PRELIMINARY THEORY OF THE WAIRAKEI GEOTHERMAL FIELD

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Summary

By making quantitative comparisons between theory and observations it is shown that hydrostatic theories cannot account for the pressure changes which have occurred in the Wairakei geothermal field. The pressures at three representative dates can, however, be obtained on the theory that the hot water flows according to Darcy's law under the influence of buoyancy forces resulting from the surrounding cold ground water, allowance being made for the effect of the uncased parts of the bores in increasing the vertical permeability of the hot region. According to this theory a steady rate of drawoff from the bores will result after a period of the order of one year in an almost steady pattern of pressure with depth, the greater the drawoff the lower being the pressure at any depth. The ultimate source of most of the water withdrawn is the cold ground water surrounding the hot region, which is heated by the hot rock through which it flows and in so doing cools the outer parts of the hot region. The steady reduction in width of the hot region caused by this lateral inflow extends much deeper than the botom of the bores, and at the 1963 rate of drawoff will lead to a reduction of 10% in the radius of the hot region over a period of about 70 years. This contraction of the hot region will cause a slow fall in pressure if the rate of drawoff is held constant.

INTRODUCTION

During the past decade, the rate of artificial drawoff of hot water from the Wairakei field has shown an irregularly increasing trend (Fig. 1), reflecting the increasing capacity of the installed power-generating plant and the fluctuation in the load. Over the same period the pressure of the hot water at various points, all at a depth of about 2,200 ft (670 m) below the ground surface, has steadily fallen (Fig. 2). These pressures were all measured in bores from which no surface discharge was being taken, and at a sufficient depth to be below the water level (that is, there was no steam present). By assuming a pressure gradient corresponding to the hydrostatic pressure of water at the appropriate temperature, they were adjusted to give the pressure at one particular depth, RL–900.

There have been no continuous measurements in any one bore, but the fragmentary records fall naturally into certain groups indicated by the letters A to D in Fig. 2. Group A consists of five bores all within 0·17 km of one another and within the same distance of power-producing bores. Group C consists of two bores about 1 km apart and each about 1·5 km south of the production bores. D represents a single bore about 1 km east of the production area, and about 0·3 km from the Waikato River. Group B consists of eight bores drilled during 1959 to the west of the production area, spread

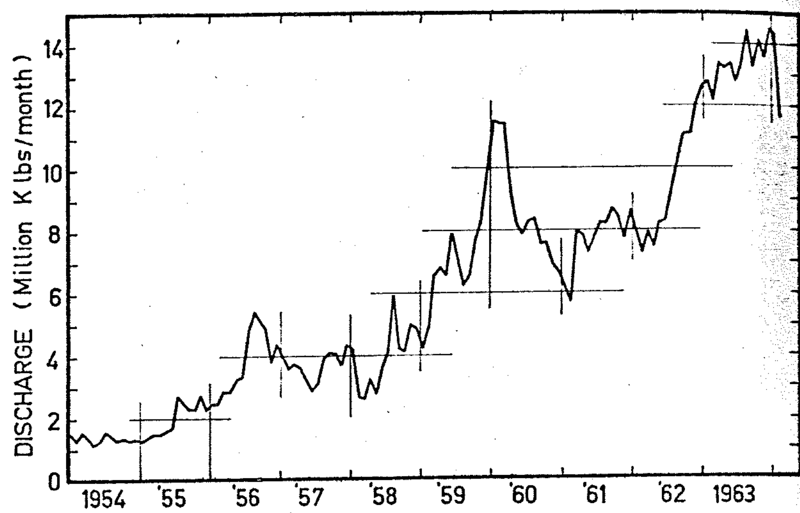

FIG. 1—Total discharge from all Wairakei bores. (Reproduced from Ministry of Works report by R. S. Bolton.)

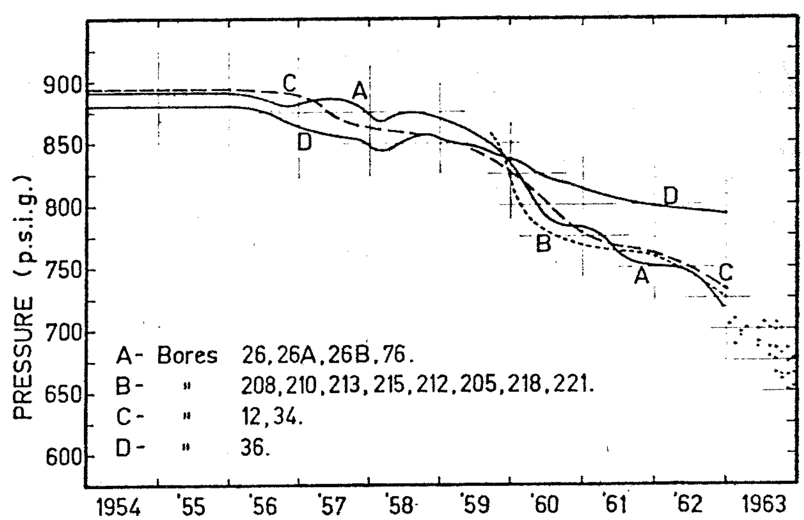

FIG. 2—Aquifer pressures at Wairakei, at reduced level (R.L.) —900, i.e., 900 ft below the survey datum. (Reproduced from Ministry of Works report by R. S. Bolton.)

over a kite-shaped area about 3·4 km long by 3·0 km wide, the mean distance from the nearest production bores being about 2·6 km.

It is obviously desirable to find some theory which will explain these observed pressure changes, in order that the geothermal field may be exploited to best advantage, and that an estimate can be made of its useful life. It will be shown in this paper that the analysis of existing records disproves certain hypotheses which come to mind, but that a satisfactory theory can be developed, which can be refined as more data are obtained.

CLOSED TANK HYPOTHESES

At depths more than a few hundred metres below ground level, where there is no steam present, the Wairakei geothermal field consists of an area in which the ground water is at a temperature of approximately 250°C, surrounded by a boundary layer a few hundred metres wide in which the temperature falls steadily to normal values.

The first question that arises is whether there is any hydrological connection between the regions with hot and cold ground water: that is, is there an impermeable wall around the hot region? Consider the hypothesis that the hot region is surrounded on all sides and below by such an impermeable barrier.

Since the water inside this barrier is to a first approximation all at the same temperature, convection currents will be negligible and the pressure at any point will be given by the theory of hydrostatics:

$$\text{Pressure, } p = -\int_0 \rho(z) g \, dz$$

Applying this formula to the temperature-versus-depth observations on bore 19 in 1953 (Banwell, 1957), assuming the presence of liquid water at all depths, one obtains the value 801 p.s.i.g. for the pressure at depth 670 m. Figure 2, however, records pressure measurements at this depth which are more than 90 p.s.i. higher than this value.

When the calculated hydrostatic pressure at other depths is compared with the minimum pressure required to keep water liquid at the temperature observed at the same depth, the calculated pressure for all depths between 150 m and 400 m is found to fall short of that required, the greatest deficit being about 60 p.s.i.

This hypothesis must therefore be discarded since it cannot account for pressures as high as those observed. Consider next what might be called the "pressurised tank" hypothesis, in which we assume not only impermeable sides and bottom to the tank but a very low permeability lid. In this case some constant may be added to all pressures below this lid, so accounting for the observed high pressures.

The difficulty with this hypothesis is how to explain the observed rate of natural outflow of hot water and steam from the field, which is of the order of 400 kg/sec, or 5×10^5 cm^3/sec of water at 250°C. Since we have postulated that there is no inflow, the outflow can come only from the expansion of water (and rock) in association with a steady decrease in pressure. Now estimate this rate of pressure drop.

If the volume of the tank is taken as 25 km³, corresponding to radius and depth each about 2 km, and the porosity is given the rather high value of 0·2, the volume of water is 5 km³. Taking the compressibility,

$$\beta = \frac{1}{V}\left(\frac{\delta V}{\delta p}\right),$$

of water at 250°C and 40 bar pressure as 70×10^{-6} bar^{-1}, we have

$$\text{Rate of pressure change} = \frac{dp}{dt} = \frac{1}{\beta V} \cdot \frac{dV}{dt}$$

$$= \frac{1}{70(10^{-6})5(10^{15})}(-5 \times 10^{5})31{\cdot}56 \times 10^{6}\ \text{bar/yr} = -45\ \text{bar/yr} \simeq -650\ \text{p.s.i./yr}$$

Thus even if the volume of the field were 10 times greater than assumed here, and the effective compressibility of the water also 10 times greater, the observed pressure excess over hydrostatic values would disappear in less than 20 years—much less than the known historical age of the field. This hypothesis also is therefore untenable.

