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Theory of the Wairakei geothermal field

1. Single-phase, drawdown model

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Time-varying solutions of this model were obtained by computer after steady-state solutions were found inadequate. When the history of mass discharge from 1953 to 1968 is used as input, the observed pressures can be closely matched from 1961 to 1967, but the fit is not good at other dates. Three of the four parameters adjusted to achieve the best fit have values in the expected range but the fourth, the compressional diffusivity, is inconsistent with other physical parameters in the model. Possible reasons for this are two-phase effects and lateral inflow in the liquid-only region.

INTRODUCTION

Earlier papers (Marshall 1966, 1970) showed that hydrostatic models are incompatible with measurements and observations at the Wairakei geothermal field and discussed steady-state solutions of 2 simple hydrodynamic models. The present paper first completes the discussion of steady-state solutions in the 1970 paper and then goes on to describe time-varying solutions of one of these models.

STEADY-STATE SOLUTIONS

Using the 'simplest cases first' approach (Marshall 1970), the two models both consider a fluid in a single phase with the properties of water at 250°c, which initially flows vertically upwards parallel to the axis in a circular cylindrical region in the porous ground. Fluid is removed from a distributed sink with a mean depth of 510 m. For comparatively weak sink strengths, the sink removes a portion of the upward flow while the remainder continues to flow out at the ground surface, but for greater sink strengths this flow is reversed. The surface of the fluid is then assumed to draw down below the ground surface, and it was found that it would come to an equilibrium level for any given sink strength. In this case, the entire flow to the sink comes from below, the decreased pressure at the sink inducing greater than natural upflow.

The boundary conditions at the cylindrical surface of the hot region distinguish the 2 different models. In one case this surface is assumed to be an impermeable wall down to some depth at which the pressure remains constant. In the other case, lateral inflow of water from the surrounding cold region is assumed to occur at a rate proportional to the fall in pressure at any depth. This cold water will heat up as it flows through the thermal boundary, causing this boundary to move slowly inwards.

In most impermeable wall solutions, the depth of the wall required to make the model fit the field measurements is between 1 and 2 km. It is not suggested that there is in fact a constant pressure source of hot water at such a shallow depth. This assumption was made

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as the easiest way of handling the lower boundary condition in this simple model. It is mathematically equivalent to a variation of the lateral inflow model in which the horizontal permeability in the cold region is zero down to the bottom of the wall and infinite below that. In the case actually described as the lateral inflow model this horizontal permeability is taken to be constant from the ground surface down to infinite depth.

To use steady-state solutions to model the actual discharge and pressure history at Wairakei it was assumed that steady-state conditions existed in 1955 and 1961. The solution to the lateral inflow model was given by Marshall (1970). In the impermeable wall model, the permeability k in the cold region, and therefore also the parameter b in Marshall's (1970) equation 2 are zero. Whereas the lower boundary condition in the lateral inflow model was that the potential $\phi_3 = 0$ at infinite depth, in the impermeable wall model this condition is that $\phi_3 = 0$ at the bottom of the wall, $z = z_b$.

(The dependent variable used here is the fluid potential $\phi = gz + (p - p_0)/\rho$. This is the sum of the gravity and pressure potentials. It may be thought of as the pressure with the effect of gravity removed. The particular potential ϕ_3 is measured with respect to the cold water surrounding the geothermal field. The possibility of some downflow in this cold water is allowed for in this potential by including a parameter C, expected to be positive and small compared with g, the acceleration of gravity.)

