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# HEAT DISPERSION EFFECT ON THERMAL CONVECTION IN A POROUS MEDIUM LAYER

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## ABSTRACT

Rubin, H., 1974. Heat dispersion effect on thermal convection in a porous medium layer. *J. Hydrol.*, 21: 173–185.

Thermal convection resulting from vertical temperature gradients in porous media is analyzed. The effect of heat dispersion is taken into account. It is found that heat dispersion increases the thermal stability of the flow field and may inhibit the appearance of convection currents, which would appear if dispersion effects are omitted.

The longitudinal as well as the lateral dispersivities affect the thermal stability and the dimensions of the convection cells. As a result of the convection currents the horizontal streamlines in the steady state are distorted. The thermal convection exhibits internal waves in the field.

## INTRODUCTION

In some situations associated with geothermal activity it is possible that groundwater motion is influenced by convection currents due to large temperature gradients (Lapwood, 1948; Wooding, 1957). Such groundwater motions may happen in the aquifer of Lake Kinnereth springs in Israel (Dagan and Kahanovitz, 1968). Heat transfer in the porous layer is affected by the thermal diffusivity of the liquid as well as the conduction properties of the solid fracture. Usually groundwater is under conditions of steady flow. In previous investigations it was found that if Peclet number of the flow field is small heat transfer can be characterized by convection and by diffusion expressed through the scalar heat diffusivity of the saturated porous layer. At large Peclet numbers the scalar heat diffusivity should be exchanged by the dispersion tensor which depends on the intrinsic dispersivity of the porous layer. The need for the application of the dispersion tensor is typical for inhomogeneous porous layers where the characteristic length for heat diffusion is large.

The aim of the present study is to analyze the effect of heat dispersion on thermal stability of the flow field in cases of large Peclet numbers.

BASIC EQUATIONS OF THE FLOW FIELD

According to the Boussinesq approximation the basic equations for the flow field are:

$$\frac{\partial q_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\rho_0}{\epsilon}\left(\frac{\partial q_i}{\partial t} + \frac{q_j}{\epsilon}\frac{\partial q_i}{\partial x_j}\right) = -\rho g n_i - \frac{\partial p}{\partial x_i} - \frac{\mu}{K} q_i \tag{2}$$

$$\frac{\partial T}{\partial t} + \beta \frac{q_j}{\epsilon}\frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_i}\left(E_{ij}\frac{\partial T}{\partial x_j}\right) \tag{3}$$

$$\rho = \rho_0 [1 - \alpha(T - T_0)] \tag{4}$$

where $q_i$ = the specific flux vector component; $T$ = the temperature; $K$ = the permeability of the porous layer; $n_i$ = the component of a unit vector in the $z$ direction; $E_{ij}$ = the component of the dispersion tensor; $\beta$ = the ratio between volumetric heat capacity of the fluid fracture to that of the saturated formation (if the solid matrix does not conduct heat then $\beta = 1$); $\epsilon$ = the medium porosity; $\alpha$ = the volume coefficient of thermal expansion; $\rho_0$ = a density of reference. Eq. 1–4 were used in a similar manner by other investigators (Lapwood, 1948; Nield, 1968). However, in their analyses dispersion effects were neglected; therefore, in eq. 3 instead of the dispersion tensor they expressed heat conductivity by the scalar molecular diffusivity of the saturated porous medium.

The dispersion tensor according to previous investigations (Bear, 1961; Pfankuch, 1963; Poreh, 1965) is a second order anisotropic tensor which depends on another fourth order tensor expressing the intrinsic dispersivity of the porous medium. In an isotropic medium the dispersion tensor is axisymmetric and can be expressed by the following equation:

$$E_{ij} = F_1 \delta_{ij} + F_2 u_i u_j \tag{5}$$

where $u_i = (q_i/\epsilon)$, is the component of the barycentric flow velocity vector; $\delta_{ij}$ is Kronecker's delta; $F_1$ and $F_2$ are functions of the pore size of the porous medium and Peclet and Reynolds numbers of the field.

