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FRICTIONAL HEAT DURING METAMORPHISM

2. *Quantitative evaluation of concentration of heat generation in space*

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Deformation of the crust is commonly of different intensity in different parts of dynamothermally metamorphosed volumes. Consequently, if deformation results in the conversion of mechanical energy to heat within the crust, this heat production will not be uniformly distributed. This paper examines the effects of degree of spatial concentration of conversion of mechanical energy to heat on the temperature rise in and near the region where heat is generated. The treatment holds time duration of heat generation constant ($t = 3 \times 10^{12}$ sec), total energy converted to heat constant (energy averaged over 15 km thickness of crust, $E = 4.4$ cal g^{-1}) and varies antithetically the thickness (i.e., volume) of crust and rate of heat production within that thickness. Resulting temperature profiles at the end of the fixed time period have been computed; exact data for eight models are presented and compared.

It is concluded that significant temperature increases—approaching or in excess of 100 degrees C—may be possible in zones of intense deformation when these zones are 2 km or less across, even though only a modest amount of energy (represented by E) is converted to heat. When conditions of deformation are appropriate, abnormally steep and 'reversed' geothermal gradients, as sometimes are observed, may be attributable to mechanical energy locally converted to heat.

Introduction

One aspect of the total energy balance in deformed portions of the earth's crust is the conversion of mechanical energy to heat. As temperature is one of the most important of the intensive thermodynamic variables for an understanding of metamorphism, it is important to try to obtain an understanding of those factors which contribute to the determination of the temperature changes with time at different points within the earth's crust. Attempts to evaluate these variable factors must isolate individual variables, and determine theoretically the consequences of their variation while other parameters are held constant. By comparing the differences indicated as they are derived from constructed model situations, one may obtain information on the influences of the variables which is potentially useful in the interpretation of real situations.

The effects of the degree of spatial concentration of a fixed quantity of energy converted to heat within a portion of the earth's crust will be examined with the object of evaluating (to a first degree of approximation) the influence of absolute and relative degree of concentration of heat generation on temperatures. In order to isolate this variable, other possible variables will be held constant, e.g., such factors as time, conductivity, thermal diffusivity, uniformity of rate of heat generation within the time period and volume considered (and thereby the uniformity of deformation within this time and volume).

It will be implicitly assumed that heat is generated locally by conversion of mechanical energy to heat. Some limitations inherent in this assumption that affect application of the results will be discussed later (see p. 273). However, it should be kept in mind that this assumption is not necessary to the validity of the solution of the heat flow problem. Neither is it essential to the temperature distributions in space arising from the models that heat generation be exclusively by conversion of mechanical energy to heat. (The models depend on values of rate of heat generation, time duration of heat generation, the relative positions of the boundaries of the region within which heat is generated, conductivity, thermal diffusivity, etc., the values of which are independent of the means by which heat is generated.)

It is evident that mechanical deformation of the earth's crust is not uniform throughout deformed orogenic belts, and, in consequence, any conversion of mechanical energy to heat during deformation will likewise not be uniform. In regions of intense deformation, the likelihood of a relatively high rate of conversion of mechanical energy to heat during a limited time period is enhanced. It is also possible, when the mechanical energy of deformation is locally converted to heat for a limited period of time, that the temperature at one point in the crust may be higher than temperatures either above or *below*, a case not expected to be realized if consideration is restricted to upward transfer of heat from the mantle into and through the crust.

Mathematical problem

A general physical model was devised which would be amenable to geologically significant variation, rigorous mathematical treatment, and quantitative evaluation. This model may be most easily visualized by restricting attention to a column of the crust 1 cm by 1 cm by 2×10^6 cm within which heat flows linearly in the direction of the long dimension of the column. This would be most nearly approximated by selecting a vertical column of one cm^2 cross-section and 20 km length near the center of an orogenic belt, adjacent columns thereby being essentially similar so that lateral heat flow may be ignored as negligibly small. In this case the degree of concentration of heat generation within segments along the long dimension of the column would depend on the intensity of deformation crossing the column at different depths. The column would be representative of the central portion of an orogenic belt.

