VOL. 77, NO. 35 JOURNAL OF GEOPHYSICAL RESEARCH DECEMBER 10, 1972

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A Theoretical Study of Geothermal Energy Extraction

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Efficient extraction of geothermal energy from a dry well depends on the ability to establish a closed pressurized circuit of water through a large zone fractured in hot impermeable rock. Long-term perpetuation of significant power extraction depends, in addition, on the ability to extend the initial fracture zone through the effects of thermal stress cracking of the adjacent hot rocks. In support of an experimental program to test the feasibility of using this type of energy source, we have solved numerically the combined equations describing the coupled processes of fluid flow, heat transport, and rock fracture. The results show a strong dependence on the extent to which underground pressure can be maintained and the fracture zone continuously extended. They indicate that under favorable, but perhaps not unreasonably exotic, circumstances the extraction of significant thermal power from each well can be expected to continue for many decades.

The extraction of geothermal energy has generally relied on the relatively rare occurrence of natural steam and has contributed an almost insignificant fraction of the world's power needs [*United Nations*, 1964]. *Robinson et al.* [1971] give a large annotated bibliography on this matter and related matters. With the expected depletion of available fossil fuels in the foreseeable future, however, it is necessary to investigate other possible sources for the expanding power requirements that even today are sometimes not adequately fulfilled. As a result, there are active programs to develop for this purpose the capabilities of atomic reactors, controlled thermonuclear fusion, and the concentration of solar energy.

In addition, there is renewed interest in the possibility of developing better techniques for extracting energy from the earth in ways that are not dependent on rare and fortuitous combinations of circumstances. One proposal describes the deposition of energy by powerful underground nuclear bombs and the gradual extraction of this energy as it is needed. Another, developed by *Robinson et al.* [1971], is based on a technique for the removal of natural heat energy in the absence of natural steam. This paper describes a supporting theoretical analysis for this second method.

DESCRIPTION OF PROCESS

The report by *Robinson et al.* [1971] describes the geothermal energy extraction technique in considerable detail in conjunction with a new method for drilling the necessary deep holes into hot rock. For our present purpose the process in its most idealized form can be summarized briefly as follows.

At a place where the underlying rock is sufficiently hot a hole is drilled. Typically, the temperature at the bottom would be 250°–450°C, but at present the depth necessary to achieve this temperature should be no greater than about 5 km.

Water is forced into the hole under pressure to crack the rock by a process called hydraulic fracturing.

A parallel hole is drilled to the top part of the fracture. Connection with the fracture can be made by directional drilling or by a second hydraulic fracture if necessary.

A closed pressurized circuit of water flow is established at the rate of several million gallons per day, with influx to the hot rock through the deeper hole and a return of the heated water through the shallow hole. The pressure assures a liquid phase throughout and can be adjusted to lie just below the threshold for the continued propagation of hydraulic fracturing if desired.

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If the initial fracture zone is large enough, appreciable energy can be extracted for many years before the conduction flux from the adjacent rock exhausts the nearby reservoir of heat.

There is, however, a beneficial secondary effect from cooling the rock. Thermal stresses are induced by the tendency to contract, the result being a significant number of new cracks opening a progressively greater volume of hot rock to the circulation of water.

As the volume increases, through both extended rock cracking and the removal of dissolved minerals, the power extraction rate is enhanced by several features: (1) greater exposed area of rock-water interface; (2) enhancement of heat transport by convection of water directly to and from the hotter rocks; (3) enlarged area at the edge of the cracked region for conduction from far distances; and (4) preferential penetration of cracks into deeper rocks where the temperature is greater.

Eventually, a nearly steady-state power extraction rate, which could persist for many years, is possible. Also possible would be an enhanced exploitation through deepening of the cold-water downflow well into the hotter rocks or nearby drilling of satellite wells into the cracked heat source region.

The two greatest uncertainties concern the possibilities that rock contraction will not actually result in a significant increase in permeability and that underground leakage will preclude the maintenance of sufficient pressure for pumping up the hot water. For these and other reasons, an extensive experimental program, for which effective planning and design require, among other things, a realistic theoretical investigation of the processes likely to occur during the various phases of water-rock interaction, will be required. In this paper we describe the results of numerical solutions on a high-speed electronic computer of the equations of fluid dynamics, including the effects of buoyancy, pressure gradients, and the drag exerted on water flowing through cracks with variable porosity; the heat balance equation, including the effects of convection and conduction; and a porosity equation describing the process of hydraulic fracturing, the contraction because of cooling, the buildup of tension, cracking when tension exceeds ultimate strength, and the corresponding changes in volume accessible to the water.

The goals of our investigation have been to use the best available data for water and rock properties and to calculate for a variety of possible circumstances both the early-time developments (through the first few years of water circulation) and the late-time approach to steady state or depletion (on the scale of many decades). The results are as realistic as available fracturing theory and material properties data allow and apply directly to the practical assessment of geothermal energy extraction potentialities as well as to the planning and design of both experimental and production facilities.