The conclusion to be drawn at this stage is that no hydrostatic theory is satisfactory, whether the pressure at ground level is taken as atmospheric or some higher value. The only alternative is that there must be an inlet into the hot region somewhere, in which case there must be a potential gradient (that is, a pressure gradient additional to the hydrostatic one) which drives the hot water through the region from the inlet to the natural outlets of the ground surface. Before considering hydrodynamic models of the Wairakei field, however, it is desirable to consider in general terms what happens when hot and cold water are present together in one porous medium.

BEHAVIOUR OF AN ISOLATED HOT REGION IN A GROUND WATER FIELD

To fix our ideas, consider a roughly spherical region about 4 km in diameter in which the temperature everywhere is 250°C, situated in a saturated isotropic porous medium at 20°C (Fig. 3). Thermal conduction will immediately begin to reduce the steep temperature gradient at the boundary. The time for the thickness of the boundary layer to reach about 20% of the radius is given by

$$t = (0{\cdot}005r^{2})/\alpha \qquad \text{(Carslaw \& Jaeger, 1947)}$$

Taking the thermal diffusivity α as 0·003 cm²/sec,

$$t = \frac{0{\cdot}005\ (2 \times 10^{5})^{2}}{0{\cdot}003}\ \text{sec} \simeq 6{\cdot}7 \times 10^{10}\ \text{sec} \simeq 2{,}000\ \text{years}$$

It takes eight times longer for the temperature at the centre to begin to fall. Evidently the hot region will not diffuse away while we are considering what else may befall it.

If all the water were at rest the isobars would be as shown schematically in Fig. 3. The density of water at 250°C being only 0·8 g/cm³, the isobars in the hot region are spaced 25% further apart than in the cold, and the pressure at A is greater than B which is at the same depth. The state of rest is therefore unstable, being upset in the first place by inflow of cold water below the hot region. One can see that a quasi-steady state will arise, shown schematically in Fig. 4. Upflow in the hot region compresses the spacing of the isobars there, while downflow in the surrounding cold water widens the spacing, still leaving an outward pressure gradient above the hot region and an inward gradient below. Water circulates in the direction ABCDA.

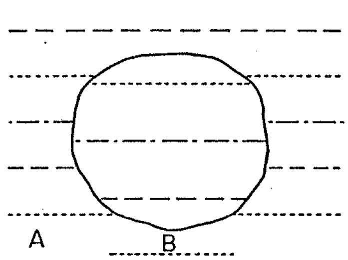

FIG. 3—Hydrostatic isobars in and near an isolated hot region (schematic).

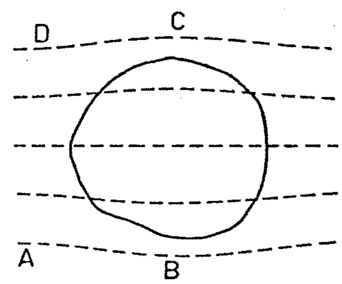

FIG. 4—Isobars in the presence of vortex flow (schematic).

This state is described as quasi-steady because the inflow of cold water into the hot region at B, and the outflow of hot water at C must disturb the original boundaries of the hot region. One might expect at first sight that the effect of this flow across a thermal boundary would be to cause a degradation of the high temperature as cold water permeates hot rock, or as hot water permeates cold rock, resulting in a steady broadening out of the boundary layer at the top and the bottom of the hot region. Provided, however, that the thermal diffusivity is not too small in relation to the linear dimensions of the pores in the porous medium and the flow rate of the water, i.e., provided the Péclet number, length × velocity ÷ thermal diffusivity, is not too large, the water and the rock in any locality will come into thermal equilibrium with each other in a time short compared with the time for the water to traverse the thermal boundary layer. In this case the effect of an upward flow of water through the hot region is to displace the hot region bodily upward without affecting its shape or boundary layer thickness. In a study of the heat-pulse method of measuring sap flow in conifers (Marshall, 1958), I verified experimentally that the same phenomenon was occurring (on a much smaller scale). In that study I gave an equation relating the velocity of the heat pulse to what I called the "sap flux", which is identical to the "filter velocity" or "macroscopic velocity" q used in the expression for Darcy's law:

$$q = -(k\rho/\mu) \text{ grad } \phi \tag{1}$$

k is the permeability of the porous medium, ρ and μ the density and viscosity respectively of the water, and ϕ the potential of the water defined by:

$$\phi = gz + (p - p_0)/\rho \tag{2}$$

where z is the height and p the pressure.

That equation is

$$\text{Heat-pulse velocity} = (\rho c/\rho' c')q$$

where c is the specific heat of the water, and ρ' and c' are respectively the density and the specific heat of the saturated porous medium. Typical values of $\rho' c'$ for various types of rock lie between 0·52 and 0·55 c.g.s. units, which will also be the value for saturated rock if the porosity, f, is small. The velocity of a hot region in saturated ground may be nearly twice the filter velocity measured in the cold region. In ground with high porosity the multiplying factor decreases but is still greater than 1·0. The actual mean velocity of the water is given by q/f, which is much greater than the filter velocity if the porosity is small.

This mechanism leads to some rather surprising results. Consider the situation when the 4 km diameter hot region has risen a distance equal to its diameter, whether due to the vortex induced by its own buoyancy or to some other upward stream of ground water. This might take about 10^3 years. The major part of the region is still at 250°C, although the width of the boundary layer has increased somewhat, due to thermal diffusion. The rock which was originally at 250°C is now cold, while other rock which was cold is now 250°C. And if the porosity is 5% there will have been about 10 changes of water in the hot region during this period. The heat "pulse" has an existence of its own apart from the matter which contains the heat at any time; the flowing ground water can transport this heat without degrading its temperature. This process has important implications for the Wairakei field.

When the Péclet number increases beyond the values for which water and rock at any point have the same temperature, the first effect is for the heat pulse in the water to be displaced slightly downstream from the heat pulse in the rock; temperature changes in the water lag behind those in the surrounding rock, the temperature difference being proportional to the rate of flow. (The Péclet number considered here is based on the pore size, and is therefore a "microscopic" Péclet number which must be distinguished from the "macroscopic" Péclet number considered by Wooding (1963) which is based on the size of the whole hot region.)

As the hot region rises under the influence of its buoyancy-induced vortex the question arises whether or not it will retain its original shape. Expressions for the rate of movement of a hot region of ellipsoidal shape can be derived from the solutions for a fluid mass imbedded in another fluid in a porous medium (Yih, 1965). The situation treated by Yih is not the same as that considered here, since in his case the two fluids are different and not interconvertible (e.g., oil and water) and he consequently uses the purely kinematic boundary condition that the normal components of the filter velocities on each side of the interface must be the same. In our case fluid flows across the boundary, being converted from one fluid into the other as it does so (e.g., hot water becomes cold water). The boundary condition appropriate to our case is that the normal component of the mass flow (density × filter velocity) must be the same on each side of the interface. When Yih's derivation is repeated with this new boundary condition, the effect on his result equations (66) is that in the denominators (only) μ_1 must be replaced by $(\rho_2/\rho_1)\mu_1$ (Yih's notation).

