Temperature Distribution in Circulating Mud Columns

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ABSTRACT

A model describing the two-dimensional transient heat transfer in and around a wellbore is developed. The model considers the fluid flowing down a drill string and returning up the annulus. Calculated results show that the use of steady-state solutions previously published give good estimates of circulating mud temperatures. The transient solution presented here is more suited for matching temperature logs. The viscous flow energy, rotational energy, and drill bit energy were found to be significant items in the energy balance and are included in the model.

Calculated vertical temperature distributions in the innermost casing string permit the determination of the average temperature increase in that pipe. This quantity may then be used to compute thermal stresses and to predict casing stability.

INTRODUCTION

In drilling deep wells (15,000 to 30,000 ft) the geothermal temperatures encountered can cause problems with drilling fluids, drill pipe, and casing. To evaluate the effects of these high temperatures on the drill pipe and casing, it is necessary to know the temperature distributions in these pipe strings.

Previous work includes the approximate model described by Edwardson *et al.*[1] for estimating formation temperature disturbances resulting from mud circulation. This work formed the basis for the calculation method that Crawford *et al.*[2] proposed for estimating mud temperatures. Ramey[3] proposed a model for solution of the wellbore heat transmission problem. He assumed that heat transfer in the wellbore is steady-state while heat transfer in the earth is due to transient radial heat conduction. Ramey's model gave results comparable to the more exact method of Squier *et al.*[4] and formed the basis for the solution of other wellbore circulation problems.[5,6]

Two recent treatments of the mud circulation problem[7,8] are based on models that assume steady-state conditions in both the wellbore and the surrounding earth. The treatment of this problem by Raymond[9] is very good with the exception that he neglected the presence of the casing strings and the effects of energy sources in the system.

The prediction of these temperature distributions can best be accomplished with a two-dimensional thermal model that accounts for the dynamic flow of mud down the drill pipe and back up through the annulus around the drill pipe, with appropriate heat interchange by convection and conduction. Such a model is described in this paper.

DESCRIPTION OF THE PROBLEM

This problem consists of determining the temperature distribution in and around a wellbore during the drilling operations on that well. Drilling fluids are pumped down the drill pipe and recirculated up the annulus surrounding the drill pipe. Energy is added to the fluid columns by the frictional flow losses in the drill pipe and annulus, the shear work done in rotating the drill string, and frictional work at the drill bit.

Since the temperature in the earth's crust increases with depth, the drill fluids encounter increasingly higher temperatures with increased depth. This heated fluid then flows to the surface and tends to heat the casing as it passes through it. In deep wells this casing heating can be sufficient to cause excessive thermal stresses and result in pipe buckling. Casing stability predictions can be made if the average temperature increase in the pipe is known.[10] A method for calculating this quantity is presented here.

Fig. 1 is a representation of the system considered in this model. We assume that fluid enters the drill string at the surface at a constant rate and known temperature. The fluid flowing down the drill pipe has a vertical temperature distribution resulting from convective heat transfer within the fluid, heat generated by fluid friction, and heat exchange with the drill pipe. A vertical temperature distribution is calculated for the drill pipe by accounting for the vertical conduction of heat in the pipe and the convective heat exchange between the pipe and the fluid columns surrounding it. An energy balance for the drill fluids in the annulus accounts for the convective heat transfer within the fluid, heat

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[1]References given at end of paper.

generated by fluid friction and pipe rotation, and convective heat exchange between the fluid, and the drill pipe and casing. The vertical temperature distribution in the casing string is calculated from an energy balance equation that accounts for the vertical conduction of heat in the casing, the convective exchange of heat with the drill fluid inside the casing, and the conduction of heat between the casing and the annulus surrounding it. Energy balance equations are written for three additional radial increments that are based on the sizes of the casing strings present. Provision is made for another three radial increments of arbitrary size. In these six outer increments, the flow of heat is considered to be by conduction only in both the vertical and radial directions.