HYDRAULIC FRACTURE PHASE

After the drill hole has penetrated a sufficient depth into hot relatively impermeable rock, the technique of hydraulic fracturing is used to generate a cracked zone through which the water can be circulated. The center for this process may be located somewhat above the bottom of the hole. The usual result of hydraulic fracturing in thick-bedded sedimentary rock is a radiating disk-shaped crack, which ideally could be up to several kilometers in radius but only a few centimeters wide. Because the initial vertical-stress component generally exceeds the horizontal components, the crack radii will lie in a vertical plane, the orientation being dependent on the chance state of nonisotropy at the point of origin. Without the additional effects of thermal stress, a single crack is likely to occur, enlarging in its original plane because of the smaller pressure required for preferential cleavage at the extremities in comparison with that necessary to initiate secondary cracking on the faces. Discussions of hydraulic fracturing experience in sedimentary rocks have been given by *Sun* [1969] and *Craft et al.* [1962].

There has been, however, no field experience in hydraulic fracturing under the somewhat different circumstances necessary for geothermal energy extraction. For two reasons we expect that the results might differ from the idealized single-crack fracture described above. One of these reasons arises from the nonisotropy and the inhomogeneity associated with the joints and the fissures in the (probably) granitic material. The second reason is a consequence of thermal stress induced by using cold water to hydraulically fracture hot rocks. Whereas the stress component normal to the face of the initial fracture is increased by the water pressure required for hydraulic fracture, the tangential stresses are not so affected. Indeed, the horizontal tangential component (and the somewhat greater vertical tangential component) will be relieved by contraction of the cooling crack faces near the water inlet to the extent that within a few hours the thermal stress can be expected to penetrate to a depth as great as 10 cm. If the difference between rock and water temperature is great enough, a reduction of the horizontal tangential stress component to below the threshold for hydraulic fracture initiation (i.e., to below the progressively decreasing stress level necessary to propagate the expanding tip of the initial crack) will result. As a result, the zone of hydraulic fracture would propagate in all directions in a series of vertically oriented cracks arranged in a more or less rectangular pattern of vertically elongate blocks. Ideally, the zone would be roughly cylindrically symmetric about the axis of the drill hole and would have a porosity that decreases both radially and with vertical distance from the water source. Because the ratio of the hydrostatic pressure to the horizontal components of the lithostatic stress is roughly independent of depth, we expect the fracture zone to propagate upward and downward at approximately equal rates. This propagation, however, is modified somewhat by the effects of buoyancy, which can be expected to induce a preferentially downward tendency.

Our theoretical modeling of this process and the subsequent process associated with energy extraction is, indeed, based on the assumption of a cylindrically symmetric fracture zone. Solution of the equations requires the use of a high-speed electronic computer, and this, too, necessitates such a restriction because the achievement of sufficient resolution with available computers requires that we avoid truly three-dimensional variations of the flow field configuration.

A brief analysis is also presented of the noncylindrical case, in which there is a single initial crack in one plane, as was originally assumed by *Robinson et al.* [1971], but in this case the thermal fracture propagation has had to be omitted from the analysis. It appears that the likelihood of one or the other of these idealized configurations occuring is dependent on the temperature and the rate of flow of water into the rock during the earliest stages of the process. A rapid flow of cold water may favor the occurrence of a roughly cylindrical multicrack region, whereas the slow flow of hot water will more likely induce a single-plane crack. At present this contention is supported only qualitatively by the following.

1. Rapid water flow will result in loss of dynamic head at the extremities of the crack system, which tends to impede extension.
2. Whereas the idealized picture of hydraulic fracturing requires a continuously decreasing pressure for extension as the radius increases, the actual process is likely to proceed through a series of lenticular breakthrough steps, and thus a lower limit for the necessary extension pressure that may not be much less than that required to initiate new cracking near the water source position is indicated.
3. A rapid flow of water assures maximum cooling near the water source, with consequent large thermal stress and a low threshold for fracture initiation.

Thus, to calculate the hydraulic fracture phase, we use the necessary equations to describe the heat transport, the water flow, and the rock cracking that occur when a single source of cold water is injected into the hot rocks. During this phase there is no return flow of heated water to the surface; an exact balance is maintained between the inflow rate and the rate of porosity increase from cooling contraction and hydraulic pressure fracturing.

EXTRACTION OF GEOTHERMAL ENERGY

When the hydraulic fracture phase has been completed, a second hole is drilled to the top of the fracture zone while the walls of the first hole are sealed at the level where the hydraulic fracturing was centered. The goal is to establish a pathway for the flow of water through the hot cracked rock from the cold-water source near the bottom of the zone to the hot-water sink near the top. For this purpose the techniques of directional drilling and additional hydraulic fracture from the second hole are likely to be effective means for establishing the

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Professor R. W. Rex (personal communication, 1971) has suggested to us, however, that it may be difficult to establish a proper flow path between the source and the sink points. He anticipates the possibility of a short-circuit path from which there would be negligible heating of the water or, alternatively, that despite all efforts it would not be possible to establish any flow path at all. In either case his remedy would be to use a single-source 'huff-puff' system with cycles of cold water inflow and warm water withdrawal. During the inflow periods the water must be pumped at sufficient pressure to open new volume for itself by hydraulic fracture. During withdrawal this effort can be partially recovered through the elastic rebound of the rocks, but part of the newly opened volume will remain open because of contraction of the cooled rocks. To calculate the huff-puff system of geothermal energy extraction would require relatively minor modifications to the cylindrically symmetric calculations described in this report; however, we have deferred a study of this approach until there is additional evidence that it will be required.