In every point of the flow field it is possible to refer to a coordinate system which one of its axes coincides with the flow direction. The dispersion tensor components in such a case are:



$$E_{11} = \eta_1 U + \kappa$$

$$E_{22} = E_{33} = \eta_2 U + \kappa$$

$$E_{ij} = 0 \quad (i \neq j)$$

where $E_{11}$ is the longitudir
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$$\eta_1/\eta_2 = 10 \div 30$$

From eq. 5–8 we get the

$$F_1 = \eta_2 U + \kappa$$

$$F_2 = (\eta_1 - \eta_2)/U$$

THE UNPERTURBED FIELD

As a model that describes
We refer to a Cartesian coor
lateral and vertical direction
$z = 0$ and $z = d$. In this field
ponents are:

$$u_* = u_0 \qquad v_* = 0 \qquad w$$

where $u_*$, $v_*$ and $w_*$ are the
respectively. We assume that
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$$E_{11} = \eta_1 U + \kappa \tag{6}$$

$$E_{22} = E_{33} = \eta_2 U + \kappa \tag{7}$$

$$E_{ij} = 0 \qquad (i \neq j) \tag{8}$$

where $E_{11}$ is the longitudinal dispersion; $E_{22}$ and $E_{33}$ are the lateral dispersion components; $U$ is the absolute magnitude of the velocity vector; $\kappa$ is the molecular thermal diffusivity of the saturated porous medium.

If Peclet number of the flow is small, then $\eta_1$ and $\eta_2$ are small, dispersion effects are small and the scalar molecular diffusivity expresses heat conduction.

However, in a nonhomogeneous porous medium the typical length of heat diffusion may be large. Therefore, Peclet number may attain large values. Then $\eta_1$ and $\eta_2$ are almost constants and the thermal diffusion as well as terms depending on the thermal diffusivity in eq. 6 and 7 are negligible. In such cases it was found experimentally that (Pfankuch, 1963):

$$\eta_1/\eta_2 = 10 \div 30 \tag{9}$$

From eq. 5–8 we get the following expressions:

$$F_1 = \eta_2 U + \kappa \tag{10}$$

$$F_2 = (\eta_1 - \eta_2)/U \tag{11}$$

## THE UNPERTURBED FIELD

As a model that describes the steady state flow field we refer to Fig. 1. We refer to a Cartesian coordinate system $x$, $y$ and $z$ which are the horizontal lateral and vertical directions, respectively. The porous medium lies between $z = 0$ and $z = d$. In this field the steady state barycentric flow velocity components are:

$$u_* = u_0 \qquad v_* = 0 \qquad w_* = 0 \tag{12}$$

where $u_*$, $v_*$ and $w_*$ are the longitudinal lateral and vertical components, respectively. We assume that the medium is made of a poor conductor. Therefore, $\beta \simeq 1$ in eq. 3. We assume that the horizontal boundaries of the porous layer ($z = 0$ and $z = d$) are impermeable having constant temperature ($T_0$ and $T_1$, respectively). The temperature distribution in the steady state is linear as follows:

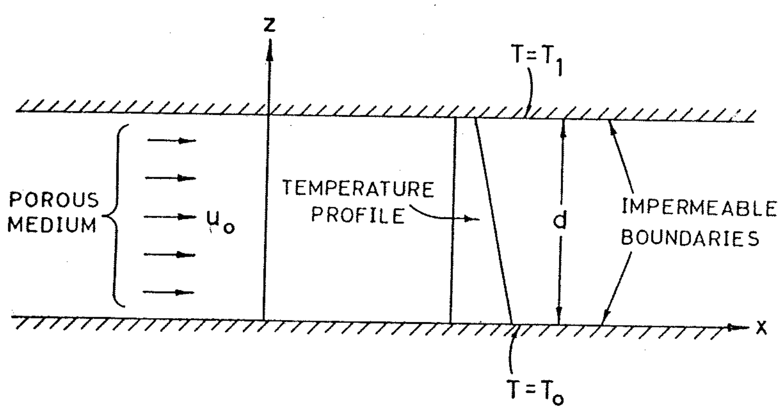


Fig. 1. Schematical description of the steady state flow field.

$$T = T_0 - \frac{\Delta T}{d} z \tag{13}$$

where:

$$\Delta T = T_0 - T_1$$

For the unperturbed density and pressure fields we obtain from eq. 1–4:

$$\rho_* = \rho_0 \left(1 + \frac{\alpha \Delta T}{d} z\right) \tag{14}$$

$$p_0 - p_* = \rho_0 \left(z + \frac{\alpha \Delta T}{2d} z^2\right) + \frac{\mu \epsilon}{K} u_0 x \tag{15}$$

where $\rho_0$ is the density at $z = 0$; $p_0$ is the pressure at the coordinate origin.