MATHEMATICAL MODEL

The description of the problem presented above leads to the following finite difference equations. The energy balance inside the drill string is given by:

$$\dot{Q}_1 - q_m \rho_m C_m \frac{\left(T^{N+1}_{1,j} - T^{N+1}_{1,j-1}\right)}{\Delta Z_j} - 2\pi r_1 h_i \left(T^{N+1}_{1,j} - T^{N+1}_{2,j}\right) = \pi r_1^2 \rho_m C_m \frac{\left(T^{N+1}_{1,j} - T^{N}_{1,j}\right)}{\Delta t} \quad \ldots\ldots (1)$$

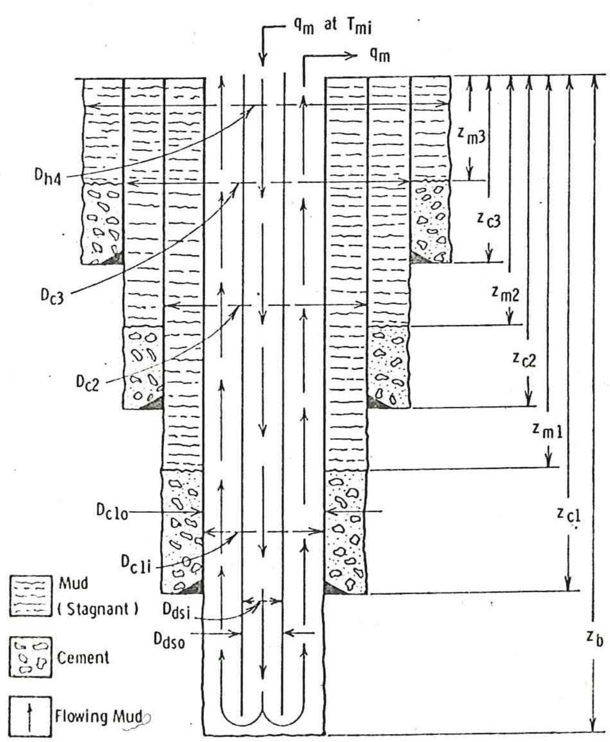

FIG. 1—SCHEMATIC DIAGRAM OF WELL COMPLETION.

The terms on the left represent, respectively, the energy source inside the drill string, the vertical convective heat transport, and the radial transfer of heat from the fluid to the drill string. The right-hand term represents the accumulation of energy in an element of the fluid inside the drill string. Vertical heat conduction in the fluid is neglected. The energy source term, $\dot{Q}_1$, accounts for fluid friction losses inside the drill pipe and is constant for all points except at the bottom of the hole. At this point additional heat may be added due to pressure losses through the bit and the work done by the bit.

The energy balance in the drill string is expressed by

$$\frac{k_{hd}}{\Delta z_j}\left[\frac{T^{N+1}_{2,j+1} - T^{N+1}_{2,j}}{\Delta z_{j+\frac{1}{2}}} - \frac{T^{N+1}_{2,j} - T^{N+1}_{2,j-1}}{\Delta z_{j-\frac{1}{2}}}\right] + \frac{r_1 h_i}{r_2 \Delta r_2}\left(T^{N+1}_{1,j} - T^{N+1}_{2,j}\right) - \frac{r_{2p} h_e}{r_2 \Delta r_2}\left(T^{N+1}_{2,j} - T^{N+1}_{3,j}\right) = \rho_d C_d \frac{\left(T^{N+1}_{2,j} - T^{N}_{2,j}\right)}{\Delta t} \quad \ldots\ldots (2)$$

The first term on the left-hand side of Eq. 2 accounts for the vertical conduction of heat in the drill pipe. The second and third terms represent the radial exchange of heat between the drill pipe and the fluid inside and outside the string. The right-hand term again represents the accumulation term. Terms in the following equations have similar physical significance.

The energy balance in the flow annulus is given by

$$\dot{Q}_2 + q_m \rho_m C_m \frac{\left(T^{N+1}_{3,j+1} - T^{N+1}_{3,j}\right)}{\Delta z_j} + 2\pi r_{2p} h_e \left(T^{N+1}_{2,j} - T^{N+1}_{3,j}\right) - 2\pi r_{3p} h_c \left(T^{N+1}_{3,j} - T^{N+1}_{4,j}\right) = 2\pi r_3 \Delta r_3 \rho_m C_m \frac{\left(T^{N+1}_{3,j} - T^{N}_{3,j}\right)}{\Delta t} \quad \ldots (3)$$

The energy source term, $\dot{Q}_2$, accounts for fluid friction losses in the annulus and the work required to rotate the drill string. It is uniform for all points.