The mathematical problem may then be stated as that of evaluating the temperature, T, at any point, z, along the representative column when heat is generated at a specified uniform rate, Q, between any selected depths, *a* (upper) and *b* (lower), during a given time period, t. Total energy converted to heat may be kept constant by appropriate variations of the rate of heat production (Q, expressed in cal cm^{-3} sec^{-1}), the volume of the representative column (expressed in cm^3) bounded by the selected depths (*a* and *b*) between which heat is produced, and the time duration of the rate of heat production (t, expressed in sec.). Appropriate antithetic variation of rate of heat production which either time duration or volume involved, while the other remains constant, allows for selective evaluation of the effect of one of the variables. Time, i.e., the relative degree of concentration of a fixed amount of energy converted to heat in time, as the variable of interest has been examined by Reitan (1968). The models considered here allow space, i.e., the relative degree of concentration of a fixed amount of energy converted to heat in space, to be examined as the variable of interest.

A rigorous statement of the mathematical problem setting forth the applicable differential equations and the boundary conditions may be found in Reitan (1968). An explanation of the solution and the derivation of the equations by which temperature increases over any pre-existing gradient may be calculated has been given by Clark (Appendix *in* Reitan in press).

The models

The models presented, by which the effects of degree of spatial concentration of conversion of mechanical energy to heat are examined, were selected from among 824 for which numerically computed solutions are available. (T, as calculated from the final equations used in the computation, varies directly with Q for any combination of the other variables. As only order of magnitude differences of Q were used in making the computations for the 824 basic models, the intermediate models which may be desired for comparisons are obtained by simple multiplication of Q and the associated values of T. Therefore there are, in fact, many more than 824 models available. Selection was based on considerations of maintaining constant total heat production within a group of 118 basic models for which time is constant.) Time duration of heat generation, t, in the models compared is constant at 3×10^{12} sec., i.e., about 100,000 years. The total energy converted to heat, which is the product of the time duration, t; the rate of heat production, Q; and the volume within which heat is produced (which may be represented by $\triangle$, the thickness of the one cm^2 cross-section representative column, where $\triangle$ is the separation of the upper, *a*, and lower, *b*, boundaries of the region of heat production), is held constant and designated $\overline{E}$. The value of $\overline{E}$ in all of these models is 1.8×10^7 cal (which ,when averaged throughout 15 km thickness of the representative column, equals 4.4. cal g^{-1}, assuming an average density of about 2.7 g cm^{-3}). This allows several values of $\triangle$ to be selected for comparison of temperatures at the end of the given time period. In the models presented, $\triangle$ has a maximum value of 15 km and a minimum value of 0.5 km. The values of $\triangle$, the values of Q, the implied strain rates in the deforming region of the crust (see

Reitan, 1968 and in press, for an explanation of the correlation of a rate of heat production with the strain rate in the deforming region), the energy converted to heat *in the deforming region* (expressed in cal g^{-1}), and the computed maximum temperature increase, T_{max}, at the center of the deforming region are given in Table 1.

The distribution of temperature increases, as a function of distance from the center of the deforming region where the maximum temperature increase occurs, is illustrated in Fig. 1 for eight models. Table 2 gives exact

Table 1.

Model No.	Δ in km	Q in cal cm^{-3} sec^{-1}	Strain rate in sec^{-1}	T_{max} in deg C	Energy→Heat (deformed region) in cal g^{-1}
15-2B	0.5	1.2×10^{-10}	9.5×10^{-12}	122.0	133
13-2B	1	6×10^{-11}	4.7×10^{-12}	113.4	67
11-2B	2	3×10^{-11}	2.4×10^{-12}	98.1	33
9-2B	3	2×10^{-11}	1.6×10^{-12}	84.8	22
6-2B	4	1.5×10^{-11}	1.1×10^{-12}	73.6	17
1-2B	5	1.2×10^{-11}	9.5×10^{-13}	64.1	13
3-2B	10	6×10^{-12}	4.7×10^{-13}	38.1	6.7
4-2B	15	4×10^{-12}	3.1×10^{-13}	24.0	4.4