The dispersivity tensor components in the unperturbed flow field are according to eq. 6–8:

$$D_{xx} = \eta_1 u_0 + \kappa \tag{16}$$

$$D_{yy} = D_{zz} = \eta_2 u_0 + \kappa \tag{17}$$

$$D_{xy} = D_{xz} = D_{yz} = 0 \tag{18}$$

## THE PERTURBED FIELD

The flow field is now subjected to small perturbations in the velocity $(u, v, w)$, temperature $(\theta)$, pressure $(p')$ and the dispersion tensor $(E'_{ij})$.

For stability analysis we may refer to a two-dimensional flow field (Kuo, 1961; Veronis, 1965, 1968). Thus the velocity perturbation in the longitudinal $(u)$ and vertical $(w)$ directions may be expressed with the aid of the stream function $\psi$:

$$u = \frac{\partial \psi}{\partial z} \qquad w = -\frac{\partial \psi}{\partial x} \tag{19}$$

Our analysis is referred to large Peclet numbers. In such cases the velocity perturbations would usually be smaller than the steady state horizontal flow velocity $u_0$. Hence in such cases the absolute magnitude of the barycentric velocity vector in the perturbed flow field is approximately given by:

$$U + U' = \sqrt{(u_0 + u)^2 + w^2} \simeq u_0 + u \tag{20}$$

By applying eq. 5, 10, 11 and 16–20, we get after neglecting small quantities the value of the dispersion tensor perturbation components as follows:

$$E'_{xx} = \eta_1 u = \eta_1 \frac{\partial \psi}{\partial z} \tag{21}$$

$$E'_{zz} = \eta_2 u = \eta_2 \frac{\partial \psi}{\partial z} \tag{22}$$

$$E'_{xz} = E'_{zx} = (\eta_1 - \eta_2)\, w = -(\eta_1 - \eta_2) \frac{\partial \psi}{\partial x} \tag{23}$$

Thus in the perturbed flow field the horizontal and vertical directions are no more the principal directions of the dispersion tensor.

Substituting the various perturbation components in eq. 1–4, neglecting second order terms, eliminating the pressure perturbation and applying eq. 21–23 we obtain:

$$\frac{1}{\epsilon}\left(\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x}\right)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\psi = -\frac{g\alpha}{\epsilon}\frac{\partial \theta}{\partial x} - \frac{\nu}{K}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\psi \tag{24}$$

$$\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial x}\right)\theta=-\frac{\partial\psi}{\partial x}\frac{\Delta T}{d}+\eta_1 u_0\frac{\partial^2\theta}{\partial x^2}+\eta_2 u_0\frac{\partial^2\theta}{\partial z^2}$$

$$-\frac{\Delta T}{d}\left[-(\eta_1-\eta_2)\frac{\partial^2\psi}{\partial x^2}+\eta_2\frac{\partial^2\psi}{\partial z^2}\right] \qquad (25)$$

In eq. 25 we have assumed that $\kappa$ is much smaller than $\eta_2 u_0$.

We may refer the flow field to a moving coordinate system in which:

$$\bar{z}=z \qquad (26)$$

$$\bar{x}=x-u_0 t \qquad (27)$$

This method was similarly applied by Prats (1966). However, as he ignored the flow field accelerations his analysis did not require the assumption that the solid fracture is a poor conductor. In his analysis the frame of coordinates moves with the velocity of the heat in the porous layer. In our analysis it moves with the steady state barycentric flow velocity.

Substitution of eq. 26–27 in eq. 24–25 yields:

$$\frac{1}{\epsilon}\frac{\partial}{\partial t}\bar{\nabla}^2\psi=-\frac{\alpha g}{\epsilon}\frac{\partial\theta}{\partial\bar{x}}-\frac{\nu}{K}\bar{\nabla}^2\psi \qquad (28)$$

$$\frac{\partial\theta}{\partial t}=-\frac{\partial\psi}{\partial\bar{x}}\frac{\Delta T}{d}+\eta_1 u_0\frac{\partial^2\theta}{\partial\bar{x}^2}+\eta_2 u_0\frac{\partial^2\theta}{\partial\bar{z}^2}$$

$$-\frac{\Delta T}{d}\left[-(\eta_1-\eta_2)\frac{\partial^2\psi}{\partial\bar{x}^2}+\eta_2\frac{\partial^2\psi}{\partial\bar{z}^2}\right] \qquad (29)$$

where:

$$\bar{\nabla}^2=\frac{\partial^2}{\partial\bar{x}^2}+\frac{\partial^2}{\partial\bar{z}^2}$$