For the particular case of a spheroidal hot region with axis of revolution of length $2a$ vertical, and maximum radius r, the upward filter velocity when the surrounding cold water is at rest is given by

$$q_1 = \frac{k(\rho_2 - \rho_1)\, g}{\mu_1 + \varepsilon\mu_2} \qquad (3)$$

where subscript 1 refers to the hot region and subscript 2 to the cold, and

$$\varepsilon = \frac{\rho_1}{\rho_2}\left(\frac{\alpha_0}{2 - \alpha_0}\right)$$

$$\text{where } \alpha_0 = ar^2 \int_0^\infty \frac{d\lambda}{(a^2 + \lambda)^{3/2}\,(r^2 + \lambda)} \; .$$

ε goes from zero to infinity as the ratio r/a does the same, and in particular when r/a has the values of 0·5, 1, and 2, ε is respectively 0·168, 0·400, and 0·892

It may be remarked in passing that Yih wrongly identifies the velocity of the fluid mass with the (filter) velocity in that mass. In fact the velocity of the mass is equal to the filter velocity divided by the porosity. A simple numerical example will emphasise the difference between Yih's case and ours. For a porous medium with 5% porosity in which $\rho_1 c_1/\rho' c'$ has the value 2, a mass of some fluid other than water having the same density and viscosity as water at 250°C will move at 20 times the filter velocity within the mass, but if the part of the medium containing this fluid is heated to 250°C and the fluid itself replaced by water at 250°C, this hot region will move at only twice the filter velocity of the hot water, and this filter velocity is not the same as that in the previous fluid mass because of the difference in boundary condition just discussed. The physical reason for this contrasting behaviour is that in the case of the two fluids the porous medium is simply an unchanging framework within which the action occurs, whereas in the case of the hot region the porous medium is involved in the temperature changes as well as the fluid, and the fluid has to transport the heat of the medium as well as its own heat.

Returning now to the question of possible changes in the shape of the hot region as it rises, one might surmise from the expectation that the system will tend to dissipate its gravitational potential energy as rapidly as possible, that the hot region will tend to become a tall, thin column, or perhaps a number of such columns, this configuration giving the lowest value for ε and therefore the greatest value for q_1 (c/f Yih, p. 228). Equation (3) is easily verified for this extreme case, for the downward flow in the cold region, having a much larger cross-sectional area to flow through than the slender hot column, has negligible velocity. Thus from equation (1), $\partial\phi_2/\partial z = 0$. By eliminating the pressure term p between the two forms of equation (2) corresponding to hot and cold water, one obtains for the relation between the two potentials at any depth,

$$\rho_1\phi_1 = \rho_2\phi_2 - (\rho_2 - \rho_1)gz$$

Differentiating, $\rho_1(\partial\phi_1/\partial z) = \rho_2(\partial\phi_2/\partial z) - (\rho_2 - \rho_1)g$

Hence since $\partial\phi_2/\partial z = 0$, $\partial\phi_1/\partial z = -[(\rho_2 - \rho_1)/\rho_1]g$ (4)

Since there is no horizontal flow except near the end of the column the values of the potential and therefore its gradient must be the same in both the hot and the cold regions. Substituting (4) in (1),

$$q_1 = -\frac{k\rho_1}{\mu_1}\frac{\partial\phi_1}{\partial z} = -\frac{k\rho_1}{\mu_1} \times -\left(\frac{\rho_2 - \rho_1}{\rho_1}\right)g = \frac{k(\rho_2 - \rho_1)g}{\mu_1} \qquad (5)$$

When an initially spherical hot region becomes tall and rod-like as it rises, the effect of thermal conduction becomes increasingly important and it will eventually dissipate by thermal diffusion.

Equation (3) indicates an interesting contrast between the behaviour of a hot spot in an infinite cold region, and a cold spot in an infinite hot region. Since the viscosity of water at 250°C is only one-tenth of its value at 20°C ($\mu_1 = 0{\cdot}001$ poise as opposed to $\mu_2 = 0{\cdot}01$) the rate of rise of the hot spot is quite sensitive to its shape, whereas the rate of fall of the cold one is not. When $r/a = 1$ the value of ε is 0·4 so q_1 for the hot spot is only one-fifth of its maximum value. In contrast, for a cold spot with $r/a = 1$, q_2 is only 4% less than its maximum.

It is interesting to consider the case of a small cold spot inside a hot region which in turn is contained in an infinite cold region. If the hot region is rising at the maximum rate, so that its filter velocity is given by equation (5), then if the cold spot is tall and thin it will remain stationary. For in this case ϕ_2 is constant everywhere, and there is zero flow in the cold spot just as there is in the outer cold region. The cold spot, however, will soon begin to warm up by thermal conduction and begin to rise. If on the other hand r/a for the cold spot is significantly greater than zero, it will rise but at a rate which is slow compared with the filter velocity in the hot region. This is because most of the upwelling hot water bypasses the cold spot which is a high resistance region because of the high viscosity of the cold water. The upward velocity of the cold water within the spot is consequently much lower than that of the surrounding hot water. Thus a cold spot must either remain stationery or rise slowly in a hot region which is rising at the maximum rate. If, however, the hot water is rising at a rate more than a few percent lower than the maximum a cold spot in it will fall.

Somewhat different from the case of an isolated hot region is that of a steady source of hot water. The steady-state upward filter velocity of the hot water is determined by the potential gradient in the cold region and is not affected by the strength of the source. If this strength is suddenly increased, the increased pressure causes an outward, horizontal flow and the greater magnitude of the source is accommodated by a greater area of the hot column, the actual velocity being the same as before.

This discussion so far has assumed uniform permeability. If the upwelling hot column encounters a horizontal layer of lower permeability the velocity must decrease, by equation (1), and the area increase to provide for the constant total flow. These changes cannot occur suddenly right at the boundary, and consequently for some distance either side of a boundary between layers of different permeability there is a departure from the pattern of uniform potential gradient and parallel, upward flow.

A typical example is shown in Fig. 5 for the case of a hot cylindrical column encountering a layer 200 m thick in which the permeability is only a quarter of its value elsewhere. When the continuity equation is combined with Darcy's law, equation (1), one obtains Laplace's equation, $\nabla^2\phi = 0$,

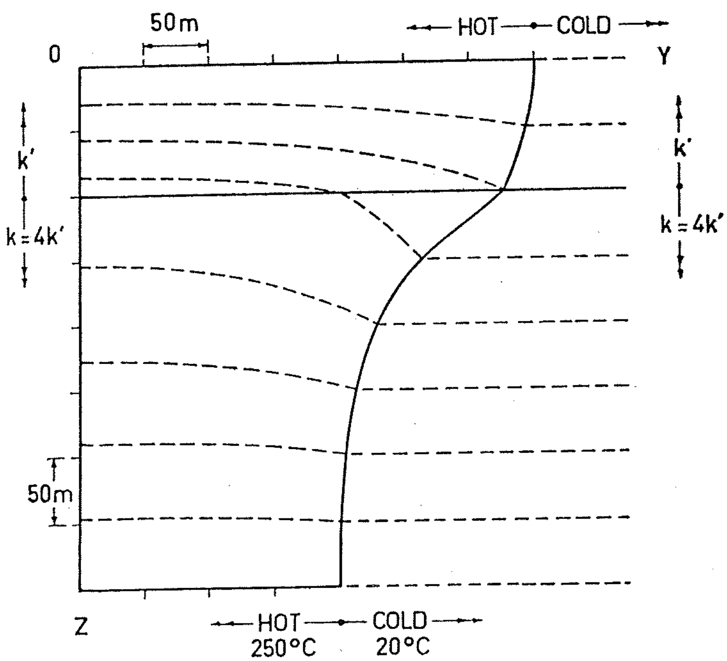

FIG. 5—Theoretical equipotentials for the steady Darcy flow of a column of water at 250°C, where it encounters a layer with permeability only one quarter of that of the rest of the aquifer. OY is the mid-plane of this layer; OZ the axis of the column.

(If the horizontal unit of length were 500 m instead of 50 m, this would be the distribution of potential for ground in which the horizontal permeability is 100 times greater than the vertical permeability.)

for the steady flow case (e.g., Flügge, 1962). The solution in Fig. 5 was obtained by numerical analysis. Notice that the diameter of the column begins to expand some 200 m below the boundary with the less permeable layer. Although the surface of the hot column is everywhere in equilibrium with the cold water at rest which surrounds it, the potential at the central axis builds up to higher magnitudes which have the effect of adding a radial outward component to the flow, increasing the diameter of the column, and at the same time increasing the vertical potential gradient through the less permeable layer. In this particular case the pressure on the axis rises up to 21 p.s.i. above the pressure at the same depth in the cold region. In spite of this outward pressure gradient the whole system is in equilibrium, and if it is disturbed, for instance by withdrawing hot water from the high pressure region, causing even quite a small drop in pressure there, the diameter of the column must contract. That is, cold water will flow laterally into the hot region even though there is an outward horizontal pressure gradient there.