The energy balance in the first casing string is:

$$\frac{k_{hs}}{\Delta z_j}\left[\frac{T^{N+1}_{4,j+1} - T^{N+1}_{4,j}}{\Delta z_{j+\frac{1}{2}}} - \frac{T^{N+1}_{4,j} - T^{N+1}_{4,j-1}}{\Delta z_{j-\frac{1}{2}}}\right]$$

$$+ \frac{r_{3p} h_c}{r_4 \Delta r_4}\left(T^{N+1}_{3,j} - T^{N+1}_{4,j}\right) - \frac{K_{45,j}}{r_4 \Delta r_4}\left(T^{N+1}_{4,j} - T^{N+1}_{5,j}\right) = \rho_s C_s \frac{\left(T^{N+1}_{4,j} - T^{N}_{4,j}\right)}{\Delta t} \quad (4)$$

For $z(j) \leq z_{c1}$:

$$k_{hs} = k_{hst} \; ; \; \rho_s = \rho_{st} \; ; \; C_s = C_{st}$$

For $z(j) > z_{c1}$:

$$k_{hs} = k_{he} \; ; \; \rho_s = \rho_e \; ; \; C_s = C_e$$

For the second annulus, the energy balance is:

$$k_{h2,j+\frac{1}{2}} \frac{\left(T^{N+1}_{5,j+1} - T^{N+1}_{5,j}\right)}{\Delta z_j \Delta z_{j+\frac{1}{2}}} - k_{h2,j-\frac{1}{2}} \cdot \frac{\left(T^{N+1}_{5,j} - T^{N+1}_{5,j-1}\right)}{\Delta z_j \Delta z_{j-\frac{1}{2}}} + \frac{K_{45,j}}{r_5 \Delta r_5}\left(T^{N+1}_{4,j} - T^{N+1}_{5,j}\right) - \frac{K_{56,j}}{r_5 \Delta r_5}\left(T^{N+1}_{5,j} - T^{N+1}_{6,j}\right) = \rho_2 C_2 \frac{\left(T^{N+1}_{5,j} - T^{N}_{5,j}\right)}{\Delta t} \quad \ldots . (5)$$

For $z(j) \leq z_{m1}$:

$$k_{h2,j} = k_{h2m} \; ; \; \rho_2 = \rho_{2m} ;$$

$$C_2 = C_{2m}$$

For $z_{m1} < z(j) \leq z_{c1}$:

$$k_{h2,j} = k_{hc} \; ; \; \rho_2 = \rho_c \; ; \; C_2 = C_c$$

For $z(j) > z_{c1}$:

$$k_{h2,j} = k_{he} \; ; \; \rho_2 = \rho_e \; ; \; C_2 = C_e$$

Similar evaluations of physical properties were used in the third and fourth annuli as given below.

For the third annulus, the energy balance is:

$$k_{h3,j+\frac{1}{2}} \frac{\left(T^{N+1}_{6,j+1} - T^{N+1}_{6,j}\right)}{\Delta z_j \Delta z_{j+\frac{1}{2}}} - k_{h3,j-\frac{1}{2}} \cdot \frac{\left(T^{N+1}_{6,j} - T^{N+1}_{6,j-1}\right)}{\Delta z_j \Delta z_{j-\frac{1}{2}}} + \frac{K_{56,j}}{r_6 \Delta r_6}\left(T^{N+1}_{5,j} - T^{N+1}_{6,j}\right) - \frac{K_{67,j}}{r_6 \Delta r_6}\left(T^{N+1}_{6,j} - T^{N+1}_{7,j}\right) = \rho_3 C_3 \frac{\left(T^{N+1}_{6,j} - T^{N}_{6,j}\right)}{\Delta t} \quad \ldots . (6)$$

For the fourth annulus, the energy balance is:

$$k_{h4,j+\frac{1}{2}} \frac{\left(T^{N+1}_{7,j+1} - T^{N+1}_{7,j}\right)}{\Delta z_j \Delta z_{j+\frac{1}{2}}} - k_{h4,j-\frac{1}{2}} \cdot \frac{\left(T^{N+1}_{7,j} - T^{N+1}_{7,j-1}\right)}{\Delta z_j \Delta z_{j-\frac{1}{2}}} + \frac{K_{67,j}}{r_7 \Delta r_7}\left(T^{N+1}_{6,j} - T^{N+1}_{7,j}\right) - \frac{K_{78,j}}{r_7 \Delta r_7}\left(T^{N+1}_{7,j} - T^{N+1}_{8,j}\right) = \rho_4 C_4 \frac{\left(T^{N+1}_{7,j} - T^{N}_{7,j}\right)}{\Delta t} \quad \ldots (7)$$