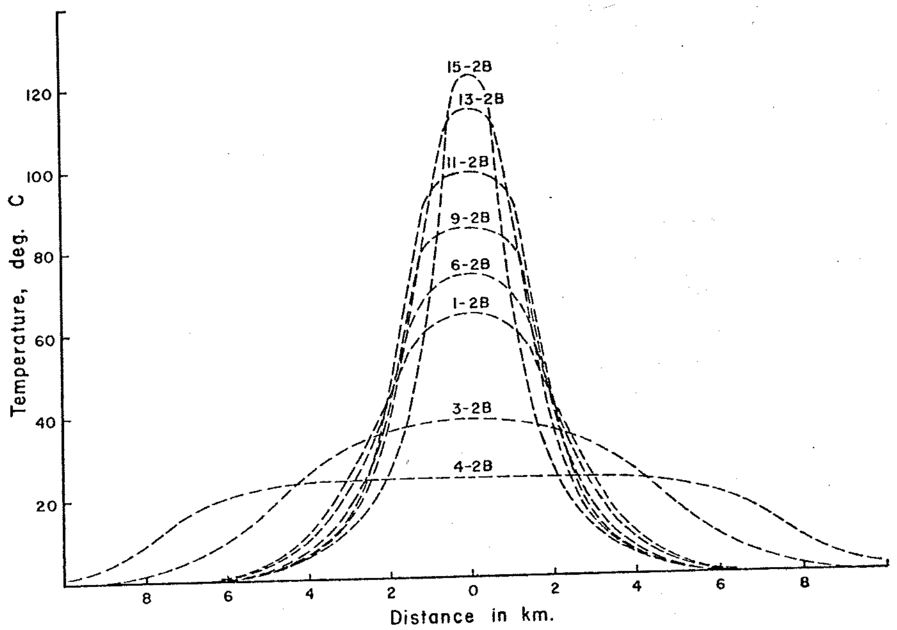

Fig. 1. Distribution of temperature increases over any pre-existing gradient related to distance from the center of the volume within which heat is generated, for each of eight models. Table 2 summarizes specific data and gives exact values of computed temperature increase at each of eight different depths for these models, assuming the center of the deforming region to be at 12.5 km depth.

In all models the time duration of heat generation, t, equals 3×10^{12} sec, and the total energy converted to heat averaged over 15 km thickness of crust, E, equals 4.4 cal g^{-1}.

computed values of temperature increases for each of eight depths for the eight models. The maximum temperature increase, T_{max}, increases as the thickness, $\triangle$, of the region within which heat is generated decreases, i.e., as the energy converted to heat is concentrated in space.

Relative degree of concentration, i.e., the ratio of $\triangle$'s (which may be called the concentration factor), might be anticipated to increase linearly with the ratio of corresponding maximum temperature increases, i.e., the ratio of T_{max}'s, but this is not the case. Examination of Table 3 reveals that there is about as much relative difference in T_{max} ratios when equal ratios of $\triangle$'s are compared (i.e., when the relative degree of concentration of heat in space – the concentration factor – is the same) as there is when the ratios of $\triangle$'s

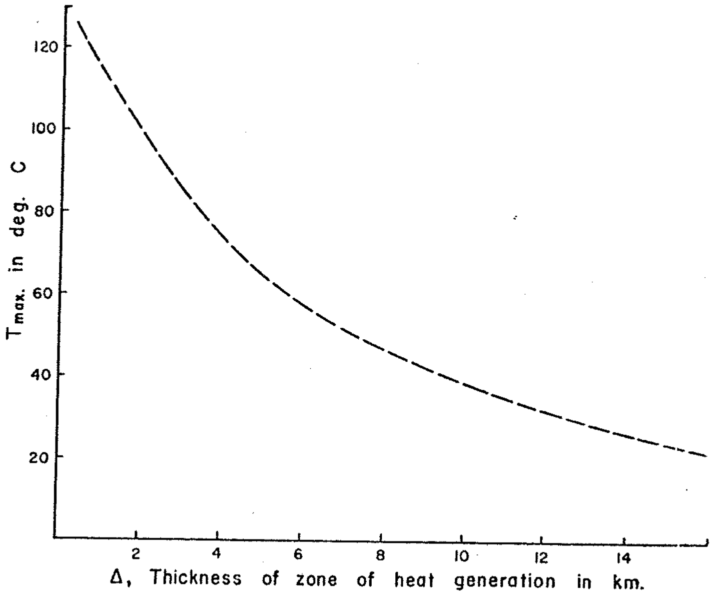

Fig. 2. The value of the maximum temperature increase, T_{max}, related to the thickness of the region, Δ, within which heat is generated.

Table 2.