Some other solutions for particular cases indicate the general pattern: with values as in the first example except that high and low permeability layers 400 m thick alternate, the maximum pressure rise on the axis increases slightly to 23 p.s.i., and the nature of the solution shows that further thickening of the layers will not affect this value. (The equipotentials have become parallel and horizontal as at the bottom of Fig. 5.)

When one takes the two-dimensional case of a vertical wall of rising hot water of the same minimum thickness as the minimum radius of the previous cases (namely, 200 m), and also makes the low permeability half of the high (rather than a quarter) the same maximum pressure rise 23 p.s.i., is obtained. When the minimum thickness is increased to 500 m the maximum pressure rise increases to 34 p.s.i., in this two-dimensional case.

FORMULATION OF A DIFFERENTIAL EQUATION

It was concluded in the section on closed tank hypotheses above that pressure measurements at Wairakei indicate the existence of an inflow into hot water region. What is more, the general similarity of the pressures in virtually all bores at any one time (Fig. 2) indicates that the inlet to the hypothetical tank consists of the whole bottom of the tank; in other words, the region penetrated by the bores is the top end of a rising column of hot ground water. Although there can therefore be no impermeable bottom to the tank, the question is still open whether there might be an impermeable wall round the sides down to the depth of the deepest bores. In the absence of such a wall the observed fall in hot water pressures in recent years would induce a lateral inflow from the surrounding cold ground water. If the Péclet number of this inflow (based on pore size) were small enough, however, the incoming water would heat up to the prevailing high temperature in the geothermal field as it passed through the boundary layer, so such inflow would not necessarily cause any observable lowering of the temperature at the bores. The long term effect of any lateral inflow would be a steady decrease in the width of the field; and any bore in the thermal boundary layer (i.e., where there is a horizontal temperature gradient) would become steadily cooler. It may be remarked that such a process would sweep up the heat stored in the peripheral rock of the field with 100% efficiency.

In attempting a quantitative treatment of the flow through an actual ground water field one immediately faces the difficulty that the permeability may vary considerably from place to place in a manner that cannot be simply determined from other physical measurements. This is in contrast to theoretical hydrostatic pressures deduced above which depend only on gravity and water density, the latter being a known function of temperature which itself does not change unpredictably from place to place. In dealing with porous flow one has no option but to choose the simplest configuration of permeability which does not violate known facts, and which one judges will not oversimplify to the extent of removing the cause of the phenomena one hopes to explain. For the purpose of this mathematical treatment I postulate the configuration shown in Fig. 6. There are two regions in each of which the temperature is uniform, and the thickness of the boundary layer is neglected. The hot region is a solid of revolution about the z-axis. In the surrounding cold region the permeability k is uniform and isotropic. (The hypothesis of an impermeable wall corresponds to $k = 0$.) In the hot region the horizontal permeability is taken to be infinite and the vertical permeability in the undisturbed, natural state to be uniform with depth and equal to k_{30}.

My justification for the rather drastic assumption of infinite horizontal permeability is the similarity of the curves A, B, and C in Fig. 2. This assumption removes the possibility of explaining the slight differences between these curves, but the aim of the present work is to account for the features they have in common.

The assumption of uniform initial vertical permeability is not as restrictive as it might seem because, as will be shown below, the effect of drilling the

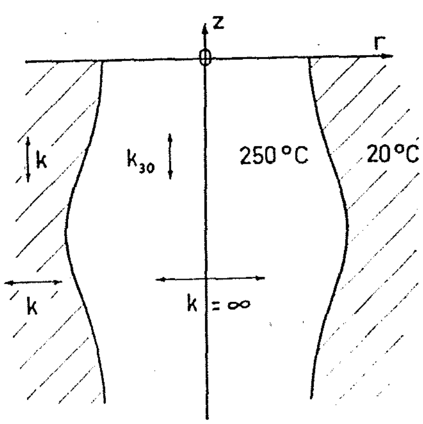

FIG. 6—The configuration of permeabilities used in the mathematical analysis. k_{30} is the original vertical permeability in the hot region, assumed constant. In general, when this vertical permeability varies with depth, as after drilling the bores, it is denoted by k_3.

bores has been to increase the effective vertical permeability very considerably, making any variations in the initial permeability of secondary importance.

It is at shallow depths that the model in Fig. 6 most obviously differs from reality. In the first place it does not allow for the observed fall in temperature of the hot region as the ground surface is approached. The assumption, however, of uniform temperature and vertical permeability would be mathematically duplicated if the actual permeability were such that $k_3\rho/\mu$ remained constant with depth, and as just remarked the precise variation of k_3 with depth is not of critical importance.

More important is the assumption that the ground remains saturated with hot water right up to the surface, whereas at depths less than 400 m pressures are in fact not great enough to prevent some water flashing off as steam. Some implications of the resulting two-phase flow will be discussed below but for the present its effect is ignored.

In a region where permeability and viscosity are constant the equation for fluid flow is (Flügge, 1962, chap. 88)

$$k/(\mu f \beta)\, \nabla^2 p = \partial p / \partial t$$

which has the same form as the heat conduction equation with the diffusivity replaced by $k/(\mu f \beta)$. With $f = 0{\cdot}2$ and $\beta = 70 \times 10^{-6}$ per bar as used above, $\mu = 10^{-3}$ poise for water at 250°C, and $k = 2{\cdot}5 \times 10^{-11}$ cm², this diffusivity equals

$$\kappa = \frac{2{\cdot}5(10^{-11})}{10^{-3}(0{\cdot}2)70(10^{-12})} \text{cm}^2/\text{sec} = 1{\cdot}8 \times 10^3 \text{ cm}^2/\text{sec} = 0{\cdot}015 \text{ km}^2/\text{day}$$

If the pressure is suddenly changed at a point distant x from another point where the pressure is held constant, the time taken for a uniform pressure gradient, and therefore steady flow, to be established between the two points is given approximately by $t = \dfrac{0{\cdot}2\, x^2}{\kappa} = 13\, x^2$ days, in the present case (for x in km). Thus for $x = 2$ km, t is less than two months. As will be shown below, the value taken for k is of the right order for the vertical permeability in the hot region. In the cold region μ is 10 times greater, but so also is the permeability derived below using steady-state theory. The assumption of steady-state conditions now made in obtaining the differential equation seems therefore not unreasonable in retrospect. For even in the cold region it should not take more than a few months to establish steady flow conditions over a region within a few kilometres of some point of disturbance to a previous steady state, and we are dealing here with pressure changes occurring over a period of some years.

Let v and w be the filter velocities in the r and z directions respectively, and subscript 1 refer to the hot region, and subscript 2 to the cold.

The continuity condition requires that:

$$\pi R^2 \rho_1 w_1 + 2\pi R\delta R\ \rho_2 w_2 - \pi(R+\delta R)^2 \rho_1 (w_1 + \delta w_1) - 2\pi R \delta z \rho_2 v_2 = 0$$

where $R(z)$ is the radius of the boundary between the hot and cold regions. This reduces to:

$$2\rho_2 v_2 + 2(\mathrm{d}R/\mathrm{d}z)\ (\rho_1 w_1 - \rho_2 w_2) + R\rho_1\ (\partial w_1/\partial z) = 0 \qquad (6)$$

Because of the infinite horizontal permeability in the hot region ϕ_1 there must be a function of z only and w_1 also is therefore constant for a given value of z. The v_2 and w_2 in equation (6) refer to their values at the boundary, R. Using Hubbert's notation (1940), that $\phi_i)_k$ is the value of ϕ of the ith fluid in the region occupied by the kth fluid,

$$\left.\begin{aligned} &\text{we have} \quad v_2 = -\frac{k\rho_2}{\mu_2}\left(\frac{\partial \phi_2}{\partial r}\right)_2 \\ &\text{and} \quad w_2 = -\frac{k\rho_2}{\mu_2}\left\{\left(\frac{\partial \phi_2}{\partial z}\right)_1 - \frac{\mathrm{d}R}{\mathrm{d}z}\left(\frac{\partial \phi_2}{\partial r}\right)_2\right\} \\ &\text{together with} \quad w_1 = -\frac{k_{30}\rho_1}{\mu_1}\left(\frac{\partial \phi_1}{\partial z}\right)_1 \end{aligned}\right\} \qquad (7)$$

Before equations (6) and (7) can be used to obtain a differential equation relating the potential to z, some expression must be found for $(\partial\phi_2/\partial r)_2$, the radial gradient of ϕ_2 on the cold side of the boundary. This can be done by assuming that at some outer radius r_e the cold water is unaffected by disturbances in the hot region. If the cold water at $r = r_e$ is at rest, the potential ϕ_2 there is zero.