The steady-state solution to the impermeable wall model is:

$$\phi_3 = \{(g - C)z_0 - F(z_0)\}\left(\frac{z - z_b}{z_0 - z_b}\right) + F(z_b)\left(\frac{z - z_0}{z_0 - z_b}\right) + F(z)$$

where $$F(z) = \frac{Q_0\sigma}{\sqrt{2}}\left(u \operatorname{erf} u + \frac{1}{\sqrt{\pi}} \exp(-u^2)\right)$$

where $$u = \frac{z - \bar{z}}{\sigma\sqrt{2}}$$

Fig. 1 shows solutions for the 2 models in which 2 parameters have been chosen in each model to make the pressures in 1955 and 1961 equal the observed values at the contemporary rate of mass discharge. It will be noticed that for discharge rates less than 2.2 tonne/sec, a value exceeded for only 2 years in the history of the field (Fig. 4), there is little difference between the pressures given by the 2 models. Both models have been used in the expectation that at some stage comparison with observation would decisively favour one or the other. It is evident that no choice could be made at this point.

It is also evident that these solutions do not fit the history of the field. It is clear that steady-state conditions did not exist in 1963, but by 1968 conditions might possibly have been approaching equilibrium. Yet the steady-state pressure at the 1968 discharge rate is at least 5.5 bar* (80 lb in^{-2}) higher than the observed pressure (Figs 1 and 4).

Two possible reasons for this discrepancy were now investigated. The first is that the assumption of steady-state conditions in 1955 and 1961 is invalid. This question is the subject of the rest of this paper. The other possibility concerns two-phase effects, which have been ignored up to now. As the pressure falls, the fraction of steam at shallower depths will rise, causing the effective permeability to fall. While outflow persists this should lift the pressures compared with a single-phase model, but depress the pressures after draw-down begins. This will be discussed in detail in Part 2.

* Throughout this paper conversions to lb in^{-2} are given because values actually used were in lb in^{-2} gauge, e.g. the pressure in 1961 was kept as close as possible to 750 lb in^{-2} gauge, not the equivalent in bars.

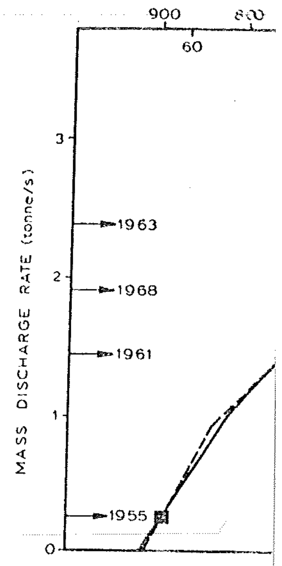

FIG. 1—Steady-state solut (dashed lines) whi inflow model are C = 43.25 cm s^{-2}

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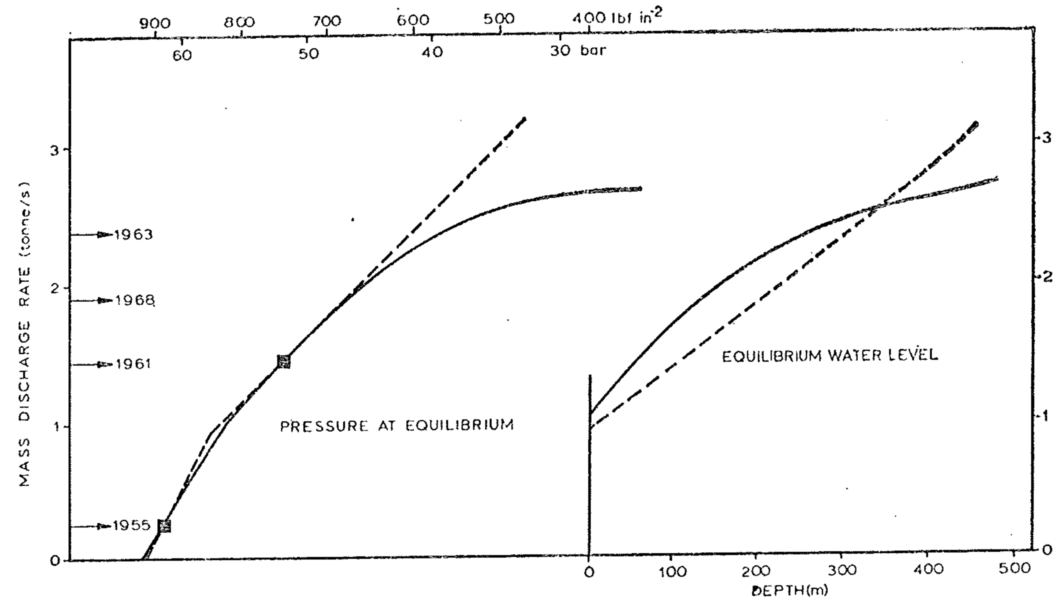


