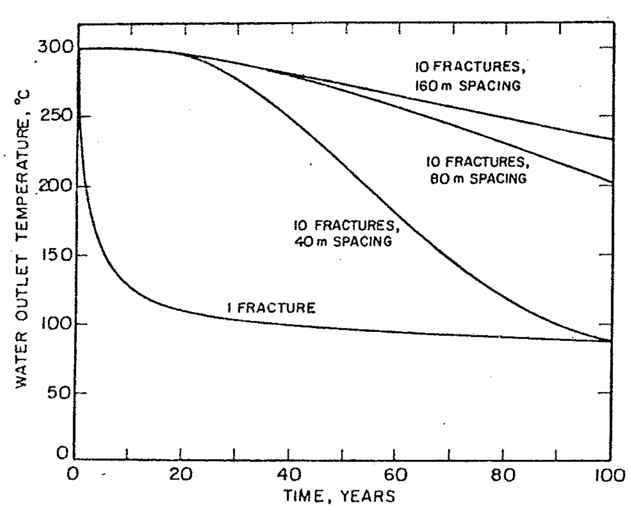

Fig. 5. Water outlet temperature versus time for 1-km² fractures with T_{R0} = 300°C and T_{W0} = 40°C.

CONCLUSION

Although it is much simplified, the mathematical model used in this study shows that the multifracture concept could greatly increase the economic utilization of hot dry rock geothermal systems. If fracture propagation occurs through the effects of thermal stress cracking, as is expected by some authors, geothermal energy extraction will be even greater than that reported in the present study.

APPENDIX A: DERIVATION OF WATER OUTLET TEMPERATURE

Special cases of the mathematical problem considered in the present paper have been studied by previous authors. *Bodvarsson* [1969] solved the problem of constant flow with sinusoidal temperature through laminated solid, and solutions for $x_E = \infty$ were published by *Lauwerier* [1955] and *Carslaw and Jaeger* [1959].

An exact analytical solution for finite x_E was recently presented by *Romm* [1972]. In the present paper the problem is solved in a slightly different way, and the results for infinite

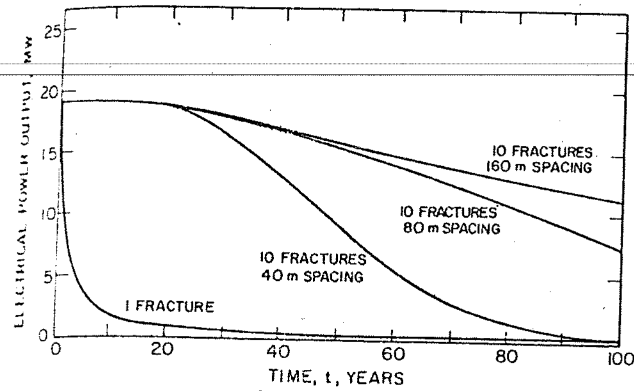

Fig. 6. Electrical power output from 1-km² fractures.

and finite x_E values are presented on the same graph in terms of suitable dimensionless quantities (Figure 3).

A solution is obtained as follows: we first substitute t' in place of t into (1)–(4):

$$\rho_W c_W b v \frac{\partial T_W(z, t')}{\partial z} = K_R \frac{\partial T_R(x, z, t')}{\partial x}\bigg|_{x=b} \quad \text{(A1)}$$

$$\frac{\partial^2 T_R(x, t')}{\partial x^2} = \frac{\rho_R c_R}{K_R} \frac{\partial T_R(x, z, t')}{\partial t'} \quad \text{(A2)}$$

$$\lim_{t' \to 0} T_R(x, z, t') = \lim_{t' \to 0} T_W(z, t') = T_{R0} - \omega z \quad \text{(A3)}$$

$$\lim_{z \to 0} T_W(z, t') = T_{W0} \qquad t' > 0 \quad \text{(A4)}$$

We then introduce the following dimensionless quantities:

$$\alpha = 2K_R H / \rho_W c_W Q b \quad \text{(A5)}$$

where H is an arbitrary length and Q is the volumetric flow rate per fracture per unit length of the system in the y direction:

$$\beta^* = \omega H / (T_{R0} - T_{W0}) \quad \text{(A6)}$$

$$x_D = x/b \quad \text{(A7)}$$

$$z_D = z/H \quad \text{(A8)}$$

$$t_D^* = [(\rho_W c_W)^2 / 4 K_R \rho_R c_R] / (Q/H)^2 t' \quad \text{(A9)}$$

$$T_{RD}(x_D, z_D, t_D') = \frac{T_{R0} - T_R(x, z, t')}{T_{R0} - T_{W0}} \quad \text{(A10)}$$

Equations (A1)–(A4), (5), and (6) thus become

$$\frac{1}{\alpha} \frac{\partial T_{WD}(x_D, t_D^*)}{\partial z_D} = \frac{\partial T_{RD}(x_D, z_D, t_D^*)}{\partial x_D}\bigg|_{x_D=1} \quad \text{(A11)}$$

$$\frac{\partial^2 T_{RD}(x_D, z_D, t_D^*)}{\partial x_D^2} = \frac{1}{\alpha^2} \frac{\partial T_{RD}(x_D, z_D, t_D^*)}{\partial t_D^*} \quad \text{(A12)}$$

$$\lim_{t_D^* \to 0} T_{RD}(x_D, z_D, t_D^*) = \lim_{t_D^* \to 0} T_{WD}(z_D, t_D^*) = \beta^* z_D \quad \text{(A13)}$$

$$T_{WD}(z_D, t_D^*) = T_{RD}(1, z_D, t_D^*) \quad \text{(A14)}$$

$$\lim_{z_D \to 0} T_{WD}(z_D, t_D^*) = 1 \qquad t_D^* \geq 0 \quad \text{(A15)}$$

$$\frac{\partial T_{RD}(x_D, z_D, t_D^*)}{\partial x_D}\bigg|_{x_D = x_{ED}} = 0 \quad \text{(A16)}$$