The energy balance in the earth is given by:

$$\frac{k_{he}}{\Delta z_j}\left[\frac{T^{N+1}_{i,j+1} - T^{N+1}_{i,j}}{\Delta z_{j+\frac{1}{2}}} - \frac{T^{N+1}_{i,j} - T^{N+1}_{i,j-1}}{\Delta z_{j-\frac{1}{2}}}\right] + \frac{K_{im}}{r_i \Delta r_i}\left(T^{N+1}_{i-1,j} - T^{N+1}_{i,j}\right) - \frac{K_{ip}}{r_i \Delta r_i}\left(T^{N+1}_{i,j} - T^{N+1}_{i+1,j}\right) = \rho_e C_e \frac{\left(T^{N+1}_{i,j} - T^{N}_{i,j}\right)}{\Delta t} \quad . (8)$$

The boundary and initial conditions are given by:

$$T^{N}_{1,0} = T_{mi}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0$$

$$T^{N}_{i,JMX} = T_s + m z_{JMX}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_e} = 0$$

$$T^{1}_{i,j} = T_s + m z_j$$

The second condition describes the assumption of no heat flow between the earth's surface and the atmosphere. The fourth condition is used to check that r_e is sufficiently large that the temperature at r_e will not change during the time of interest.

The geometric and thermal quantities appearing in the equations above are defined as follows.

For $z(j) \leq z_{dc}$:

$$r_1 = \frac{D_{dsi}}{2} \; ; \quad r_{2p} = \frac{D_{dso}}{2} \; ; \quad \Delta r_2 = \frac{D_{dso} - D_{dsi}}{2} \; ; \quad \Delta r_3 = \frac{D_{cli} - D_{dso}}{2}$$

$$k_{hd} = k_{hdp} \; ; \rho_d = \rho_{dp} \; ; \quad C_d = C_{dp}$$

For $z(j) > z_{dc}$:

$$r_i = \frac{D_{dci}}{2} \; ; \quad r_{2p} = \frac{D_{dco}}{2} \; ;$$

$$\Delta r_2 = \frac{D_{dco} - D_{dci}}{2} \; ;$$

$$\Delta r_3 = \frac{D_{clo} - D_{dco}}{2}$$

$$k_{hd} = k_{hdc} \; ; \rho_d = \rho_{dc} ; \quad C_d = C_{dc}$$

For all $z(j)$:

$$r_2 = \frac{r_1 + r_{2p}}{2}$$

$$r_3 = \frac{r_{2p} + r_{3p}}{2}$$

$$r_4 = \frac{D_{cli} + D_{clo}}{4}$$

$$r_i = r_{i-1} + \frac{\Delta r_{i-1} + \Delta r_i}{2}$$

$$r_{im} = r_i - \frac{\Delta r_i}{2}$$

$$r_{ip} = r_i + \frac{\Delta r_i}{2}$$

$$r_e = r_{IMX} + \frac{\Delta r_{IMX}}{2}$$

$$\Delta r_4 = \frac{D_{clo} - D_{cli}}{2}$$

$$\Delta r_5 = \frac{D_{c2} - D_{clo}}{2}$$

$$\Delta r_6 = \frac{D_{c3} - D_{c2}}{2}$$

$$\Delta r_7 = \frac{D_{h4} - D_{c3}}{2}$$

$$D_{c2} = \frac{D_{c2o} + D_{c2i}}{2}$$

$$D_{c3} = \frac{D_{c3o} + D_{c3i}}{2}$$

$$\frac{k_{hi,j+\frac{1}{2}}}{\Delta z_{j+\frac{1}{2}}} = \frac{2k_{hi,j}\, k_{hi,j+1}}{k_{hi,j}\Delta z_{j+1} + k_{hi,j+1}\Delta z_j}$$

$$\frac{k_{hi,j-\frac{1}{2}}}{\Delta z_{j-\frac{1}{2}}} = \frac{2k_{hi,j}\, k_{hi,j-1}}{k_{hi,j}\Delta z_{j-1} + k_{hi,j-1}\Delta z_j}$$