FIG. 1—Steady-state solutions for the lateral inflow model (full lines) and the impermeable wall model (dashed lines) which give the observed pressures at two points (squares). Parameters for the lateral inflow model are $C = 39.26$ cm s^{-2} and $k = 4.778 \times 10^{-10}$ cm^2, and for impermeable wall model, $C = 43.25$ cm s^{-2} and $z_b = -0.9923$ km.

TIME-VARYING SOLUTIONS

To avoid assumptions about steady-state conditions, it is necessary to obtain and solve the differential equation for the model that contains time as a variable. Because of the similarity of the equilibrium pressure curves in Fig. 1, and because of the additional computation involved in introducing the time variable and two-phase effects, it was decided to concentrate on the impermeable wall model at this stage.

Substituting Darcy's law in the form $q = -\dfrac{\rho k}{\mu}$ grad ϕ into the continuity equation one obtains:

$$-f\frac{\partial \rho}{\partial t} = \operatorname{div}(\rho q) = \operatorname{div}\left(-\frac{\rho^2 k}{\mu}\operatorname{grad}\phi\right).$$

Keeping to vector notation and using the identity:

$$\operatorname{div}(S.\mathbf{V}) = (\operatorname{grad} S).\mathbf{V} + S \operatorname{div}\mathbf{V}$$

where S is a scalar and $\mathbf{V}$ a vector, gives

$$f\frac{\partial \rho}{\partial t} = \operatorname{grad}\left(\frac{\rho^2 k}{\mu}\right).\operatorname{grad}\phi + \frac{\rho^2 k}{\mu}\operatorname{div}(\operatorname{grad}\phi)$$

$$= \operatorname{grad}\left(\frac{\rho^2 k}{\mu}\right).\operatorname{grad}\phi + \frac{\rho^2 k}{\mu}\nabla^2\phi \qquad (1)$$

The time derivative of density must now be changed into terms of potential. From the definition of compressibility β, we have $d\rho = \rho_0 \beta\, dp$ and from the definition of potential

$\mathrm{d}p = \rho \mathrm{d}\phi$. Thus $f \dfrac{\partial \rho}{\partial t} = \rho^2 \beta f \dfrac{\partial \phi}{\partial t}$. Substituting in Equation 1:

$$\nabla^2 \phi + \frac{\mu}{\rho^2 k} \operatorname{grad}\left(\frac{\rho^2 k}{\mu}\right) . \operatorname{grad} \phi = \frac{\mu f \beta}{k} \frac{\partial \phi}{\partial t} \qquad (2)$$

This brief derivation is essentially a formal manipulation of symbols. De Wiest (1969) examines the derivation of a closely similar equation in considerable detail. (As Philip (1970) remarks, flow in porous media is a much fragmented field with subtly different forms and terminology.) De Wiest refers to a dispute over the derivation, but himself neglects certain quadratic terms without comment.

If the derivation above is repeated using De Wiest's version of Darcy's law:

$q = -\dfrac{k}{\mu} (\rho g \operatorname{grad} z + \operatorname{grad} p)$ one obtains:

$$-\operatorname{div}(\rho q) = \frac{k}{\mu}\left[g \operatorname{grad}(\rho^2).\operatorname{grad} z + \operatorname{grad}\rho.\operatorname{grad} p + \rho \nabla^2 p\right]$$

$$= \frac{k}{\mu}\left[g(2\rho)\rho\beta \operatorname{grad} p.\operatorname{grad} z + \rho\beta \operatorname{grad} p.\operatorname{grad} p + \rho \nabla^2 p\right].$$

It is the middle term on the right which De Wiest neglects, but since, as he remarks earlier, the hydrostatic pressure is usually dominant in ground-water flow, $\operatorname{grad} p \simeq -\rho g \operatorname{grad} z$ and the second term will cancel out half of the first term, removing the controversial factor 2.

