Geophys. J. R. astr. Soc. (1973) 34, 193–201.

Temperature Gradients and the Convective Velocity in the Earth's Core

M. C. Frazer

(Received 1973 April 11)

Summary

Investigation of the methods used to calculate the adiabatic temperature gradient in the fluid core of the Earth leads to the conclusion that none of them is valid. This is because they all, in effect, apply to the liquid core theory that is strictly applicable to solids only, without providing any justification for such a step. Thus claims based on such calculations that the core is stable against convection cannot be accepted. Simple dynamical arguments are then used to obtain an estimate for the mean vertical component of the convective velocity, on the assumption that the core is undergoing large-scale convection. The figure obtained is $3 \times 10^{-4}\,\mathrm{m\,s^{-1}}$, about twice previous estimates which were based on the westward drift of the geomagnetic secular variation.

1. Introduction

Heat may be lost from the core by radiation, conduction or convection. The net outward radiative flux is expected to be low, largely because of an anticipated low opacity and the comparatively low temperature gradient. The heat losses due to conduction and convection are difficult to estimate, in part because of the present uncertainty as to whether the core does in fact convect. The crucial quantities involved in this estimate are the melting point of the core material as a function of radius, and the adiabatic temperature gradient throughout the core. For convection to occur in a fluid such as the core, which is effectively inviscid (except in thin boundary layers), the adiabatic temperature gradient must be less than the melting temperature gradient—if this is not the case, the fluid will be stable against the convective instability.

If we follow the usual assumption, that the boundary between the inner and outer cores represents a change from the solid to the liquid state of the core material, then at that boundary ($1{\cdot}2 \times 10^6$ m from the centre of the Earth), the melting point temperature and the actual temperature are equal. (If a change in composition occurs across the boundary, these temperatures may not be equal, and the following arguments must be modified slightly.) The actual temperature throughout the outer core must then be at or above the melting temperature. If the adiabatic gradient is greater than the melting gradient, there can then be no convection in the fluid. There can also be no convection if the actual temperature is above the melting temperature and the actual temperature gradient is less than the adiabatic gradient; convection is not then a more efficient process of heat transfer than conduction.

Convection is in fact a very efficient process for transporting heat, so that if a fluid system is convecting, the actual temperature will at all points in the fluid be very nearly the same as the adiabatic temperature.

The precise thermal state of the core therefore clearly depends upon the relative positions of the actual, adiabatic and melting temperatures. Until recently, it was generally assumed that the adiabatic curve lay above the melting curve, and that the fluid convected, so that the actual temperature was very close to the adiabatic temperature. But Higgins & Kennedy (1971; see also Kennedy & Higgins 1973, where they essentially repeat the same arguments related to the adiabatic gradient) re-estimated both the melting temperature for pure iron and the adiabatic temperature throughout the core, and concluded that the latter lay below the former in most of the outer core. That is, their estimate of the adiabatic temperature lay below their estimate of the melting temperature, so that the core is, on their view, stable against thermal convection. Their estimate of the melting temperature is for pure iron, not mixed with any lighter elements, and is open to criticism on several grounds. Birch (1972) has found rather different figures and concludes that with the presently available information, no estimate of the melting temperature of iron at core pressures can claim greater accuracy than ± 500 K. The effects of constituents other than iron are not well known, save only that they will certainly decrease the melting temperature. For the purposes of this paper, however, we will accept the melting temperatures of Higgins & Kennedy, and consider some of the consequences.

2. The adiabatic temperature gradient in the core

Thermodynamically, the adiabatic gradient is a well-defined concept, and it is obtained quite generally from one of Maxwell's thermodynamic relations (Pippard 1957, p. 45). If T is the temperature, p the pressure, V the volume and S the entropy of a system, then in an adiabatic process, the appropriate one of Maxwell's relations is:

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p.$$

On applying this to a self-gravitating fluid system in hydrostatic equilibrium, we find that:

$$\left(\frac{\partial T}{\partial r}\right)_S = -\frac{\alpha}{C_p} g T, \tag{1}$$

where r is the radius, α the coefficient of thermal expansion, g the acceleration due to gravity, and C_p the specific heat at constant pressure. This equation defines the adiabatic temperature gradient through the fluid. To apply this relation to the liquid outer core of the Earth, we must either know α, C_p and g as functions of r throughout the outer core, and T at any one point in the outer core or on one of its boundaries, or transform equation (1) into some other form involving other known properties of the core. Higgins & Kennedy use two methods, both of which fall into the second category.