$$K_{45,j} = \frac{k_{hs,j}\, k_{h2,j}}{k_{hs,j} \ln \frac{r_5}{r_{5m}} + k_{h2,j} \ln \frac{r_{5m}}{r_4}}$$

$$K_{56,j} = \frac{k_{h2,j}\, k_{h3,j}}{k_{h2,j} \ln \frac{r_6}{r_{6m}} + k_{h3,j} \ln \frac{r_{6m}}{r_5}}$$

$$K_{67,j} = \frac{k_{h3,j}\, k_{h4,j}}{k_{h3,j} \ln \frac{r_7}{r_{7m}} + k_{h4,j} \ln \frac{r_{7m}}{r_6}}$$

$$K_{78,j} = \frac{k_{h4,j}\, k_{he}}{k_{h4,j} \ln \frac{r_8}{r_{8m}} + k_{he} \ln \frac{r_{8m}}{r_7}}$$

$$K_{im} = \frac{k_{he}}{\ln \frac{r_i}{r_{i-1}}}$$

$$K_{ip} = \frac{k_{he}}{\ln \frac{r_{i+1}}{r_i}}$$

For $i = IMAX$: $K_{ip} = 0$.

In calculating the average conduction coefficient between radial nodes, we have neglected the presence of the steel casing and considered only the conductivities of the adjacent annuli. The formulae given are the well known equations for conductances in series.

To calculate the convective heat transfer coefficients for turbulent flow, we have used the Seider-Tate[11] equation with the term $(\mu/\mu_w)^{0.14}$ set equal to 1. For laminar flow, we have used an asymptotic value of 4.12 for the Nusselt number.[12] Thus,

For $N_{Rei} \leq 2,100$:

$$h_i = \frac{(4.12)\ k_{hm}}{D_{dsi}}$$

For $N_{Rei} > 2,100$:

$$h_i = \frac{(0.027)\ (N_{Rei})^{0.8}\ (N_{Pr})^{0.333}\ k_{hm}}{D_{dsi}}$$

For $N_{Ree} \leq 2,100$:

$$h_e = \frac{(4.12)\ k_{hm}}{D_{dso}}\ ; \quad h_c = \frac{(4.12)\ k_{hm}}{D_{cli}}$$

For $N_{Ree} > 2,100$:

$$h_e = \frac{(0.027)\ (N_{Ree})^{0.8}\ (N_{Pr})^{0.333}\ k_{hm}}{D_{dso}}$$

$$h_c = \frac{(0.027)\ (N_{Ree})^{0.8}\ (N_{Pr})^{0.333}\ k_{hm}}{D_{cli}}$$

At the bottom of the drill string, it is necessary that $T_1 = T_2 = T_3$, due to mixing of the two mud columns. We arbitrarily multiplied the convective coefficients, h_i and h_e, by 10^8 at this point to satisfy this condition.

SOLUTION OF THE PROBLEM

The energy equations may be rearranged into the form:

$$A_{ij} T^{N+1}_{i,j-1} + AA_{ij} T^{N+1}_{i-1,j} + B_{ij} T^{N+1}_{i,j} + CC_{ij} T^{N+1}_{i+1,j} + C_{ij} T^{N+1}_{i,j+1} = d_{ij} \quad \text{. . . . . . (9)}$$

These simultaneous equations are then solved using a modified alternating direction line successive overrelaxation method. Eq. 9 is first rewritten as

$$A_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i,j-1}} + B_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i,j}} + C_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i,j+1}} = d_{ij} - AA_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i-1,j}} - CC_{ij} \underset{k}{T^{N+1}_{i+1,j}}, \quad \text{. . . . . . (10)}$$

where k denotes the iteration number. We now solve Eq. 10 for each successive value of i from $i = 1$ to $i = 3$ using Thomas' algorithm[13] at each column. Thus, we get

$$\underset{k+\frac{1}{2}}{T^{N+1}_{i,j}} = \gamma_{i,j} - \frac{C_{ij}\ T^{N+1}_{i,j+1}}{\beta_{i,j}},$$

where

$$\gamma_{i,j} = \frac{D_{i,j} - A_{ij}\ \gamma_{i,j-1}}{\beta_{i,j}}$$

$$\beta_{i,j} = B_{ij} - \frac{A_{ij}\ C_{i,j-1}}{\beta_{i,j-1}}$$

$$D_{i,j} = d_{ij} - AA_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i-1,j}} - CC_{ij} \underset{k}{T^{N+1}_{i+1,j}}$$