If the establishment of a suitable pathway for water flow is assumed, it is expected that the system will be self-pumping as a result of buoyancy effects. Indeed, the calculations confirm this expectation. Although the buoyancy within the fracture zone may be insufficient to overcome the Darcy resistance at late times (after many decades), it is more than overcome by the pumping effects of the density difference in the two drill holes. It is, of course, essential that the rock be sufficiently competent for the retention of pressure, this requirement being one of the crucial points to be demonstrated by field experience.

Buoyancy will produce a second, even more important effect, namely, the preferential tendency for the input cold water to circulate downward into the lower hot rocks, which tends to assure the passage through a heating circuit rather than an inefficient short circuit to the withdrawal pipe. At the same time, this tendency also assures an optimal direction for the propagation of additional thermal fracture downward and laterally into adjacent rocks, which are likely to be even warmer. This propagation continues to enlarge the cracked zone and allows for very-long-range energy withdrawal at perhaps an increasingly greater rate. This tendency is even further enhanced by the fact that water viscosity decreases significantly during heating, which helps to assure preferential circulation through the hottest extremities of the fracture zone.

The propagation of fracture from thermal stress effects has been described to us by Professor G. Kennedy (personal communication, 1971) as one of the the most questionable of the postulates, requiring considerable experimental confirmation. His experience suggests the possibility that contraction of the rock will take place predominantly through pore distortion, the resulting distance between cracks being very small. Equation 5 below shows that the corresponding permeability depends very strongly on the crack spacing d_c, and our calculations show that for $d_c \lesssim 1.0$ cm the water circulation is so strongly impeded as to preclude significant extension of the power extraction by this mechanism. Our hope is that the continued pressurization to just below the threshold for hydraulic fracture will assure the formation of relatively widely spaced cracks. The calculations show that for $d_c \gtrsim 5.0$ cm the permeability will be great enough to allow for a high level of power extraction for many decades, so that a principal goal of the experimental effort will be to devise ways to encourage this wide spacing.

Whereas the water flow circuit cannot be considered strictly closed, we have included in the calculations only the water necessary for hydraulic fracture and thermal contraction. In practice, it will also be necessary to account for the removal of dissolved minerals and convected particulates and possibly to replenish the losses from underground leakage through cracks leading away from the energy extraction zone.

THERMAL FRACTURE PROPAGATION EQUATIONS

The combined processes of water circulation, heat transport, and rock fracture are described by the following equations [*Harlow and Amsden*, 1971].

The conservation of mass for water is given by

$$\partial \rho_w \theta/\partial t + \nabla \cdot \rho_w \mathbf{u} \theta = \rho_w S \qquad (1)$$

The conservation of momentum for water is given by

$$\nabla p - \mathbf{g}\rho_w + (\mu\theta/k)\mathbf{u} = 0 \qquad (2)$$

The conservation of heat energy in both rock and water is given by

$$\begin{aligned}\partial\{[b_R\rho_R(1-\theta) + b_w\rho_w\theta]T\}/\partial t \\ + \nabla \cdot \rho_w b_w \theta \mathbf{u} T \\ = \nabla \cdot [\kappa_R(1-\theta)\nabla T] + \rho_w b_w T_s S \qquad (3)\end{aligned}$$

The nomenclature is as follows:

θ	porosity, open volume for water flow (per unit volume).
ρ_w	density of water, a function of temperature.
ρ_R	density of rock.
$\mathbf{u}$	water velocity.
S	source or sink of water from surface pipes (volume per unit volume per unit time).
p	water pressure.
$\mathbf{g}$	acceleration of gravity.
μ	coefficient of water viscosity, a function of temperature.
k	permeability, a function of porosity and crack spacing.
b_R, b_w	specific heats of rock and water, respectively.
T	temperature.
T_s	source or sink temperature.
κ_R	heat conduction coefficient for rock.

These equations are supplemented by the points discussed below.

Conservation of water mass accounts for the inlet source and the outflow sink as well as for changes in accessible crack volume.

The momentum equation neglects inertial and advection terms, $\rho\partial\mathbf{u}/\partial t + \rho\mathbf{u}\cdot\nabla\mathbf{u}$, and thus a local balance of three stresses is implied. These stresses are produced by pressure gradients, gravitation (including buoyancy effects), and the Darcy drag.

For the buoyancy terms we put

$$\rho_w = \rho_0[1 - \beta_w(T - T_c)] \qquad (4)$$

where β_w is the volumetric expansion coefficient for the water and T_c is a constant reference temperature. In all terms but the gravity term the correction to ρ_0 is neglected.