In the present work I have followed the usual practice in thermal convection studies and neglected all changes in density apart from those caused by temperature differences. Since these differences are not explicit in the impermeable wall model, the flow considered being entirely in the hot region, this is equivalent to taking ρ constant (as has already been done in deriving Equation 2 from Equation 1). Since the viscosity μ is also constant in the hot region, Equation 2 becomes:

$$\nabla^2 \phi + \frac{1}{k} \operatorname{grad} k . \operatorname{grad} \phi = \frac{\mu f \beta}{k} \frac{\partial \phi}{\partial t} \qquad (3)$$

This is the equation used in Part 2, but here in Part 1 the permeability is considered constant and the second term vanishes. The equation then becomes identical with the heat-conduction equation, with the quantity $k/\mu f \beta$ playing the part of the thermal diffusivity. Muskat (1946) was evidently the first to write down the differential equation for a compressible liquid. He assigned to the quantity $k/\mu f \beta$ the symbol κ, often used for thermal diffusivity, but did not name it. It seems reasonable to call it the "compressional diffusivity".

To visualise what a satisfactory solution must do, it is useful to refer to the diagram of hot water potential versus depth (Fig. 2). It is assumed at present that the permeability is uniform, and the undisturbed condition is therefore represented by a straight line. The significance of the parameter C is that if C is zero this initial straight line coincides with the cold hydrostatic potential. If C is positive the initial hot water potential lies below the cold hydrostatic line, and if C is negative, above.

To reproduce the observed pressures, the potential at the reference depth 670 m must have prescribed values at particular times. The values at the end of 5 particular years are shown in Fig. 2. The sink centred at 510 m is handled as in Marshall (1970) with changes

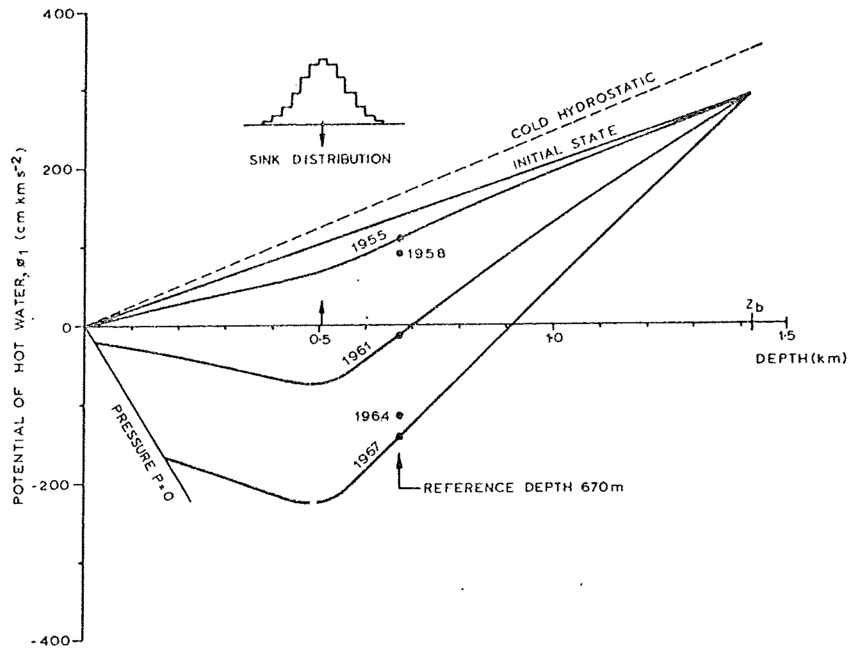


FIG. 2—Typical potential versus depth curves. The solid circles correspond to the observed pressures at the dates shown.

appropriate to numerical methods of solution rather than analytical—the Gaussian distribution is replaced by a histogram with constant values within each depth step. At each time step the magnitude of each local sink value is given a value corresponding to the mean rate of mass discharge during that time interval.