Next we rewrite Eq. 9 as

$$AA_{ij} \underset{k+1}{T^{N+1}_{i-1,j}} + B_{ij} \underset{k+1}{T^{N+1}_{i,j}} + CC_{ij} \underset{k+1}{T^{N+1}_{i+1,j}} = d_{ij} - A_{ij} \underset{k+1}{T^{N+1}_{i,j-1}} - C_{ij} \underset{k+\frac{1}{2}}{T^{N+1}_{i,j+1}} \quad \text{. . . . . . (11)}$$

This equation is then solved for each successive value of j from $j = 1$ to $j = JMAX - 1$, using Thomas' algorithm again at each row. Thus, we get

$$T_{i,j \atop k+1}^{N+1} = \gamma_{i,j} - \frac{CC_{ij} T_{i+1,j}^{N+1}}{\beta_{i,j}}.$$

Problems involving dynamic flow conditions generally exhibit numerical instabilities at early times unless very small time-step sizes are used. For this reason provision was made in our program to subdivide the first few time steps into smaller time steps. This was done after each trip cycle as well as at the beginning of the problem. This proved to be an effective means for eliminating numerical instabilities while utilizing reasonable time-step sizes. Table 1 presents a comparison of results obtained for various time-step sizes with and without this startup resizing.

CALCULATION OF AVERAGE CASING TEMPERATURE INCREASE

Having attained a temperature distribution in the first casing string at a particular time step, we can calculate the average temperature increase in the casing string. We assume that the casing was initially at the same temperature as the geothermal temperature in the earth. Thus, we can calculate

$$\Delta T_{4,j}^{N+1} = T_{4,j}^{N+1} - T_{4,j}^{0}.$$

Let $z(JM1) = z_{m1}$ and then calculate

$$\overline{\Delta T}_{4}^{N+1} = \frac{\sum_{j=1}^{JM1} \Delta T_{4,j}^{N+1}}{JM1}$$

This is the average temperature increase in the casing string above the cement top and is the quantity required for casing stability calculations.

RESULTS

Since there are two presumably significant physical features included in this model that have not been considered in previous models, i.e., energy sources and transient heat transfer, we have made calculations aimed at evaluating their effect.

The data of Holmes and Swift, Table 2, were used to calculate our curves on Fig. 2 with the exception that h_e and h_c were 7.5 and 5.9 Btu/hr-sq ft-°F, respectively. Our curve showing the temperature after 144 hours of drilling is essentially the steady-state solution. The agreement of this curve with the steady-state curve of Holmes and Swift indicates that both models are formulated correctly. One might presume then that the steady-state solution is adequate. However, the temperature distributions for earlier times indicate that a higher temperature exists early in the drilling cycle. Also, it is convenient to have a transient solution available for matching temperature logs during nondrilling periods.

The effect of including heat in the calculations is shown in Fig. 3. The lower curve gives the temperature distribution at 24 hours for no energy sources. This curve was calculated using the calculated value of 126.5 Btu/hr-sq ft-°F for the convective heat transfer inside the drill string. The center curve was calculated with a uniform energy source of 8.5 Btu/hr-ft inside the drill string and a uniform source of 16.4 Btu/hr-ft in the annulus. The upper curve was calculated using the same energy sources used for the center curve

TABLE 1 — BOTTOM-HOLE TEMPERATURE

Time (hours)	0.5	1.0	2.0	3.0	6.0
	With Start-Up Time-Step Resizing				
	Time-Step Size (hours)				
6.0	190.9	191.5	192.4	193.0	192.9
12.0	186.1	186.3	186.4	186.9	187.4
18.0	184.1	184.2	184.0	184.6	185.0
24.0	182.8	183.0	182.4	183.2	183.5
	Without Start-Up Time-Step Resizing				
6.0	190.9	191.7	193.5	195.5	201.3
12.0	186.0	186.2	186.6	187.1	188.9
18.0	183.7	183.8	184.0	184.2	185.0
24.0	182.1	182.3	182.4	182.6	183.0