We define dimensionless variables as follows:

$$\Psi_1=\frac{K}{\nu d^2}\psi \qquad \Theta_1=\frac{K\eta_2 u_0}{\nu\Delta T d^2}\theta$$

$$\zeta=\bar{z}/d \qquad \xi=\bar{x}/d \qquad \tau=\frac{\nu}{K}t \qquad (30)$$



Substituting these variable

$$\frac{1}{\epsilon}\frac{\partial}{\partial\tau}\nabla^2\Psi_1=-R\frac{\partial}{\partial\xi}\Theta_1$$

$$\frac{1}{r}\frac{\partial}{\partial\tau}\Theta_1=-\frac{\partial\Psi_1}{\partial\xi}+\frac{\eta_1}{\eta_2}\ldots$$

$$+\frac{\eta_1-\eta_2}{d}\ldots$$

where:

$$\nabla^2=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\zeta^2} \qquad R=\ldots$$

The parameter $R$ is Rayle... flow field perturbations m...

$$\Psi_1=\Psi(\zeta)\exp[ik\xi+\sigma\tau\ldots$$

$$\Theta_1=\Theta(\zeta)\exp[ik\xi+\sigma\tau\ldots$$

In eq. 34 and 35 $k$ is the h...

$$\sigma=\sigma_r+i\omega$$

where $\sigma_r$ expresses amplific... perturbation oscillations. ...

From eq. 31, 32, 34 and...

$$(1+\sigma/\epsilon)(D^2-k^2)\Psi=\ldots$$

$$(D^2-l^2-\sigma/r)\Theta=ik\Psi$$

where:

$$l=k\sqrt{\eta_1/\eta_2} \qquad D=\ldots$$

The boundary condition...

$\eta_2 u_0 \frac{\partial^2 \theta}{\partial z^2}$

$\frac{\partial^2\psi}{\partial x^2} + \eta_2 \frac{\partial^2 \psi}{\partial z^2}\Big]$ (25)

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Substituting these variables in eq. 28 and 29 we obtain:

$$\frac{1}{\epsilon}\frac{\partial}{\partial \tau}\nabla^2\Psi_1 = -R\frac{\partial}{\partial \xi}\Theta_1 - \nabla^2\Psi_1 \tag{31}$$

$$\frac{1}{r}\frac{\partial}{\partial \tau}\Theta_1 = -\frac{\partial \Psi_1}{\partial \xi} + \frac{\eta_1}{\eta_2}\frac{\partial^2\Theta_1}{\partial \xi^2} + \frac{\partial^2\Theta_1}{\partial \zeta^2} + \frac{\eta_1 - \eta_2}{d}\frac{\partial^2\Psi_1}{\partial \xi^2} - \frac{\eta_2}{d}\frac{\partial^2\Psi_1}{\partial \zeta^2} \tag{32}$$

where:

$$\nabla^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \zeta^2} \qquad R = \frac{\alpha g \Delta T d}{\eta_2 u_0 \epsilon \nu} \qquad r = \frac{K\eta_2 u_0}{\nu d^2} \tag{33}$$

The parameter $R$ is Rayleigh number; $r$ is a modified Prandtl number. The flow field perturbations may be expanded as follows:

$$\Psi_1 = \Psi(\zeta)\exp[ik\xi + \sigma\tau] \tag{34}$$

$$\Theta_1 = \Theta(\zeta)\exp[ik\xi + \sigma\tau] \tag{35}$$

In eq. 34 and 35 $k$ is the horizontal wave number; $\sigma$ is a complex number:

$$\sigma = \sigma_r + i\omega \tag{36}$$

where $\sigma_r$ expresses amplification of the flow field perturbations; $\omega$ expresses perturbation oscillations.