Applying the Laplace transform with respect to t_D^* and solving for the water outlet temperature yield

$$\bar{T}_{WD}(z_D, s) = \frac{1}{s}\left[1 + \left(\frac{\beta^*}{s^{1/2} \tanh [(x_{ED} - 1)/\alpha] s^{1/2}}\right)\right] \cdot \exp\left(-z_D s^{1/2} \tanh \frac{x_{ED} - 1}{\alpha} s^{1/2}\right) + \frac{\beta^* z_D}{s} - \frac{\beta^*}{s^{3/2} \tanh [(x_{ED} - 1)/\alpha] s^{1/2}} \quad \text{(A17)}$$

where $\bar{T}_{WD}(z_D, s)$ is the Laplace transform of the dimensionless water outlet temperature.

Equation (A17) is most difficult to invert analytically except in the case of a single fracture ($x_E = \infty$), where inversion of (A17) shows that T_{WD} can be expressed as a function of β (equation (8)) and t_D' (equation (10)) only:

$$T_{WD}(t_D') = 1 - 2\beta (t_D'/\pi)^{1/2} [1 - \exp(-1/4t_D')] - (1 - \beta) \operatorname{erf} [\tfrac{1}{2}(t_D')^{1/2}] \quad \text{(A18)}$$

Parameter T_{WD} has been plotted versus t_D' in Figure 7 for various β values. The curves are very similar except at early t_D' values, because the initial temperature at the top of the fracture depends upon the geothermal gradient. For high geothermal gradients or large fracture height the temperature at the production well increases slightly and then decreases down to the injection temperature at very large time. For simplification in the solution of (A17) the geothermal gradient has been neglected, T_{R0} being taken as the average temperature along the fracture height. Setting $\beta^* = 0$ into (A17) and neglecting the 1 following x_{ED} yield

$$\bar{T}_{WD}(z_D, s) = \frac{1}{s} \exp\left(-z_D s^{1/2} \tanh \frac{\rho_W c_W Q x_E}{2 K_R H} s^{1/2}\right) \quad \text{(A19)}$$

Equation (A19) was inverted by means of a numerical method [*Papoulis*, 1957], and the result expressed as a function of x_{ED} (equation (9)) and t_D' (equation (10)) is plotted in Figure 3.

APPENDIX B: ELECTRICAL POWER OUTPUT

The total amount of heat in the hot rock per unit length of the system in the y direction, which could be recovered by cooling from T_{R0} to T_{W0}, is

$$H_R = 2\rho_R c_R z x_E (T_{R0} - T_{W0}) \quad \text{(B1)}$$

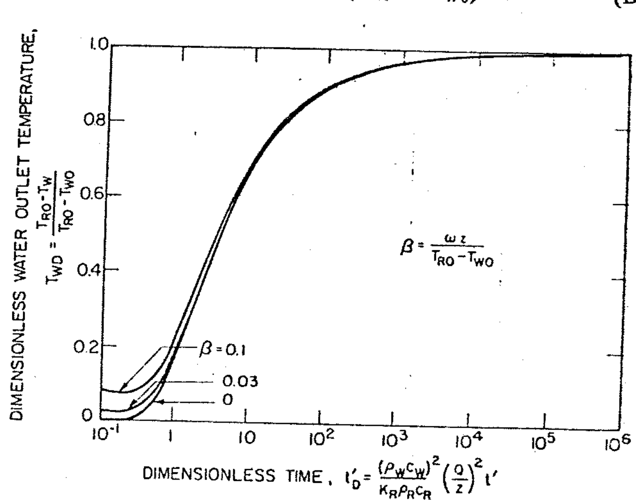

Fig. 7. Dimensionless water outlet temperature versus dimensionless time for a single fracture ($X_{ED} = \infty$) showing effect of geothermal gradient.

The heat extraction rate or thermal power output by the water between T_W and T_{W0} is

$$\rho_W c_W Q(T_W - T_{W0})$$

or

$$\rho_W c_W Q(T_{R0} - T_{W0})\left(1 - \frac{T_{R0} - T_W}{T_{R0} - T_{W0}}\right)$$

The cumulative heat extraction by water over a period t' is given by

$$H_W = \int_0^{t'} \rho_W c_W Q(T_{R0} - T_{W0}) \cdot\left(1 - \frac{T_{R0} - T_W}{T_{R0} - T_{W0}}\right) dt' \qquad \text{(B2)}$$

The ratio of the amount of heat extracted by the water to the initial heat content in the rock is thus equal to

$$\frac{H_W}{H_R} = \int_0^{t'} \frac{\rho_W c_W Q}{2\rho_R c_R z x_E}\left(1 - \frac{T_{R0} - T_W}{T_{R0} - T_{W0}}\right) dt'$$

or

$$\frac{H_W}{H_R} = \frac{1}{2X_{ED}} \int_0^{t_D'} [1 - T_{WD}(t_D')]\, dt_D' \qquad \text{(B3)}$$

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