The more important method used by them uses some equations obtained by Valle (1952). He starts from the form of equation (1), and in effect calculates the ratio α/C_p from an elementary theory of adiabatic processes in solids. Applying this theory to a liquid, and assuming that the only difference between the solid and the liquid is that the velocity of transverse waves v_t is zero in the latter, he obtains the expression used by Higgins & Kennedy:

$$\frac{T_d}{(\rho v_l^3)^{1/3}} = \text{constant}, \tag{2}$$

where T_d is the adiabatic tempe
the fluid.
In obtaining equation (2),
becomes so like a solid that we
latter. But if a liquid is so near t
applied to both, then the liquid
least to some extent, transverse c
is thus derived under assump
Although Valle's approximatior
similarities to a solid) might hav
experimental support, and we
The second method used by
use of a thermodynamic parame
a liquid, the parameter is:

where K_S is the isentropic bulk
is not the thermodynamic Grün
of state, used in the reduction
thermal or isentropic equations
unless $d\Gamma/dT$ is zero, and in ger
dynamic parameters (Knopoff
In terms of Γ, equation (1)

and it is from this that Higg
gradient. No attempt is made
which would allow Γ to have s
Both of the methods used
temperature gradient within th
in effect applying to a liquid t
the behaviour of solids. While
liquid and a solid to be not
remains: that they are indeed
not be applied to the other unl
extension of the applicability c
given for the use of one or otl
curves obtained from them sh
The difficulty is made ev
temperature through the core a
must be, in effect, integrated n
side of the equation generates
tion, so that the final calcula
seems clear that the conclusio
stable against convection, is n
The difficulties of calculati
outer core, and the importance
of the numerical integration,
To do this, we need values of α
adiabatic, actual and melting

boundary (again assuming no change in composition at that boundary). Values of $g = g(r)$ can be obtained from any standard Earth model—we will use the model HB1 as given in Stacey (1969).

The values of α and C_p in the core are not, in fact, well known. Values that have been used or quoted by various authors range from $3{\cdot}6 \times 10^{-6}\,\mathrm{K}^{-1}$ to $10 \times 10^{-6}\,\mathrm{K}^{-1}$ for α, and from $4{\cdot}61 \times 10^{2}\,\mathrm{J\,kg^{-1}\,K^{-1}}$ to $7{\cdot}54 \times 10^{2}\,\mathrm{J\,kg^{-1}\,K^{-1}}$ for C_p. The range of values of the ratio α/C_p is thus:

$$0{\cdot}48 \times 10^{-8} \leqslant \frac{\alpha}{C_p} \leqslant 2{\cdot}2 \times 10^{-8}.$$

The most likely values of α and C_p are probably those estimated by Bullard (1950), with the modification of halving his correction of C_v to get C_p. This is required to allow for his use of a central temperature of 10 000 K, rather than of about one-half that as is now believed. We then have:

$$\alpha = 4{\cdot}5 \times 10^{-6}\,\mathrm{K}^{-1}$$

$$C_p = 7{\cdot}12 \times 10^{2}\,\mathrm{J\,kg^{-1}\,K^{-1}},$$

so that

$$\frac{\alpha}{C_p} = 0{\cdot}63 \times 10^{-8}.$$

With these figures and the above assumption about the temperature at the inner/outer core boundary, we can now integrate equation (1) (using Earth model HB1) to obtain the adiabatic temperature through the core, and compare this distribution with the melting curve of Higgins & Kennedy. When this is done for a variety of values of α/C_p, we find that if the adiabatic temperature is greater/less than the melting temperature at the outer boundary, then it will also be greater/less than the melting temperature throughout most of the outer core, unless the two temperatures are very similar at the outer boundary. The relative values of the two temperatures at the outer boundary can thus be used as a rough measure of whether the adiabatic temperature is higher or lower than the melting temperature throughout most of the outer core, for the particular value of α/C_p chosen. The melting curve of Higgins & Kennedy gives a melting point of 4020 K at the outer boundary, and it is with this that the adiabatic temperature must be compared.

Table 1 gives these adiabatic temperatures for the minimum, maximum, and most probable values of the ratio α/C_p as given by the above figures, and for the value of this ratio that gives an adiabatic temperature equal to the melting temperature on the boundary. For values of $\alpha/C_p < 0{\cdot}70 \times 10^{-8}\,\mathrm{kg\,J^{-1}}$, the adiabat through all or most of the outer core lies above the Higgins & Kennedy melting curve, allowing the possibility of convection, and for values greater than that, the adiabat lies in the solid phase so that convection is not then possible. These are of course mean values of α and C_p, as no radial dependence has been included.