In the steady state the potential like the pressure satisfies Laplace's equation. Solution of this equation with the boundary conditions:

$$\begin{aligned} (\phi_2(r,z))_2 &= 0 \text{ at } r = r_e \\ \text{and } (\phi_2(r,z))_2 &= (\phi_2(z))_1 \text{ at } r = R \\ \text{gives } (\partial\phi_2/\partial r)_2 &= (\phi_2)_1/(R \ln R/r_e) \end{aligned} \qquad (8)$$

Equations (6), (7), and (8), together with the equation relating ϕ_1 and ϕ_2:

$$\rho_1\phi_1 = \rho_2\phi_2 - (\rho_2 - \rho_1)gz$$

(which follows from equation (2)), lead to the differential equation

$$\frac{\mathrm{d}^2\phi_2}{\mathrm{d}z^2} + \frac{2}{R}\cdot\frac{\mathrm{d}R}{\mathrm{d}z}\left(1 - \frac{k\rho_2\mu_1}{k_{30}\rho_1\mu_2}\right)\frac{\mathrm{d}\phi_2}{\mathrm{d}z}$$

$$-\left\{1 + \left(\frac{\mathrm{d}R}{\mathrm{d}z}\right)^2\right\}\frac{k\rho_2\mu_1}{k_{30}\rho_1\mu_2}\cdot\frac{2}{R^2 \ln r_e/R}\phi_2 = \frac{2}{R}\frac{\mathrm{d}R}{\mathrm{d}z}\left(\frac{\rho_2 - \rho_1}{\rho_2}\right)g \qquad (9)$$

The partial differentials have become total differentials since ϕ_2, which is actually $(\phi_2)_1$ the potential which cold water would have in the hot region, is a function of z only.

For any particular choice of permeabilities, temperatures and boundary shape between the two regions, equation (9) can be solved to give the potential and therefore the pressure at any depth. The potential can be used to find the lateral outflow v_2 from equation (7), and therefore the new position of the boundary at some later instant.

THE IMPERMEABLE WALL HYPOTHESIS

In using equation (9) to search for values of the permeability which would correctly predict observed pressure changes, I have chosen initially three observed measurements by which to test possible theories. These are:

(i) The rate of mass discharge (steam and water) for the whole Wairakei geothermal field in its natural state. Fisher (1964) estimates this as 440 kg/sec.

(ii) The pressure at reduced level—900 ft in 1955 (Fig. 2). Most of the measurements lie between 890 p.s.i.g. and 900 p.s.i.g.

(iii) The pressure at the same depth in December 1961. The shape of the graph of total discharge (Fig. 1) during 1960 and 1961 would lead one to expect that if the time constant of the field were of the order of a few months or less, the pressure should be approaching an equilibrium value at this date. Figure 2 shows that for the bore groups A, B, and C this is indeed the case, the pressures lying between 750 and 760 p.s.i.g.

At this stage it is necessary to consider whether the drilling programme has caused any significant increase in the vertical permeability in the hot region. Banwell (1957) tabulates the bore data up to 1955. The lowest few hundred metres of most bores are uncased, so even when a bore is shut off at the surface this uncased length might be expected to serve as an important bypass for the upwelling hot water in the vicinity, which would flow into the bore near the bottom and return to the porous ground below the casing.

At the potential gradients involved the flow in the bores will be turbulent (Reynolds number $\simeq 10^6$) and given by:

$$\Delta p = f(l/d)\ (\rho_1 u_m^2/2) \quad \text{(e.g., Giedt, 1957)}$$

where f is now the friction factor, d the diameter of the bore, and u_m the mean velocity in it. Putting

$$\Delta p/l = \text{grad}\ p = \rho_1\ \text{grad}\ \phi_1$$

and $$\tfrac{1}{4}\pi d^2 u_m = \pi R^2 w_m$$

gives $$w_m = (d^5\ \text{grad}\ \phi_1/8fR^4)^{\frac{1}{2}} = \text{equivalent filter velocity of the bore.} \quad (10)$$

Since the flow in the bores is turbulent whereas the flow in the porous ground is laminar, the ratio of the two types of flow will vary with the potential gradient; as an arbitrary basis for comparison the gradient given by equation (4), corresponding to cold water at rest, was chosen. To arrive at a value for the natural upward filter velocity from the total mass discharge, an estimate for the area of the hot column (i.e., the whole geothermal field) is required. From a map showing hot and cold bores I have chosen a circle of radius 2 km as being of the right order and used it throughout these calculations. If we choose $k_{30} = (2{\cdot}5 \times 10^{-11})\text{cm}^2$, then equation (5) with the values $\rho_1 = 0{\cdot}8\ \text{g/cm}^3$, $\mu_1 = 10^{-3}$ poise for water at 250°C and $\rho_2 = 1{\cdot}0\ \text{g/cm}^3$, $\mu_2 = 10^{-2}$ poise for water at 20°C, leads to w_{10} = natural upward filter velocity $= 0{\cdot}49 \times 10^{-5}$ cm/sec. Multiplication by the area of the field and the density ρ_1 gives for the total mass discharge, 493 kg/sec, which considering the approximate nature of the estimate of the area is near enough to the mass discharge estimated from observations in the field.

In equation (10) I have chosen $f = 0{\cdot}1$ for the friction factor, which according to Rouse and Howe (1953) occurs when the depth of irregularities in the wall surface is about 10% of the bore diameter. This is almost certainly an overestimate of the friction factor for a bore with a slotted liner. In addition, in such bores I have taken the diameter d as that of the liner, neglecting any flow in the slot between the liner and the drilled hole. For these two reasons Fig. 7 is a conservative estimate of the increase in effective vertical permeability caused by the drilling programme.

The hypothesis that there is an impermeable wall between the hot and the cold regions is equivalent, from the point of view of the hot region, to the hypothesis that k, the permeability of the cold region, is zero. If in addition we assume that the radius R is constant with depth, equation (9) reduces to $d^2\phi_2/dz^2 = 0$, or $d^2\phi_1/dz^2 = 0$. (This, of course, could be obtained directly from the continuity equation and the postulated constancy

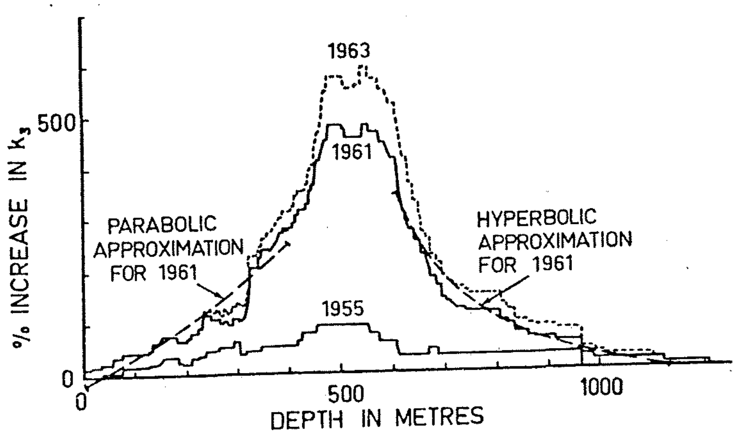

FIG. 7—The calculated increase in effective vertical permeability of the hot region due to the uncased parts of the bores.

of the vertical permeability in the hot region.) In the calculations I have approximated the effect of the bores in 1955 to three zones with a constant increase in permeability, thus:

TABLE 1—Depth Zones for 1955

z (metres)	0 to −130	−130 to −430	−430 to −590	−590 to −965
ε	0	0·48	0·94	0·32

ε is the equivalent increase of permeability due to the bores when the undisturbed permeability k_{30} is taken as $2{\cdot}5 \times 10^{-11}$ cm², and $d\phi_1/dz = -[(\rho_2 - \rho_1)/\rho_1]g = -245$ cm/sec². (For one particular solution I took ε as rising linearly from 0 at $z = 0$ to 0·48 at $z = -260$ m, the values at greater depths being as before. This much closer fit to Fig. 7 made a difference of only 1·5 p.s.i. to the pressure at RL − 900 ($z = -670$ m), indicating that the above lumping of the effect of the bores is not a very drastic approximation.)