The permeability coefficient in the Darcy drag term is given by the Kozeny formula

$$k = (4d_c^2/5)[\theta^3/(1-\theta)^2] \qquad (5)$$

where d_c is a measure of the distance between cracks. This formula, which illustrates the strong dependence that can be expected from the details of how the rock fracturing occurs, was derived from the form given by *Gray* [1963].

The variation of water viscosity with temperature is approximated by

$$\mu = \frac{0.279}{(T/°\mathrm{C}) - 3.8}\,\frac{\mathrm{g}}{\mathrm{cm\,sec}} \qquad (6)$$

which is a fit to the data given by *Gray* [1963].

Porosity is determined as a function of temperature by first calculating

$$\theta' = \beta_R(T - T_0)/[\beta_R(T - T_0) - 1] \qquad (7)$$

If $\theta' > \theta_y$, $\theta_0 = \theta'$; but, if $\theta' < \theta_y$, $\theta_0 = 0$. Here β_R is the volumetric coefficient of expansion of the rock, T_0 is the temperature (a function of depth) at which the compressional stress vanishes, and θ_y is the porosity that occurs when the ultimate tensile strength has first been exceeded. We assume that $T_0 = KT_\infty$, where K is a constant and T_∞ is the initial temperature, a function of depth. In the absence of hydraulic fracture, $\theta = \theta_0$.

A second step in calculating the total porosity is required if there is simultaneous hydraulic fracturing, as is the case during the early phase of the process before the second hole for hot-water withdrawal has been drilled. The inlet volume per unit time V must then exactly balance the rate of increase of accessible crack volume; i.e.,

$$\int \frac{\partial\theta}{\partial t}\,d\tau = V$$

where the integration extends over all the volume of rock. If the inlet source is distributed over a volume τ_s, $S(\text{source}) \equiv V/\tau_s$. Our procedure has been to propagate a constant porosity crack (with $\theta = 5.0 \times 10^{-3}$) from hydraulic fracture, which is enhanced by the calculation of θ_0 to produce the total porosity θ as a function of position and time.

The equation of heat energy conservation neglects work and dissipation terms, which,

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however, could easily have been added if calculations had shown them to be important. The equation assumes local temperature equilibrium on the basis of the fact that the heat exchange rates are very rapid in comparison with the other significant processes. Convection of heat occurs because of water motion; conduction is important only in the solid-rock region.

During the energy extraction phase the inlet volume no longer exactly balances the increase in crack volume. Hydraulic fracturing does not occur, but, if the pressure is maintained, the porosity from hydraulic fracture is not lost. Indeed, sufficient pressure must be maintained until thermal fracture catches up to the hydraulic fracture level, at which time the pressure could be released if desired. Before the release of pressure, θ remains constant until $\theta_0 = \hat{\theta}$, after which $\theta = \theta_0$. In any case, water is withdrawn from a sink region, also of volume τ_s, and over-all conservation requires that

$$S_{\text{sink}} = \frac{1}{\tau_s}\left(\int \frac{\partial \theta}{\partial t}\, d\tau - V\right)$$

The source water is at a prescribed cool inlet temperature; solution of the equations determines the temperature at the sink, one of the principal results of the calculation.

MAGNITUDE OF K

The factor K appearing in the calculation of θ_0 is chosen to describe the change in temperature required for the relief of unbalanced overburden stress. Because the horizontal components of the nonisotropic stress are less than the vertical components, the hydrostatic water pressure alone is not far below the threshold for fracture. Accordingly, we expect the value of K to be only slightly less than 1. The precise magnitude depends on circumstances partially under experimental control through variations in the water pressure maintained at the surface. Indeed, hydraulic fracture results from pressurizing to the extent that $K \geq 1.0$, whereas thermal fracture corresponds to lesser pressures whose effect must be enhanced by thermal contraction before the threshold for fracture is reached.

Because hydraulic fracture results in a relatively widely spaced cracking, we expect that operating with surface pressures just below the threshold for hydraulic fracture will likewise enhance the crack spacing, and, correspondingly, the permeability, over that which might result with low surface pressure. This control potentiality may be a significant factor in overcoming the worry that cooling contraction will be manifested in displacements only at the scale of the finest inhomogeneities, from which negligible permeability would result. The matter is complicated, however, by the fact that the required pressure for hydraulic fracture extension decreases as the radius of curvature of the tip increases. Thus, although this radius is probably bounded by the tendency to extend in circular jumps of bounded curvature, it nevertheless follows that the pressure threshold for initiation of normal cracks is greater than that required for extension of the established crack. To overcome this threshold requires the presence of a temperature gradient whereby thermal stress is greater near the water source than at the crack tips. This, fortunately, is exactly the case, and again there is control of the extent to which it is so by means of variations in the inlet flow rate of the cold water.

OUTLINE OF SOLUTION PROCEDURE

Details of the numerical methodology are very similar to those discussed by *Amsden and Harlow* [1970] and *Harlow and Amsden* [1971]; thus we present only a brief summary here.