From eq. 31, 32, 34 and 35 we obtain:

$$(1 + \sigma/\epsilon)(D^2 - k^2)\Psi = -ikR\Theta \tag{37}$$

$$(D^2 - l^2 - \sigma/r)\Theta = ik\Psi + k^2\frac{\eta_1 - \eta_2}{d}\Psi + \frac{\eta_2}{d}D^2\Psi \tag{38}$$

where:

$$l = k\sqrt{\eta_1/\eta_2} \qquad D = \frac{d}{d\zeta}$$

The boundary conditions of impervious boundaries at constant temperature yield:

$$\Psi, \Theta = 0 \quad \text{at} \quad \zeta = 0,1 \tag{40}$$

From eq. 37 and 38 we obtain the following ordinary differential equation:

$$(D^2 - l^2 - \sigma/r)(1 + \sigma/\epsilon)(D^2 - k^2)\Psi$$

$$- k^2 R\,\Psi + ik^3 R\,\frac{\eta_1 - \eta_2}{d}\,\Psi + ikR\,\frac{\eta_2}{d}\,D^2\Psi = 0 \tag{41}$$

This is the differential equation of thermal instability of flow in a porous medium layer.

According to eq. 37–40 the boundary conditions of this equation are:

$$\Psi = 0 \text{ and } \nabla^2\Psi = 0 \text{ at } \zeta = 0.1 \tag{42}$$

STABILITY ANALYSIS

The instability condition is characterized by $\sigma_r = 0$ in eq. 36. Therefore, in this case we get from eq. 41 after separating real and imaginary parts:

$$(D^2 - l^2)(D^2 - k^2)\Psi + \frac{\omega^2}{\epsilon r}(D^2 - k^2)\Psi - k^2 R\Psi = 0 \tag{43}$$

$$\frac{\omega}{\epsilon}(D^2 - l^2)(D^2 - k^2)\Psi - \frac{\omega}{r}(D^2 - k^2)\Psi$$

$$+ k^3 R\,\frac{\eta_1 - \eta_2}{d} + kR\,\frac{\eta_2}{d}\,D^2\Psi = 0 \tag{44}$$

Eq. 43 and 44 with the boundary conditions 42 form a linear eigenvalue problem.

It is possible to solve the set of eq. 43 and 44 and to substitute the boundary conditions as Eliasson (1971) did. However, a simpler variational approach is to assume that $\Psi$ may be expressed by a sine series which fulfils the boundary conditions (Chandrasekhar, 1961; Nield, 1968). In this method we obtain an independence between the various sine modes. The lowest mode of instability requires:



$\Psi = \sin\pi\zeta$

By substituting eq. 45 in ec

$(\pi^2 + l^2)(\pi^2 + k^2) - \frac{\omega^2}{\epsilon r}$

$\frac{\omega}{\epsilon}(\pi^2 + l^2)(\pi^2 + k^2) +$

According to eq. 46 and then $R$ should get complex stability (Chandrasekhar, 19 quired by the two last terms tensor perturbation. Such a motions (internal waves) wh steady state barycentric flow

By eliminating $\omega$ from ec equation for $R$:

$aR^2 + bR + q = 0$

where:

$a = \frac{(-k^3 \frac{\eta_1 - \eta_2}{d} + k\frac{\eta_2}{d}}{(\pi^2 + k^2)[\frac{1}{\epsilon}(\pi^2 + l^2) -}$

$b = \frac{\epsilon r k^2}{\pi^2 + k^2} \qquad q = -\epsilon r(\pi$

The critical Rayleigh num fies eq. 48; $a$ is very small, its about $10^{-2}$; $r$ is about $10^{-6}$ mated by:

$R = -\frac{q}{b} - \frac{aq^2}{b^3} + \ldots$

The last term in eq. 50 is e

$R \simeq \frac{(\pi^2 + l^2)(\pi^2 + k^2)}{k^2}$

$$\Psi = \sin \pi \zeta \tag{45}$$

By substituting eq. 45 in eq. 43 and 44 we obtain:

$$(\pi^2 + l^2)(\pi^2 + k^2) - \frac{\omega^2}{\epsilon r}(\pi^2 + k^2) - k^2 R = 0 \tag{46}$$

$$\frac{\omega}{\epsilon}(\pi^2 + l^2)(\pi^2 + k^2) + \frac{\omega}{r}(\pi^2 + k^2) + k^3 R \frac{\eta_1 - \eta_2}{d} - \pi^2 k R \frac{\eta_2}{d} = 0 \tag{47}$$

According to eq. 46 and 47 $\omega$ cannot vanish in the case of instability. As then $R$ should get complex values. Therefore, no possibility of true marginal stability (Chandrasekhar, 1961) does exist. Non vanishing value of $\omega$ is required by the two last terms in eq. 47. These terms result from the dispersion tensor perturbation. Such a case of instability is characterized by overstable motions (internal waves) which are observed by an observer moving with the steady state barycentric flow velocity.