The 1961 permeabilities were approximated by a hyperbola in $0 > z > -450$ m, a constant in $-450\text{ m} > z > -610$ m, and a hyperbola again in $-610\text{ m} > z >$ about -1100 m. When k_3 has the form $k_3 = [\alpha k_{30}/(z+\beta)]$, where α and β are known constants obtained by fitting hyperbolas to the appropriate parts of Fig. 7, the continuity equation gives:

$$\text{Const.} = k_3(d\phi_1/dz) = [\alpha k_{30}/(z+\beta)]\,[d\phi_1/dz]$$

By integration, $\phi_1 = c_1(\tfrac{1}{2}z^2 + \beta z) + c_2$

Thus when the permeability in a given zone is represented by a constant or a hyperbola the potential integrates to a simple function of z. The boundary conditions between zones are that ϕ_1, and $k_3(d\phi_1/dz)$ be continuous across the boundary.

Converted into the units of Fig. 1 the natural discharge of the field, 493 kg/sec, becomes 2·358 million Klb per month. From Fig. 1 the discharge from the bores for 18 months prior to mid-1955 is seen to be close to 0·5 of the natural discharge. Similarly at the end of 1961 the average artificial draw-off for the previous two years at about 8·3 millions Klb/mth is 2·9 times the natural discharge.

To estimate the mean depth and vertical distribution of the "sink" through which artificial draw-off is removed from the hot region, I used a table listing the mass flow from each bore at the test measurement nearest to but not later than December 1961, together with the number of months the bore was discharging during 1961. By weighting this data in different ways the following results are obtained:

TABLE 2—Sink Data

Weighting	A	B	C
Mean sink depth (m)	523	507	505
Mean uncased length (m)	229	196	239

A: Weighted according to the mass discharge at the time of test.
B: Weighted according to this mass discharge multiplied by the number of months discharging during 1961.
C: Equal weighting.

The mean sink depth is calculated on the assumption that the draw-off from each bore is uniformly distributed along its uncased (or slot-cased) length. Clearly the mean sink depth and length are continuously changing, but the general similarity of these three estimates encourages one to use them as typical. Before obtaining this data I made a number of calculations assuming the sink was uniformly distributed between −450 m and −610 m (mean depth = 530 m; mean length 160 m). On repeating some of these with the upper limit of the sink raised to −400 m (mean depth 505 m; mean length 210 m), the pressure at −670 m, (RL − 900) changed by only 2 p.s.i., indicating a fortunate insensitivity to the precise position of the sink.

The effect of such a distributed sink is obtained by repeating the development of equation (9) assuming Q cm³ of hot water are removed each second from a horizontal layer of the hot column 1cm thick. The differential equation is changed only by the addition to the right-hand side of a term: $(\mu_1/k_3\rho_2) \cdot (Q/\pi R^2)$. With the present impermeable wall hypothesis, this again integrates simply to a function with two integration constants. With two boundary conditions between adjacent zones and one condition at each end of the whole series there are as many equations as unknown integration constants. When written in matrix form it is easy to arrange that there is no non-zero element more than one place above the leading diagonal. The solution is obtained without excessive labour by first reducing the matrix to a lower triangle by eliminating the unwanted elements in turn.

At the ground surface $z = 0$ the boundary conditions is $\phi_1 = 0$. At the bottom of the impermeable wall ($z = z_b$) we assume the hot region is fed from a reservoir at constant potential. Since on the present hypothesis we are postulating no connections between the hot and the cold regions we can have no preconceived ideas about the value of this reservoir potential, ϕ_{1b}. Choice of a particular value determines $d\phi_1/dz$, which together with the natural discharge rate determines k_{30}. The upward filter velocity in the presence of the bores is given by:

$$\left| w_1 \right| = \frac{k_{30}\rho_1}{\mu_1}\left(\left|\frac{d\phi_1}{dz}\right| + \varepsilon \frac{2\cdot 5 \times 10^{-11}}{k_{30}} \sqrt{245 \left|\frac{d\phi_1}{dz}\right|}\right) \qquad (11)$$

To keep the set of simultaneous equations linear, the method adopted was to solve them first using the values of ε for $k_{30} = 2\cdot 5 \times 10^{-11}$ cm², and then use the mean value of $|(d\phi_1/dz)|$ for each zone of depth from this solution to get corrected values for ε from equation (11). On repeating the solution with the corrected ε the pressure at −670 m usually changed by only a few p.s.i.

A number of solutions were carried out taking the 1961 sink strength as 3·0 times the natural discharge rather than the more accurate value of 2·9. Some of these which illustrate the general pattern are shown in Fig. 8. With the initial potential gradient chosen as $(d\phi_1/dz) = -245$ cm/sec², and various depths $-z_b$ chosen for the reservoir always keeping its potential, $\phi_{1b} = -245 z_b$, the depth $-z_b$ has to be reduced to as little as 1·34 km to bring the pressure at $z = -670$ m (i.e., RL − 900) up to the observed value for this depth, 758 p.s.i.g. (For $z_b = -2$ km, the pressure is 700 p.s.i.g., and for $z_b = -\infty$, it is 628 p.s.i.g.)

For an initial potential gradient of $(d\phi_1/dz) = -175$ cm/sec² corresponding to $k_{30} = 3 \cdot 5 \times 10^{-11}$ cm², the pressure at −670 m when $z_b = -\infty$ is 689 p.s.i.g, rising to 757 p.s.i.g. when $z_b = -1 \cdot 34$ km. With $k_{30} = 3 \cdot 09 \times 10^{-11}$ cm², and $z_b = -1 \cdot 34$ km the pressure is 755 p.s.i.g.

All these solutions were carried through at a time when I was erroneously taking the 1955 sink strength as zero instead of 0·5. For zero 1955 sink strength the pressure at −670 m in 1955 rises from 882 p.s.i.g. when $k_{30} = 3 \cdot 5 \times 10^{-11}$ cm² to 928 p.s.i.g. when $k_{30} = 2 \cdot 5 \times 10^{-11}$ cm². But now using the correct sink strengths for 1955 and 1961, respectively 0·5 and 2·9 times the natural discharge, and choosing $k_{30} = 2 \cdot 5 \times 10^{-11}$ cm² and $z_b = -1 \cdot 40$ km one obtains 896 p.s.i.g. for the 1955 pressure and 755 p.s.i.g. for 1961.

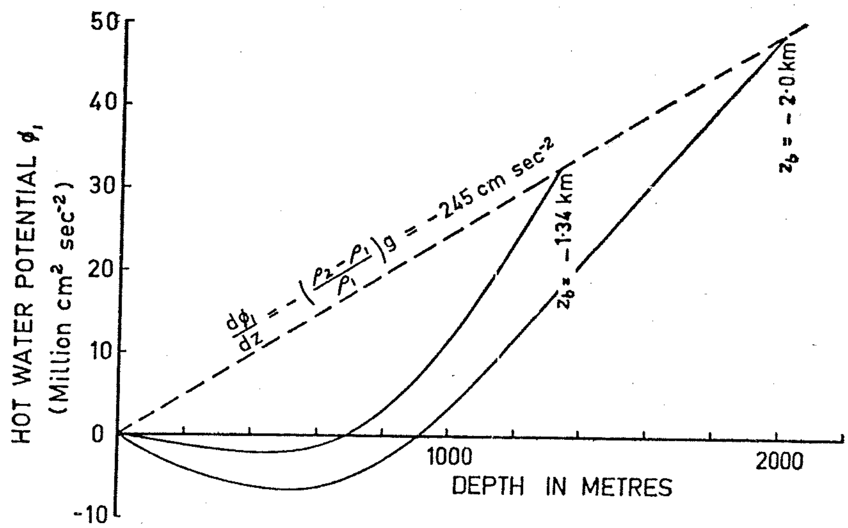

FIG. 8—Typical solutions for the impermeable wall hypothesis.