The Crank–Nicolson method is used to solve the partial differential equation (Carnahan *et al.* 1969). In this method the difference version of the time derivative is obtained from the mean of the space derivatives at each end of the time step. Given the distribution of potential with depth at any time, this requires the solution of an ordinary differential equation with a boundary condition at each end of the space (depth) range to get the potential at the next time step. Once the gradient of the potential at $z = 0$ becomes negative, drawdown begins and the upper boundary moves down the line of zero pressure, $p = 0$ (Fig. 2). The rate of downflow during a time step is taken as the mean of the rates at each end of the step, the resulting rate of drawdown of the liquid surface depending inversely on the porosity, f. The magnitude of the drawdown at each step is found by iteration. Further details of the numerical methods and a discussion of accuracy are given in Appendix 1.

RESULTS

Although many laboratory measurements of the porosity of core samples have been made, the mean porosity which determines the rate of drawdown in this model must be regarded as an unknown parameter. For the first solutions the rather low value of 0.02 was deliberately chosen to give rapid drawdown. In the impermeable wall case in Fig. 1 the permeability $k_{30} \doteqdot 2.9 \times 10^{-11}$ cm^2 which, together with $\mu = 0.001$ poise (10^{-4} N s m^{-2}) and $\beta = 70 \times 10^{-6}$ per bar (Marshall 1966) gives the diffusivity $\kappa \doteqdot 2 \times 10^4$ cm^2 s^{-1}.

For this value of diffusivity the time required to establish a uniform pressure gradient between two points 1 km apart is about 1 day, a time which was confirmed by trials of the numerical subroutines using a Wang 370 programmable calculator. Since the observed pressure response to variations in discharge rate during 1956–58 indicated that it took about 1 month to produce a response comparable to that occurring after 8 hours in these trials, the first run of the full program on the computer (an IBM 1130) used a value for diffusivity reduced by a factor of 90, namely about 219 $cm^2 s^{-1}$. In this run the pressures were still much too sensitive to variations in discharge rate, rising for instance during 1960 by 3.4 bar (50 lb in^{-2}) and responding to the partial shutdown in 1968 by rising more than 4.8 bar (70 lb in^{-2}). It was decided therefore that the compressional diffusivity must be regarded as a parameter in its own right. Possible reasons for the low values required will be considered in the discussion below.

The position now was that 4 parameters were available, which were to be varied to obtain the best fit between the observed and computed pressures when the history of the mass discharge was used as input to the model. Two of these parameters were present in the steady-state solutions, the depth of the impermeable wall $-z_b$, and the parameter C which determines the potential there. The other 2 parameters, the porosity and the diffusivity, determine the rate of response of the system to changes in discharge rate.