Table 1

$\frac{\alpha}{C_p}$	$0{\cdot}48 \times 10^{-8}$	$0{\cdot}63 \times 10^{-8}$	$0{\cdot}70 \times 10^{-8}$	$2{\cdot}2 \times 10^{-8}$
T_p	4170	4070	4020	3110

Adiabatic temperatures at the outer boundary of the outer core for various values of the ratio α/C_p. Compare these temperatures with Higgins & Kennedy's estimate of the melting temperature at that point of 4020 K.

By then taking $\kappa = 60\,\mathrm{W\,m^{-1}\,K^{-1}}$ we allow a small contribution from the ionic term as well.

From the results of Higgins & Kennedy, we estimate the mean gradient of the melting temperature as:

$$\left|\frac{\partial T}{\partial r}\right|_m = 0{\cdot}2\times 10^{-3}\,\mathrm{K\,m^{-1}},$$

so that:

$$q_n = 10{\cdot}6\times 10^{-3}\,\mathrm{W\,m^{-2}}.$$

As the radius of the outer core is $3{\cdot}49\times 10^6$ m, and the gradient of the melting temperature at the outer boundary of the outer core is $0{\cdot}3\times 10^{-3}\,\mathrm{K\,m^{-1}}$, the maximum possible conductive heat loss from the core to the mantle is

$$h_n = 2{\cdot}7\times 10^{12}\,\mathrm{W}.$$

(Stacey (1969) obtains a value of $1{\cdot}4\times 10^{11}$ W, largely because of the high value of τ he uses; Verhoogen (1961) obtains $1{\cdot}3\times 10^{12}$ and 3×10^{12} W, using the Strong and Simon values of the melting points of iron respectively.)

If the estimate of Higgins & Kennedy of the adiabatic gradient in the core should be correct, then this estimate of h_n would be close to the actual heat loss from the core, as there would be no convection and all of the heat loss would be by conduction at the melting temperature. If the adiabat lies above the melting-curve, the maximum possible conductive heat loss will be less than h_n because the adiabatic temperature gradient will be less than the figure used here. And lastly, if the adiabat lies above the melting curve and the actual temperature is the same as the adiabatic temperature, with the fluid convecting, the amount of heat lost by convection will be no less than that lost by conduction without convection (at the same temperature gradient).

4. The velocity of convection in the core

The above figure for the heat flux in the core can be used to provide an estimate of the mean convective velocity that would result in a convecting core. For if the actual temperature gradient is $|\partial T/\partial r|_c$ and the adiabatic temperature gradient is $|\partial T/\partial r|_a$, we let δT be the excess in temperature over the actual temperature gained by a fluid element in rising adiabatically through a distance δr. Then:

$$\delta T = \left(\left|\frac{\partial T}{\partial r}\right|_a - \left|\frac{\partial T}{\partial r}\right|_c\right)\delta r$$

$$\equiv \frac{\Delta T}{\Delta r}\,\delta r, \qquad (4)$$

where we define $\Delta T/\Delta r$ as the excess of the adiabatic temperature gradient over the actual temperature gradient. For $\delta r > 0$, $\Delta T/\Delta r$ must be negative for convection to occur.

If the fluid is in fact convecting, then the convective heat flux per square metre per second is q_v, where

$$q_v = \rho v_r C_p \frac{\Delta T}{\Delta r}\,\delta r, \qquad (5)$$

in which v_r is the vertical, or radial, component of the convective velocity. The same equation describes the heat flux produced by fluid moving down into the sphere, as then both v_r and δr change sign. Thus if v_r and δr stand for the average vertical

Pippard, A. B., 1957. *The elements of classical thermodynamics*, Cambridge University Press, Cambridge.

Rice, M. J., 1970. Inelastic electron scattering and the Lorenz ratio of liquid metals, *Phys. Rev. B*, **2**, 4800–4804.

Stacey, F. D., 1969. *Physics of the Earth*, John Wiley & Sons, Inc., New York.

Valle, P. E., 1952. On the adiabatic temperature gradient in the interior of the Earth (in Italian), *Ann. Geofisica*, **5**, 41–53.

Verhoogen, J., 1961. Heat balance of the Earth's core, *Geophys. J. R. astr. Soc.*, **4**, 276–281.