Combining the various items discussed above, we may write the mass and momentum equations in the following forms:

$$\partial\theta/\partial t + \nabla\cdot\mathbf{u}\theta = S \tag{8}$$

$$\nabla\frac{p}{\rho_0} - \mathbf{g} + \mathbf{g}\beta_w(T - T_c) + \frac{\mu\theta}{\rho_0 k}\mathbf{u} = 0 \tag{9}$$

We now let $p/\rho_0 \equiv (p_0/\rho_0) + \phi$, where p_0/ρ_0 is a function of depth only, determined by the equilibrium condition

$$\nabla(p_0/\rho_0) - \mathbf{g} + \mathbf{g}\beta_w(\langle T\rangle - T_c) = 0 \tag{10}$$

where $\langle T\rangle$ is the average water temperature at the depth in question. The momentum equation then becomes

$$\nabla\phi + \mathbf{g}\beta_w(T - \langle T\rangle) + (\mu\theta/\rho_0 k)\mathbf{u} = 0 \tag{11}$$

To solve these equations, we transform them to

$$\mathbf{u}\theta = -k\rho_0[\nabla\phi + \mathbf{g}\beta_w(T - \langle T\rangle)]/\mu \tag{12}$$

$$\frac{\partial\theta}{\partial t} - \nabla\left\{\frac{k\rho_0}{\mu}\left[\nabla\phi + g\beta_w(T - \langle T\rangle)\right]\right\} = S \qquad (13)$$

Numerical integration is accomplished by using finite difference representations of these equations and the temperature equation. The solution proceeds from prescribed initial conditions through a sequence of small time increments. For each cycle the following steps are required. (1) The new temperature is calculated for each finite difference cell by using a finite difference representation for (3). (2) The corresponding values of θ and $\partial\theta/\partial t$ are calculated. (3) Equation 13 is solved for the pressure-density ratio ϕ. (4) New velocities are obtained from (12).

SOME CALCULATION EXAMPLES

Three examples are described here in detail, the discussion being supplemented by a summary of the results from a number of additional calculations examining the effects of parameter variations. The basic data for the calculations are given below, and the variations are described in Table 1. The data below were assembled from various sources in an attempt to represent the properties of the rocks underlying the Jemez caldera in north central New Mexico (D. Brown, personal communication, 1971).

T (source) = 65°C
T (rock) = 300°C
$K = 0.98$
$\theta_y = 0.0$
$\kappa_R = 6.2 \times 10^{-3}$ cal/cm sec °C
$\rho_R = 2.65$ g/cm³
$b_R = 0.25$ cal/g °C
$\beta_R = 2.5 \times 10^{-5}$/°C
$b_w = 1.0$ cal/g °C
$\beta_w = 5.0 \times 10^{-4}$/°C
$d_c = 5.0$ cm
$\rho_0 = 1.0$ g/cm³
$g = 980$ cm/sec²

Each calculation proceeds through two phases. The first, which accomplishes the hydraulic fracture, has only an inflow of water at a progressively increasing rate continuously proportional to the open crack volume. The second, the energy extraction phase, has a constant inflow rate and a withdrawal rate balancing both the inflow and the increase of porosity.

TABLE 2. Rezone Data for Three Basic Examples

Example	Rezone Time, years	Source-Sink Separation after Rezone, meters
1	0.08	80
	5.97	180
2	0.08	40
	0.92	140
	9.57	120
3	0.02	25
	0.59	60
	3.85	40

As each calculation progresses, we occasionally rezone the computational region, doubling its height and radius, and reposition the water source and the sink. The criteria for choosing new inlet and outlet positions are based on locating the extent to which the porosity has penetrated. The inlet of cold water, for example, always moves downward to account for the preferential downward circulation of the cold water and the consequent contraction of the rocks in that direction. Table 2 gives the history of rezoning and source-sink separation for the three calculations.

The thermal power extraction for these examples is shown in Figures 1 and 2. The examples in Figure 1 differ only in the water flow rate. The larger rate of example 1 results in a more rapid decrease of power for the first several years than that shown for example 2. Repositioning of the source and sink positions gives the discontinuity just prior to six years, but the perturbation does not persist. It is apparent that the larger flow
thermal fracture enhance
long-term increase in pow

TABLE 1. Variations among Three Basic Examples

Example	Hydraulic Fracture Pumping Time, months	Final Flow Rate, cm³/sec
1	1.00	1.45×10^5
2	1.00	4.00×10^4
3	0.25	1.00×10^4

Example 3 represents
ment from which the conc
ture could be verified in
period of time.

All three examples exhibi
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The closely related contours
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In addition to the examples d
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summarized briefly as follows.

Increasing d_c from 5.0 to 10.0
a 10% increase in late-time ther

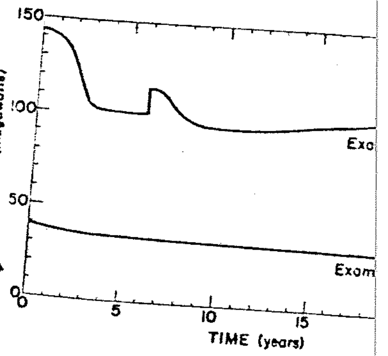


Fig. 1. Thermal power extraction as
of time for examples 1 and 2

parent that the larger flow rate produces enough thermal fracture enhancement to result in a long-term increase in power extraction.