Model No.	Q in cal cm^{-3} sec^{-1}	Upper boundary *a* in km	Lower boundary *b* in km	Temperature increase in deg. C over any pre-existing gradient at depth z, in km							
				z = 1	z = 2	z = 4	z = 6	z = 9	z = 12.5	z = 17.5	z = 21
15-2B	1.2×10^{-10}	12.25	12.75	0	0	0	0.1	7.8	122.0	1.3	0
13-2B	6×10^{-11}	12.0	13.0	0	0	0	0.1	7.9	113.4	1.3	0
11-2B	3×10^{-11}	11.5	13.5	0	0	0	0.2	8.9	98.1	1.6	0
9-2B	2×10^{-11}	11.0	14.0	0	0	0	0.3	10.6	84.8	2.1	0
6-2B	1.5×10^{-11}	10.5	14.5	0	0	0	0.6	13.0	73.6	2.8	0
1-2B	1.2×10^{-11}	10.0	15.0	0	0	0	0.7	16.3	64.1	3.9	0
3-2B	6×10^{-12}	7.5	17.5	0	0	1.0	6.4	29.0	38.1	16.8	1.0
4-2B	4×10^{-12}	5.0	20.0	0.2	0.7	5.4	18.5	23.7	24.0	22.7	5.4

Table 3.

Concentration factor; equals ratio of △'s compared	△'s compared	$\frac{T_{high}}{T_{low}}$	△'s compared	$\frac{T_{high}}{T_{low}}$	△'s compared	$\frac{T_{high}}{T_{low}}$
2	0.5 vs 1	1.1	1 vs 2	1.2	5 vs 10	1.6
5	0.5 vs 2.5	1.4	1 vs 5	1.8	2 vs 10	2.6
10	0.5 vs 5	1.9	1 vs 10	2.9	1 vs 10	2.9

compared – the concentration factor – varies from 2 to 10. In Fig. 2 T_{max} is shown plotted against △, from which is seen that the rate of increase of T_{max} becomes greater as △ becomes smaller. The more the energy converted to heat is concentrated in space the more strongly is the maximum temperature increase affected. (It is evident that if any finite amount of heat is generated in a region, the temperature increase must approach infinity as the volume of the region – represented by △ – approaches zero. Conversely, as the volume approaches infinity the temperature increase must approach zero if a finite amount of heat is generated uniformly throughout the volume.)

The computations assume that all of the heat generated which is not dispersed by thermal conduction is used to raise the temperature of the solids of the representative column and that there are no simultaneously induced heat 'sinks'. In consequence, the temperatures given must be regarded as maximum attainable temperatures. Furthermore, in attempting to correlate a given rate of heat production with deformation of the crust – and thereby strain rate – frictional generation of heat was assumed (Reitan in press). As slip along interfaces giving rise to frictional heat is the least efficient deformation, the indicated temperature increases associated with these strain rates of deformation must be regarded as maximal. Therefore, on both counts, the temperature increases indicated by these models in response to a given rate of strain for a given time period should be regarded as estimates of the probable upper limit of what might be reached in any natural process.

Conclusions

It is demonstrated by these model studies that significant temperature increases in portions of the earth's crust are possible in response to the heat generated by conversion of mechanical energy to heat during deformation. Frictionally generated heat is most likely to be realized when strain rates are not very low. Strain rates in the order of 10^{-14} sec^{-1} might well be too low for any significant slip along interfaces to occur with concomitant frictional generation of heat. But, if strain rates become rather high, the work done on the region undergoing deformation becomes proportionally larger. If therefore, considerations are to be restricted to cases in which only modest total amounts of energy are converted to heat, the volume involved in the deformation must be restricted. These model studies show that in exactly these kinds of circumstances – moderately high strain rates maintained over geologically relatively short time intervals and restricted to rather narrow zones – significant temperature increases will result which may be attributed to the conversion of mechanical energy to heat. It appears that for large temperature increases to occur *absolute* degree of concentration of energy conversion to heat is more important than *relative* degree of energy conversion to heat. In rather confined zones of intense deformation – zones 2 km or less across, for example – temperature increases approaching or in excess of 100 degrees may be possible. If these conditions are realized during deformation in tectonically active regions, mechanically generated heat may locally contribute significantly to the total heat budget in dynamothermally metamorphosed volumes of the crust and may be of significance in the attainment of the abnormally steep or the occasionally 'reversed' geothermal gradients sometimes indicated by the application of theoretical and laboratory mineralogical and assemblage equilibrium studies to the interpretation of natural occurrences.

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