TABLE 2 — WELL AND MUD PROPERTIES

Well depth, ft	15,000
Drillstem OD, in.	6-5/8
Drill-bit size, in.	8-3/8
Circulation rate, bbl/hour	300
Inlet mud temperature, °F	75
Mud viscosity, (lb/ft-hour)	110
Mud thermal conductivity, Btu/(ft°F-hour)	1.0
Mud specific heat, Btu/(lb°F)	0.4
Mud density, lb/gal	10.0
Formation thermal conductivity, Btu/(ft°F-hour)	1.3
Formation specific heat, Btu/(lb°F)	0.2
Formation density, lb/cu ft	165
Surface earth temperature, °F	59.5
Geothermal gradient, °F/ft	0.0127

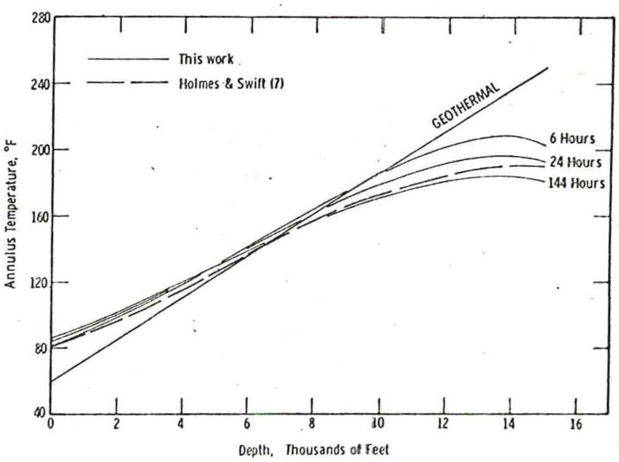

FIG. 2 — CALCULATED ANNULUS TEMPERATURE.

plus a source of 572,625 Btu/hr at the bottom of the hole. The magnitude of the difference between the curves with and without energy sources obviously should not be ignored.

The pump energy sources used were calculated from pressure drop data read from charts. These data indicated that 20 percent of the pump energy would be dissipated in the drill string, 8.5 percent in the annulus, and 70 percent across the drill bit. The mud rate of 210 gal/min and total pressure drop of 2,000 psi indicated a total pump input of 250 hp. In addition we assumed a rotary input of 125 hp divided 60 percent-40 percent between the annulus and bit, respectively. This then gave energy sources of 50 hp in the drill pipe, 96.5 hp in the annulus, and 225 hp at the drill bit.

These results were obtained using 10 radial increments and 153 vertical grid points, and a time step of 2 hours. The computing time required on a CDC 6400 is 170 seconds for 144 hours of real time.

CONCLUSIONS

From the foregoing results we make the following conclusions.

1. The energy source terms are quite significant in that they have a marked effect on the computed temperature distribution.
2. A transient solution can be useful, especially in matching temperature logs during no-flow cycles.
3. This model provides for multiple casing strings in the completion layout; this provides flexibility not available in previous models.
4. There is some uncertainty in using the heat transfer correlations cited, but lack of actual field data necessitates some assumptions in this regard.
5. The assumed initial casing temperature is arbitrary, but since the calculated average temperature increase will be at a maximum with this assumption, it appears reasonable.