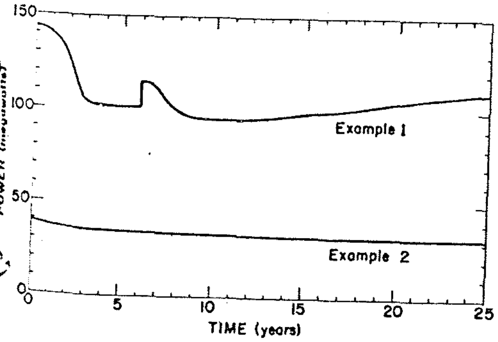
Example 3 represents a small-scale experiment from which the concept of thermal fracture could be verified in a relatively short period of time.

All three examples exhibit considerable qualitative similarity in the underground water flow pattern development and in the corresponding configurations of temperature and porosity contours. The streamlines of water flow, for example, show a strong jetting of cold water downward from the source resulting from the effects of negative buoyancy. Near the bottom of the fracture zone the water spreads laterally and then upward in the peripheral hot crack tips, where the flow resistance is significantly decreased by the lowering of viscosity with heating, and the upward migration is enhanced by positive buoyancy. With even moderate values for the permeability the flow pattern is that of a ring vortex in the lower part of the fracture zone, from which water slowly migrates upward and converges onto the outlet sink.

The closely related contours of rock temperature and porosity expand with the propagating thermal fracture into a pear-shaped zone extending predominantly in the downward and lateral directions.

In addition to the examples discussed so far, we examined the effects of varying several of the rock property parameters. The results are summarized briefly as follows.

Increasing d_c from 5.0 to 10.0 cm resulted in a 10% increase in late-time thermal power extraction. For d_c less than about 1.0 cm the power extraction enhancement from thermal fracture becomes negligible.



ig. 1. Thermal power extraction as a function of time for examples 1 and 2.

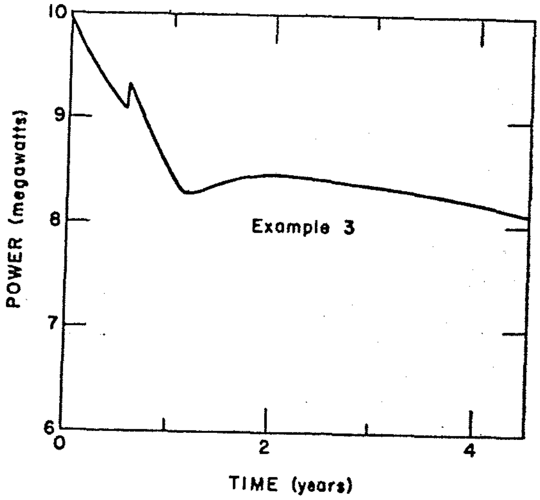


Fig. 2. Thermal power extraction as a function of time for example 3.

Decreasing K from 0.98 to 0.96 produced an 18% drop in late-time power extraction, which shows again the strong dependence on susceptibility to thermal fracture and the corresponding necessity for maintaining the pressure at nearly the hydraulic fracture threshold so that K will be as near 1.0 as possible.

Although we believe that $\theta_v = 0.0$ most nearly represents physical reality, we investigated the effects of varying this parameter. For $\theta_v \lesssim 1.0 \times 10^{-4}$ there was no significant change, but for $\theta_v \gtrsim 5.0 \times 10^{-4}$ we observed significant decreases in late-time power extraction. For $\theta_v \gtrsim 3.0 \times 10^{-3}$ the extraction rate has essentially lost its thermal fracture enhancement.

Two calculations were performed in which the uniform initial rock temperature (300°C) was modified to have a uniform gradient of ±20°C/km, 300°C being the average value. In both cases the power extraction is negligibly different at early times (~10 years) and differs by <5% at late times.

Calculations produced with water flow rates even higher than those of examples 1 and 2 show that the late-time power extraction increases very nearly linearly with increasing water flow rate.

ENERGY EXTRACTION FROM FLAT-CRACK HYDRAULIC FRACTURE

Up to this point the analysis has been restricted to cylindrically symmetric examples. In contrast, if the initial hydraulic fracture is confined to a single planar crack, the calculations must be modified somewhat. This could be the case if thermal fracture propagation turned out to produce negligible enhancement in the lateral permeability. Our purpose in this section is to show that, even under this unfavorable circumstance, it appears possible to extract appreciable geothermal energy.

In the mid-plane of the crack we utilize a Cartesian coordinate system, x and y being horizontal and vertical coordinates and z denoting distance into the rock from the edge of the crack, not from the mid-plane position.