By eliminating $\omega$ from eq. 46 and 47 we get the following second order equation for $R$:

$$aR^2 + bR + q = 0 \tag{48}$$

where:

$$a = \frac{\left(-k^3 \frac{\eta_1 - \eta_2}{d} + k \frac{\eta_2}{d} \pi^2\right)}{(\pi^2 + k^2)\left[\frac{1}{\epsilon}(\pi^2 + l^2) - \frac{1}{r}\right]^2}$$

$$b = \frac{\epsilon r k^2}{\pi^2 + k^2} \qquad q = -\epsilon r(\pi^2 + l^2) \tag{49}$$

The critical Rayleigh number ($R_c$) is the minimum value of $R$ which satisfies eq. 48; $a$ is very small, its order of magnitude is about $10^{-12}$ (as $\eta_1/d$ is about $10^{-2}$; $r$ is about $10^{-6}$; $\epsilon$ is about $10^{-1}$). Therefore, $R$ may be approximated by:

$$R = -\frac{q}{b} - \frac{aq^2}{b^3} + \ldots \tag{50}$$

The last term in eq. 50 is extremely small and negligible, thus:

$$R \simeq \frac{(\pi^2 + l^2)(\pi^2 + k^2)}{k^2} \tag{51}$$

Eq. 51 is the exact solution for $R$ if the last two terms in eq. 47 are neglected, namely, when neglecting the effect of the dispersion tensor perturbation. Thus the dispersion tensor perturbation hardly effects the criterion of instability, but it induces overstable motions in the flow field.

According to eq. 51 the critical wave number (which is related to the critical Rayleigh number) is:

$$k_c = \pi(\eta_2/\eta_1)^{1/4} \tag{52}$$

The critical Rayleigh number is:

$$R_c = \pi^2[1+(\eta_1/\eta_2)^{1/2}]^2 \tag{53}$$

An increase in the horizontal flow velocity in the steady state reduces Rayleigh number of the field and thus stabilizes the flow field. Moreover it increases the ratio $\eta_1/\eta_2$. Therefore it also increases the critical Rayleigh number of the flow field.

DISCUSSION

The length ($L$) of Benard cells (the convection cells) is connected with the value of $k_c$ by the following equation:

$$L = \frac{\pi}{k_c} d = (\eta_1/\eta_2)^{1/4}\, d \tag{54}$$

Hence the length of Benard cells increases when the ratio between the longitudinal and lateral dispersion coefficients increases.

According to eq. 34:

$$\psi_1 = A_1 \sin[\pi(\eta_2/\eta_1)^{1/4}\xi]\sin\pi\zeta\cos\omega\tau \tag{55}$$

where $A_1$ is a constant. The total stream function for the case of convection currents is:

$$\psi = u_0 z + A\sin\left[\frac{\pi}{d}\left(\frac{\eta_2}{\eta_1}\right)^{1/4}(x-u_0 t)\right]\sin\frac{\pi z}{d}\cos\left(\frac{\omega\nu}{K}t\right) \tag{56}$$

where $A$ is a constant.

According to this equation we analyze the effect of instability on the flow



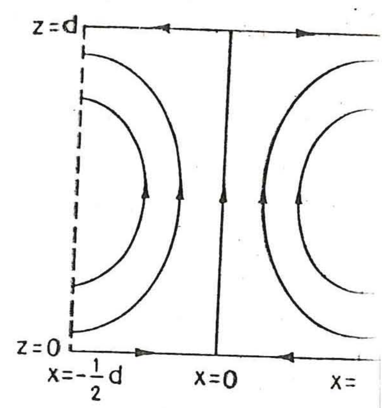


Fig. 2. Convection currents when $u_0$ =

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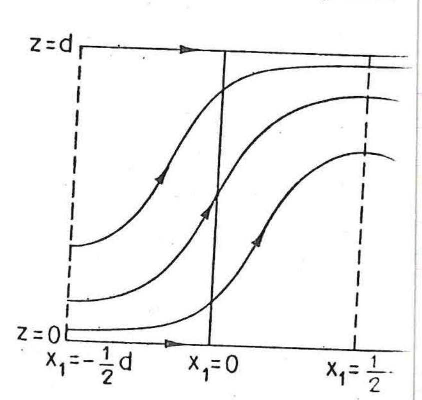