Thus this choice of k_{30} and z_b allows the impermeable wall hypothesis to be fitted to the three field measurements listed at the beginning of this section. In effect the depth of the reservoir is determined by the 1961 pressure requirement and the permeability by the 1955 pressure.

Before this hypothesis can be accepted as satisfactory, however, it must be tested against further field observations. Consider first the pressures in December 1963. From Fig. 1 the average discharge from the bores during the 10 months before this date was about 13·7 million klb/month, or 4·8 times the natural discharge. With a sink of this strength and the permeabilities as in 1963 one obtains a pressure at $z = -670$m of 675 p.s.i.g. Figure 2 shows, however, that the pressures in fact fell to about 650 to 660 p.s.i.g.—not perfect agreement. This, together with the rather unlikely predictions by this theoretical model of a zero-impedance source only 130 m below the bottom of the deepest bore to date make it desirable to examine the lateral inflow hypothesis, i.e., that the permeability in the cold region is not equal to zero.

THE LATERAL INFLOW HYPOTHESIS

The effect of the bores on the vertical permeability can be incorporated in equation (9) by replacing the constant k_{30} in the third equation (7) by $k_3(z)$, a function of z. On repeating the derivation of equation (9) with this modification, one finds the differential equation unchanged apart from the addition to the left-hand side of the term:

$$\frac{1}{k_3}\cdot\frac{dk_3}{dz}\left\{\frac{d\phi_2}{dz}-\left(\frac{\rho_2-\rho_1}{\rho_2}\right)g\right\},$$

(It is easily verified that when $k = 0$ and $k_3\,(z)$ is a hyperbola, this modified differential equation leads to the same solution obtained more directly in the previous section.) Assuming for simplicity as before that the radius of the hot region R is constant with depth (and $= 2$ km), the differential equation is now:

$$\frac{d^2\phi_2}{dz^2}+\frac{1}{k_3}\cdot\frac{dk_3}{dz}\,\frac{d\phi_2}{dz}-\frac{k\rho_2\mu_1}{k_3\rho_1\mu_2}\cdot\frac{2\phi_2}{R^2\ln r_e/R} = \frac{1}{k_3}\cdot\frac{dk_3}{dz}\left(\frac{\rho_2-\rho_1}{\rho_2}\right)g+\frac{\mu_1}{k_3\rho_2}\cdot\frac{Q}{\pi R^2} \qquad (12)$$

This equation is assumed to hold down to large depths (mathematically, to infinite depth); the undetermined parameter z_b is replaced in this hypothesis by the permeability of the cold region k, as a parameter which can be chosen to make the hypothesis fit the three field measurements listed earlier. The outer radius r_e has also yet to be given a numerical value, but since k and r_e occur only in the third term and in the form $k/\ln r_e/R$ they are not independent and changes in their values have no effect provided the value for this quotient remains unchanged. In fact I chose $r_e = 3$ km because the cold bores 32 and 33 about 1 km away from the production bores have not been affected by the pressure changes in the hot bores.

In the impermeable wall hypothesis the reservoir potential ϕ_{1b}, which determined the undisturbed vertical permeability k_{30}, was also an undetermined parmeter. We achieve a similar freedom with the present hypothesis if we allow for the possibility of a general downflow of cold water in the cold region such as might be induced if the upwelling hot water were heated ground water rather than juvenile water.

Mathematically the effect of this downflow is that at $r = r_e$, $(\phi_2)_2 = Cz$ (C = constant) instead of zero. Equation (8) then becomes:

$$(\partial\phi_2/\partial r)_2 = [(\phi_2)_1 - Cz]/[R\ln R/r_e] \qquad (13)$$

The only effect of this on the differential equation (12) is that ϕ_2 in the third term must be replaced by $(\phi_2 - Cz)$. If one then makes the substitution $\phi_2 = \phi_3 + Cz$, a differential equation in ϕ_3 is obtained which is identical in form with equation (12).

As before the variation of k_3 was handled by dividing the depths into four zones. This time the sink was assumed to be uniformly distributed between -400 m and -610 m, as being a slightly better approximation to reality than the previous zone -450 m to -610 m. In this zone the permeability was taken as $k_3 = \alpha k_{30}$ = constant, leaving equation (12) as:

$$(d^2\phi_2/dz^2) - b_1^2\phi_2 = (\mu_1/k_3\rho_2) \cdot (Q/\pi R^2) \qquad (14)$$

where $b_1^2 \equiv b^2/\alpha \equiv [k\rho_2\mu_1/\alpha k_{30}\rho_1\mu_2][2/(R^2 \ln r_e/R)]$

The solution of equation (14) is:

$$\phi_2 = c_3 e^{b_1 z} + c_4 e^{-b_1 z} - 1/b_1^2 (\mu_1/k_3\rho_2) \quad (Q/\pi R^2)$$

In the lowest zone, below the bores, the differential equation is like equation (14) except that $Q = 0$ and $\alpha = 1$. Since at $z = -\infty$, we want $\phi_2 = 0$ the solution is simply: $\phi_2 = c_7 e^{bz}$.

In each of the zones $0 > z > -400$m and -610 m $> z >$ about $-1{,}100$ m, $Q = 0$ and a fairly simple analytical solution can be obtained if the permeability k_3 has the form:

$$k_3 = \alpha k_{30} (z + \beta)^2 \qquad (15)$$

In this case equation (12) is reducible to homogeneous linear form, giving the solution:

$$\phi_2 = c_1(z + \beta)^{D_1} + c_2(z + \beta)^{D_2} + [(\rho_2 - \rho_1)/\rho_2][2g(z + \beta)/(2 - b^2/\alpha)]$$

where D_1 and D_2 are the roots of the auxiliary equation

$$D^2 + D - b^2/\alpha = 0$$

namely, D_1 and $D_2 = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + b^2/\alpha}$

While the parabolic approximation to k_3, equation (15), is not as close as the hyperbolic approximation (used in the previous hypothesis) in the zone -610 m $> z >$ about $-1{,}100$ m, it is a slightly better approximaion in the shallowest zone. (See Fig. 7.)

The search for the pair of values for k_{30} and k which would best reproduce the 1955 and 1961 pressures proceeded much as for the previous hypothesis, trial solutions gradually displaying how the pressures vary with changing permeabilities. The best result after four solutions at each date was $k_{30} = 2{\cdot}5 \times 10^{-11}$ cm², and $k = 2{\cdot}3 \times 10^{-10}$ cm². These permeabilities make the pressure at $z = -670$ m in 1955, 892 p.s.i.g. and the 1961 pressure 764 p.s.i.g. The indications are that a better fit might be obtained with $k_{30} = 2{\cdot}5 \times 10^{-11}$ cm² and $k = 2{\cdot}0 \times 10^{-10}$ cm²; these should give pressures of about 890 p.s.i.g. and 757 p.s.i.g. respectively.

As before I next checked to see how closely this lateral inflow model would predict the pessure in 1963. With $k = 2{\cdot}3 \times 10^{-10}$ cm² the 1963 pressure came to 665 p.s.i.g. and with $k = 2{\cdot}0 \times 10^{-10}$ cm² it came to 656 p.s.i.g. giving quite satisfactory agreement with Fig. 2.

DISCUSSION

In both the impermeable wall and lateral inflow hypotheses the original vertical permeability in the hot region, k_{30}, was left as a parameter to be determined by fitting the calculations to the field observations. It is significant that in each case k_{30} came out as $2{\cdot}5 \times 10^{-11}$ cm², the value which in the lateral inflow hypothesis corresponds to the cold water at $r = r_e$ being at rest, and which in the impermeable wall hypothesis corresponds to the potential at the bottom of the wall $z = z_b$ being that of cold water at rest. In fact the constant potential reservoir postulated at z_b in this latter hypothesis could mathematically be a continuation of the hot column surrounded now by a cold region with infinite permeability. For on putting $k = \infty$ in equation (9) one obtains the solution $\phi_2 = 0$.