The heat transport and the dynamics are now described by

$$\partial\xi/\partial t = (\kappa_R/\rho_R b_R)\,\partial^2\xi/\partial z^2 \tag{14}$$

$$\frac{\partial WT}{\partial t} + \nabla_{xy}\cdot WT\mathbf{u} = \frac{2\kappa_R}{\rho_0 b_w}\left(\frac{\partial\xi}{\partial z}\right)_{z=0} + WT_sS \tag{15}$$

$$\nabla_{xy}\cdot\mathbf{u}W = SW \tag{16}$$

$$\frac{\partial W\mathbf{u}}{\partial t} + \nabla_{xy}\cdot(\mathbf{u}W\mathbf{u}) = -\frac{W}{\rho_0}\nabla p + \mathbf{g}W - \mathbf{g}\beta_w W(T - T_0) - \frac{\Gamma\nu\mu}{W} \tag{17}$$

where W is the crack width, constant in time but a function of x and y; ξ is the rock temperature (at $z = 0$, $\xi \equiv T$); S is the source (or sink) of volume of water per unit volume of space per unit time; T_s is the source or sink temperature; Γ is the drag coefficient for water flow through the hydrofracture crack (Γ has magnitude in the approximate range 5–10); and ν is the kinematic viscosity of water, a function of T as in (6).

These equations are supplemented and some of the terms explained by the following comments.

Enhanced heat transport by water convection into the thermally cracked rock is here neglected, the result being underestimates of the power extraction rate. This neglect is related to the difficulty in achieving sufficient computer resolution for three-dimensional variations.

We also have neglected the changes in overall volume accessible to the water from either the thermal fracture or the removal of dissolved minerals, so that S (sink) $\equiv -S$ (source).

The value of the crack drag coefficient Γ depends on the velocity profile across the crack. Indeed, the detailed interpretation of $\mathbf{u}$ also depends on the shape of this profile. Because of uncertainties in crack geometry (both flexures in the mid-plane and irregularities of the wall), we have absorbed the questions of velocity profile into Γ. The effects of varying Γ from $\sim$5.0 to $\sim$10.0 should be within the range that can be expected from the extremes of the uncertainties described above. If, however, the crack is filled with debris or granules, the effective value of Γ is much greater.

As it was before, the solution of these equations is facilitated by putting

$$\nabla\phi \equiv \frac{1}{\rho_0}\nabla p - \mathbf{g} + \beta\mathbf{g}(\langle T\rangle - T_0) \tag{18}$$

The results for the dynamical equations are very similar to those of (12) and (13).

$$u = -(W^2/\Gamma\nu)\,\partial\phi/\partial x \tag{19}$$

$$v = -\frac{W^2}{\Gamma\nu}\frac{\partial\phi}{\partial y} + \frac{W^2\beta g}{\Gamma\nu}(T - \langle T\rangle) \tag{20}$$

$$\frac{\partial}{\partial x}\left(\frac{W^3}{\nu}\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{W^3}{\nu}\frac{\partial\phi}{\partial y}\right) = -\Gamma SW + \frac{\partial}{\partial y}\left[\frac{W^3\beta g}{\nu}(T - \langle T\rangle)\right] \tag{21}$$

where the scalar g is positive for downward gravity.

To include the effects of debris on granules filling the crack, it is sufficient to introduce a porosity θ and a permeability k in such a way that (15)–(17) become

$$\left[\theta_c + (1 - \theta_c)\frac{\rho_f b_f}{\rho_0 b_w}\right]\frac{\partial WT}{\partial t} + \nabla_{xy}\cdot WT\mathbf{u}\theta = \frac{2\kappa_R}{\rho_0 b_w}\left(\frac{\partial\xi}{\partial z}\right)_{z=0} + WT_sS \tag{22}$$

$$\nabla_{xy}\cdot\theta W\mathbf{u} = SW \tag{23}$$

$$\frac{\partial W\mathbf{u}}{\partial t} + \nabla_{xy}\cdot(\mathbf{u}W\mathbf{u}) = -\frac{W}{\rho_0}\nabla p + \mathbf{g}W - \mathbf{g}\beta_w W(T - T_0) - \left(\frac{\Gamma\nu}{W} + \frac{W\nu\theta}{k}\right)\mathbf{u} \tag{24}$$

With θ and k as
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$\kappa_R =$
$\rho_R =$
$b_R =$
$b_w =$
$\Gamma =$
$k =$
$\theta =$
$W =$
$\rho_0 =$
$g =$
T (source) $=$
$\beta_w =$
$\rho_f = 2$
$b_f = 0$
T (rock) $= 3$

Inlet volume rate	1.
Crack width	1.
Crack height	1.
Water source	75
Water sink	75

The results show an
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With θ and k as constants the solution proceeds in the way described above. For $\theta = 1.0$ and $k = \infty$ the equations reduce to those for an open crack, as is expected. An example of this type of calculation is described below.