Fig. 3. Flow field pattern when $u_0$ equals

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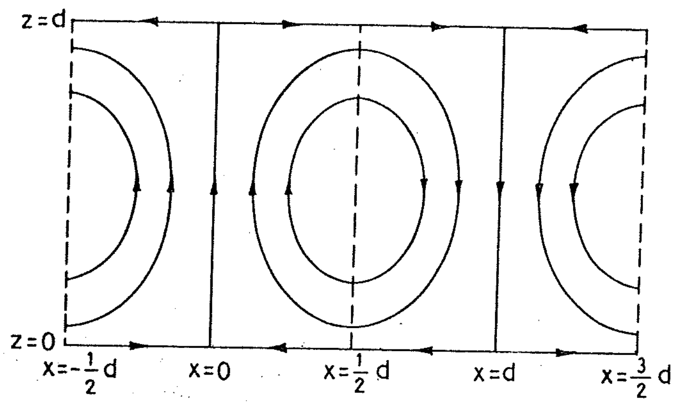


Fig. 2. Convection currents when $u_0 = 0$.

field pattern. In the absence of a steady horizontal flow ($u_0 = 0$), $\eta_1 = \eta_2$ and $\omega = 0$, then eq. 56 yields the flow pattern presented in Fig. 2. When $u_0 \neq 0$, the total velocity in the horizontal direction consists of a constant term, namely, $u_0$ and an oscillatory term due to the convection currents. The magnitude of this term changes with time and location.

In Fig. 3 a hypothetical flow pattern for $t = 0$ is presented. This figure is related to a field in which $u_0$ equals to the amplitude of the oscillatory term. A new horizontal coordinate appears in this figure:

$$x_1 = x(\eta_2/\eta_1)^{1/4} \tag{57}$$

Actually we cannot estimate the magnitude of the oscillatory term according to the linear stability analysis presented in this article. Such an estimate requires nonlinear stability analysis.

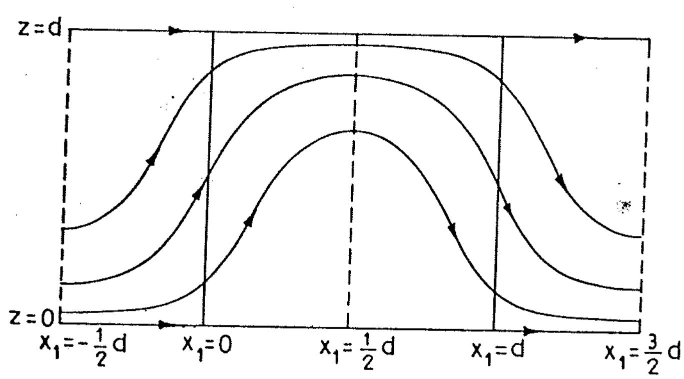


Fig. 3. Flow field pattern when $u_0$ equals to the convection velocity amplitude.

## SUMMARY AND CONCLUSIONS

The linear stability analysis can be applied for evaluating the effect of heat dispersion on the thermal stability of the flow field in porous medium.

If the horizontal flow velocity is comparatively large, the expressions for the dispersion tensor perturbation may be simplified with the aid of the velocity perturbation.

Instability of the flow field is affected by the laterl as well as by the longitudinal dispersion coefficients. The critical Rayleigh number and the length of the convection cells increase when the ratio between the longitudinal and the lateral dispersivities increases.

The convection currents appear as oscillations in the flow field. The frequency of these oscillations depends on the barycentric steady state horizontal flow velocity, the ratio between the longitudinal and lateral dispersivities and the aquifer thickness.

The dispersion tensor perturbation scarcely affects the criteria of instability but it leads to oscillations in the flow field which may be called "overstable motions" being observed by an observer who moves with the steady state barycentric flow velocity.

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