With this in mind it can be seen that the two results form part of a pattern. As shown above, if there were an inpermeable wall extending to great depths, the pressure would have to fall well below observed values in order to supply the 1961 rate of discharge. The amount of pressure drop can be relieved either by replacing the wall below a depth of 1,400 m by unimpeded inflow, or by having some inflow at all depths of an amount determined by $k = 2 \times 10^{-10}$ cm². It is almost certain that numerous other distributions of the permeability k with depth (and azimuth) could be found to fit the 1955 and 1961 pressures, and those in 1963. The value $k = 2 \times 10^{-10}$ cm² is just an average value.

The acceptability of the impermeable wall hypothesis is reduced by considering the effect of allowing for two-phase flow at shallow depths. At high Reynolds numbers the pressure gradient required for two-phase flow at the pressures involved here is of the order of 10 to a hundred times the pressure gradient for liquid flow (Owens, 1962). If this also holds true for Darcy flow, then the original vertical permeability at shallow depths in the hot region must have been 10 to a hundred times greater than assumed above, in order to transmit the natural outflow without a gross increase in the area of the hot column. In this case the relative effect of the bores on the vertical permeability at these depths must be reduced by the same factor. An indication of the effect of two-phase flow should therefore be given by ignoring the effect of the bores in the zone $0 > z > -400$ m.

If as before the depth of the impermeable wall is determined mainly by the 1961 pressure, a quick estimate of z_b in this case can be obtained by finding the value which gives the required pressure in 1961 without bothering about the fit in 1955. The result is that z_b must be raised from $-1{\cdot}40$ km to $-1{\cdot}29$ km, only 20 m below the bottom of bore number 223.

In contrast to this the lateral inflow hypothesis is not shaken by this allowance for two-phase flow, since the parameter k is not bordering on an unacceptable value. In any case a pressure measurement at the bottom of the deepest bores would decide between the hypotheses because it is there that the predicted pressures differ most (by over 100 p.s.i. at $z = -1{,}200$ m in 1963).

By giving a measure of the vertical gradient in the cold region at $r = r_e$ these calculations give an indication of the extent to which the hot water may be recirculating cold ground water. My calculations above indicate that the constant C in equation (13) is not greater than about 5. From this it is easily verified that if the permeability of the cold region is isotropic, the area over which uniform downflow must occur to supply the natural rate of outflow from the hot region is greater than 40 times the area of the hot column. If the cold ground like the hot has greater horizontal than vertical premeability this area must be even greater.

For $C \leqslant 5$, if the cold region has isotropic permeability the downward filter velocity there is not greater than 10^{-7} cm/sec. It is interesting that Wooding (1963) suggests that a downflow of this order could be induced by entrainment of cold water into the rising hot water column. Since my work gives an estimate of the ratio of the permeabilities of the hot and the cold regions, it can be used to estimate the depth of Wooding's "virtual source". The models differ in that Wooding took the permeability as isotropic but temperature dependent, but if my ratio k/k_{30} of cold horizontal permeability to undisturbed hot vertical permeability is equivalent to his permeability ratio k_0/k_1 (an assumption that needs investigation), Wooding's Table 3 shows that the depth to the virtual source is about 9 km.

One of the points of greatest interest in the lateral inflow hypothesis is the actual rate of inflow and the consequent rate of contraction of the hot column. From equations (7a) and (8) the inward filter velocity $-v_2$ in 1963 at the depth of 670 m is $5{\cdot}0 \times 10^{-6}$ cm/sec when $k = 2{\cdot}0 \times 10^{-10}$ cm^2. For $k = 2{\cdot}3 \times 10^{-10}$ cm^2 the velocity is about 10% higher. The velocity is near the peak of a broad maximum at this depth, the value at $z = -1{,}038$ m being only 7% lower, but 22 % lower at $z = -400$ m. It may be noted that this rate of inflow is about 40 times greater than that due to entrainment obtained by Wooding.

As mentioned earlier the boundary between the hot and the cold regions moves at nearly twice the filter velocity in the cold region, i.e., at about 10^{-5} cm/sec or 3·0 m/year. The contraction of a radius of 2 km at this average rate should not cause great concern to power-production authorities.

It is of interest to consider where the water removed from the ground through the bores comes from. On the impermeable wall hypothesis it can come only downward from the ground surface and upward from the reservoir at the bottom of the wall. The solution for 1963 conditions shows 36% of the total drawoff as coming down from the surface. This corresponds to a downflow of nearly twice the natural rate of outflow before drilling began, and does not agree with the observed fact that while the outflow of hot water and steam at the ground surface has been reduced it still has a positive value.

On the lateral inflow hypothesis the source of the water can only be downflow from the surface and lateral inflow from the cold region. This lateral inflow extends to much greater depths than the deepest bores, but ultimately the reduction in potential in the hot column caused by the drawoff through the bores falls to zero, leaving the rate of upflow at great depth unchanged at its natural value. The amount of inflow between two levels is given by the difference between the vertical flows at the two levels, except in the drawoff zone where the sink strength must be taken into account. In 1963 5% of the total drawoff came from lateral inflow between the surface and the depth of 400 m, a further 5% between 400 m and 610 m, and 12% between 610 m and 1,038 m. The upflow at 1,038 m depth accounted for 43% of the drawoff, this representing the lateral inflow at all depths greater than 1,038 m. The remaining 35% comes from downflow at the surface, as in the impermeable wall hypothesis.

It is not surprising that both theories show their greatest divergence from observed facts near the ground surface, since the occurrence of two-phase flow and the fall in temperature of the hot region at shallow depths were ignored in each case. The success of the theory, however, in accounting for the observed pressure changes at depth encourages the hope that shallow depth phenomena might also be predicted correctly if the theory were modified by including these two neglected aspects, and also allowing for possible variations in the permeabilities and radius of the field with depth.

It may be remarked that the author considers the main contribution of this paper to be the demonstration that some of the gross pressure changes with varying rates of drawoff at Wairakei can be reproduced by an admittedly oversimplified model in which some of the physical constants are deduced fairly directly from field observations, while others are inferred by making the model fit a portion only of the pressure-drawoff measurements. Such constants as the area of the hot column were chosen simply as being of approximately the right magnitude, and in fact the next stage in this investigation is to programme these calculations for the electronic computer and discover how sensitive the predicted pressures are to variations in such parameters as the area of the hot column. Thereafter the search must continue for a model which, while still successfully accounting for pressure changes, does not make the incorrect prediction of a considerable downflow of hot water from the surface.

REFERENCES

BANWELL, C. J. 1957: *N.Z. Dep. scient. ind. Res. Bull. 123*: 40–42.

CARSLAW, H. S.; JAEGER, J. C. 1947: "Conduction of Heat in Solids." Clarendon Press, Oxford. p. 201.

FISHER, R. G. 1964: Geothermal Heat Flow at Wairakei during 1958. *N.Z. Jl Geol. Geophys.* 7: 172–184.

FLÜGGE, W. (Ed.) 1962: "Handbook of Engineering Mechanics." McGraw-Hill.

GIEDT, W. H. 1957: "Principles of Engineering Heat Transfer." Van Nostrand.

HUBBERT, M. K. 1940: The Theory of Ground Water Motion. *J. Geol. 48*: 785–944.

MARSHALL, D. C. 1958: Measurement of Sap Flow in Conifers by Heat Transport. *Pl. Physiol. Lancaster 33 (6)*: 385–96.

OWENS, W. L. 1962: Two-Phase Pressure Gradient. Proceedings 1961–62 Heat Transfer Conference, A.S.M.E. p. 363.

ROUSE, H. ; HOWE, J. W. 1953: "Basic Mechanics of Fluids." John Wiley and Sons.

WOODING, R. A. 1963: Convection in a Saturated Porous Medium at Large Rayleigh Number or Péclet Number. *J. Fluid Mech. 15 (4)*: 527–44.

YIH, CHIA-SHUN 1965: "Dynamics of Nonhomogeneous Fluids." Macmillan.