NOMENCLATURE

C_c = heat capacity of cement, Btu/lb-°F

C_{dc} = heat capacity of drill collars, Btu/lb-°F

C_{dp} = heat capacity of drill string, Btu/lb-°F

C_e = heat capacity of earth, Btu/lb-°F

C_m = heat capacity of drill fluid, Btu/lb-°F

C_s = heat capacity of fourth radial element, Btu/lb-°F

C_{st} = heat capacity of steel, Btu/lb-°F

C_2 = heat capacity of second* annulus, Btu/lb-°F

C_{2m} = heat capacity of mud in second annulus, Btu/lb-°F

C_3 = heat capacity of third annulus, Btu/lb-°F

C_{3m} = heat capacity of mud in third annulus, Btu/lb-°F

C_4 = heat capacity of fourth annulus, Btu/lb-°F

C_{4m} = heat capacity of mud in fourth annulus, Btu/lb-°F

D_{c1i} = inside diameter of first casing string, ft

D_{c1o} = outside diameter of first casing string, ft

D_{c2i} = inside diameter of second casing string, ft

D_{c2o} = outside diameter of second casing string, ft

D_{c3i} = inside diameter of third casing string, ft

D_{c3o} = outside diameter of third casing string, ft

D_{dci} = inside diameter of drill collars, ft

D_{dco} = outside diameter of drill collars, ft

D_{dsi} = inside diameter of drill string, ft

D_{dso} = outside diameter of drill string, ft

D_{h4} = diameter of fourth hole, ft

h_c = convective coefficient inside first casing string, Btu/hr-sq ft-°F

h_i = convective coefficient inside drill string, Btu/hr-sq ft-°F

h_e = convective coefficient outside drill string, Btu/hr-sq ft-°F

$IMAX$ = total number of radial increments

$JMAX$ = total number of vertical grid points

k_{hc} = thermal conductivity of cement, Btu/hr-ft-°F

k_{hdc} = thermal conductivity of drill collar, Btu/hr-ft-°F

k_{hdp} = thermal conductivity of drill string, Btu/hr-ft-°F

k_{he} = thermal conductivity of earth, Btu/hr-ft-°F

k_{hm} = thermal conductivity of drill fluid, Btu/hr-ft-°F

k_{hst} = thermal conductivity of steel, Btu/hr-ft-°F

k_{h2m} = thermal conductivity of mud in second annulus, Btu/hr-ft-°F

k_{h3m} = thermal conductivity of mud in third annulus, Btu/hr-ft-°F

k_{h4m} = thermal conductivity of mud in fourth annulus, Btu/hr-ft-°F

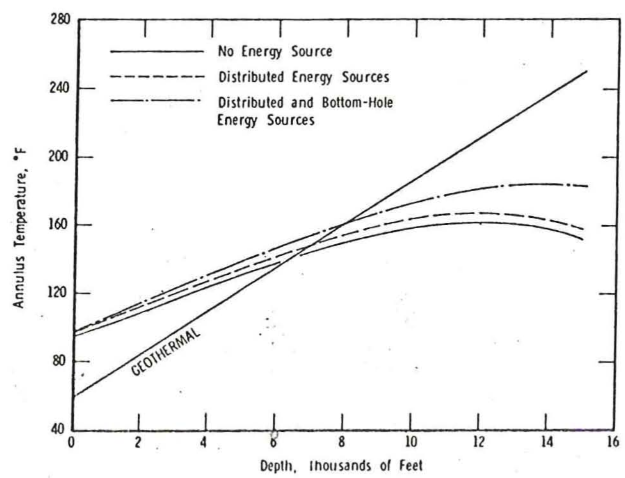

FIG. 3 — EFFECT OF ENERGY SOURCES ON ANNULUS TEMPERATURE.

*Note that numbering proceeds outward from the drill string.

m = geothermal gradient, °F/ft
N_{Pr} = Prandtl number for drill fluid, dimensionless
N_{Rei} = Reynolds number inside drill string dimensionless
N_{Ree} = Reynolds number outside drill string, dimensionless
$\dot{Q}_1$ = energy source inside drill string, Btu/hr-ft
$\dot{Q}_2$ = energy source outside drill string, Btu/hr-ft
$\dot{Q}_{bit}$ = energy source at bottom-hole, Btu/hr
q_m = drill fluid flow rate, cu ft/hr
r_e = radius at $i = IMAX$, ft
t = time, hour
$T_{i,j}$ = temperature of the cell at (i, j), °F
T_{mi} = drill fluid inlet temperature, °F
T_s = surface temperature of earth, °F
z_b = depth to bottom of hole, ft
z_{c1} = depth to bottom of first casing string, ft
z_{c2} = depth to bottom of second casing string, ft
z_{c3} = depth to bottom of third casing string, ft
z_{dc} = depth to top of drill collars, ft
z_{m1} = depth to cement-mud interface in second annulus, ft
z_{m2} = depth to cement-mud interface in third annulus, ft
z_{m3} = depth to cement-mud interface in fourth annulus, ft
ρ_c = density of cement, lb/cu ft
ρ_{dc} = density of drill collar material, lb/cu ft
ρ_{dp} = density of drill string material, lb/cu ft
ρ_e = density of earth, lb/cu ft
ρ_m = density of drill fluid, lb/cu ft
ρ_{st} = density of steel, lb/cu ft
ρ_{2m} = density of mud in second annulus, lb/cu ft
ρ_{3m} = density of mud in third annulus, lb/cu ft
ρ_{4m} = density of mud in fourth annulus, lb/cu ft

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