$$\kappa_R = 6.2 \times 10^{-3} \text{ cal/cm sec °C}$$
$$\rho_R = 2.65 \text{ g/cm}^3$$
$$b_R = 0.25 \text{ cal/g °C}$$
$$b_w = 1.0 \text{ cal/g °C}$$
$$\Gamma = 6.0$$
$$k = 10^{-4} \text{ cm}^2$$
$$\theta = 0.1$$
$$W = 0.6 \text{ cm}$$
$$\rho_0 = 1.0 \text{ g/cm}^3$$
$$g = 980 \text{ cm/sec}^2$$
$$T \text{ (source)} = 65\text{°C}$$
$$\beta_w = 5.0 \times 10^{-4}/\text{°C}$$
$$\rho_f = 2.65 \text{ g/cm}^3$$
$$b_f = 0.25 \text{ cal/g °C}$$
$$T \text{ (rock)} = 300\text{°C}$$

Inlet volume rate	1.45×10^5 cm³/sec
Crack width	1.0 km
Crack height	1.0 km
Water source	75 meters above crack bottom
Water sink	75 meters below crack top

The results show an initial thermal power extraction level of 98.4 Mw decreasing at an initial rate of about 20 Mw/yr and at a rate of about 12 Mw/yr at the end of the first year. As cool water fills more and more of the crack, the power extraction rate tends to approach a slowly decreasing level that agrees qualitatively with simple analytical calculations for conduction across a known temperature difference. A larger crack would allow for a much milder rate of decrease, and the occurrence of thermal fracture could even result in an increase in power extraction, as is shown in example 1 (Figure 1).

The flow pattern for this example is quite simple in appearance, resembling potential flow from source to sink. Without the granular filling to the crack, calculations show a comparable power extraction history but with strong secondary flow as a result of buoyancy.

We also examined the case of a crack of variable width, the width decreasing linearly from 1.0 cm at the center to zero at the edge. The results show a much more rapid decrease of thermal power extraction resulting from a confinement of the flow to a relatively narrow zone about the axis. The effect was much like that of having a much smaller crack of constant width.

These computer-generated results have been of assistance in developing an analytical approximation for the power extraction. They show that the late-time temperature distribution for the water is nearly linear between the source and sink positions. Assuming this linearity, one can solve the heat diffusion equation for the energy imparted to the water per unit time. The resulting analysis gives

$$\text{power} = 2\rho_w b_w V(T_\infty - T_s)/ \{1 + [\rho_w b_w V(\pi k_1 t)^{1/2}/\kappa_R A]\}$$

where A is the area of the crack, V is the total water flow rate (volume per unit time), $k_1 \equiv \kappa_R/(\rho_R b_R)$, and T_s is the temperature at the cold-water source. This formula is not correct near $t = 0$, but it is expected to give a progressively better estimate at late times (i.e., for times in which the denominator significantly exceeds 1). Figure 3 presents a comparison between the numerical results for the plane crack calculation described earlier and the analytical approximation. The fairly good agreement at the overlap times lends support to the utility of the approximation formula in estimating late-time trends.

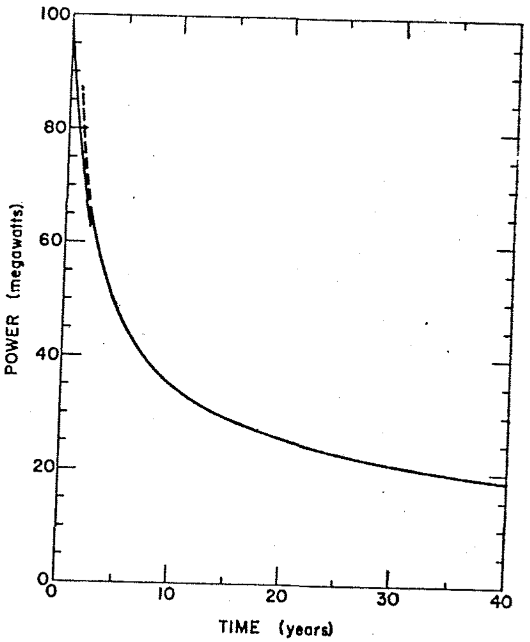

Fig. 3. Thermal power extraction as a function of time for the plane crack calculation described in the text. The curve on the left represents numerical results; the curve on the right represents the analytical approximation.

DISCUSSION

The principal conclusions from this study are that under sufficiently favorable circumstances large amounts of geothermal energy can be extracted from dry wells in hot rock. This conclusion does not depend strongly on the nature of the initial hydraulic fracture, but it does depend on the validity of the thermal fracture hypothesis and on the assumption that underground leakage does not preclude the maintenance of pressure. It is clear that these features will require field testing before this type of energy source can be considered proven.

Acknowledgments. We should like to acknowledge many valuable discussions with our associates at the Los Alamos Scientific Laboratory, who originated the concept and have studied its technical and economic feasibility, especially R. L. Aamodt, D. W. Brown, R. M. Potter, E. S. Robinson, J. C. Rowley, and M. C. Smith. We also are grateful for the valuable comments given by visitors to the Laboratory, especially R. L. Garwin, G. Kennedy, and R. W. Rex.

This work was performed under the auspices of the United States Atomic Energy Commission.

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(Received February 23, 1972;
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