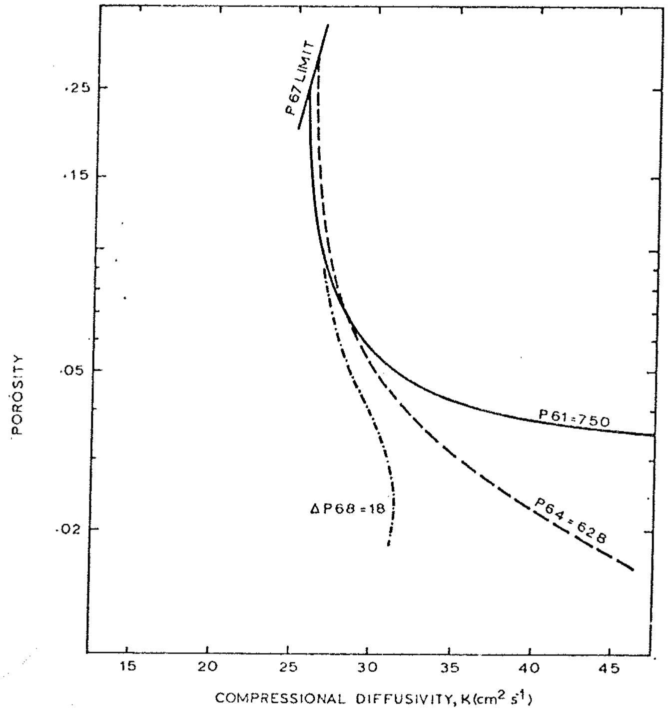
As before, attention was concentrated on certain key dates, but now without any assumption of steady-state conditions. The date 1955 was retained at the beginning of the 16-year history from 1953–68 as it heralded the first definite change in pressure, while the 9th or 10th month in 1967 was chosen to mark the latter end of the period to avoid the effect of the 1968 shutdown experiment. (Apart from 1967 all dates refer to the end of the year.) It was soon found that the 1955 pressure, P55, depended most on the parameter C, and that C had little effect on later pressures. C determines the gradient of the initial potential (Fig. 2) and the potential does not change much by 1955.

At the same time it was found that the deeper the wall depth, $-z_b$, the lower became the pressure P67. Again this is what one would expect from Fig. 2. When a more powerful computer became available (a Burroughs 6700) the program was extended to include iteration of C and z_b until both P55 and P67 came within 0.034 bar (0.5 lb in^{-2}) of the desired values, 890.5 and 590.5 lb in^{-2} gauge (61.37 and 40.70 bar) respectively. Once experience had revealed the approximate values of the differentials $\delta P55/\delta C$, etc. only about 6 iterations were needed.

Attention was then focused on 1961 which was a key date in Marshall (1970) and also is half-way between 1955 and 1967. With porosity fixed at 0.02 and the diffusivity varied between 20 and 50 $cm^2 s^{-1}$ it was found that P61 showed little change and was always more than 0.6 bar (9 lb/in^2) higher than the desired value of 750 lb in^{-2} gauge (51.69 bar). The general appearance of the computed pressure curve indicated that the diffusivity must be within this range; for values greater than 50 $cm^2 s^{-1}$ it reflects too much of the fluctuations in the mass discharge curve—during 1960–61, for instance, the pressure actually rises. When $\kappa = 20$ $cm^2 s^{-1}$ on the other hand, the pressure curve seems oversmooth.

Porosities of 0.05 and 0.15 were tried next, for which it was found possible to get pressure curves with P61 = 750. Eventually it was possible to plot the locus of such solutions on a graph of porosity versus compressional diffusivity (Fig. 3). With 4 adjustable parameters available it is in principle possible to make the theoretical pressures fit the measured ones at 4 points. Since all the solutions were a closer match to the observed pressures during the later years, 1964 was chosen as the fourth key date. Fig. 3 also shows the locus of solutions which give the observed pressure, P64 = 628 lb in^{-2} gauge (43.28 bar).

FIG. 3—Curves showing the values of compressional diffusivity and porosity required to make P61 = 750 lb in^{-2} gauge, P64 = 628 lb in^{-2} gauge, and the 1968 pressure recovery, ΔP68 = 18 lb in^{-2}. The parameters C and z_b in every case were chosen to make P55 = 890.5 lb in^{-2} gauge and P67 = 590.5 lb in^{-2} gauge.



In the range of porosities up to 0.25 in which the two curves are nearly parallel, neither pressure deviates more than 0.1 bar (1.5 lb in^{-2}) from its desired value when the other is correct. At porosity 0.05 the discrepancy is 0.14 bar (2 lb in^{-2}), but for lower porosities it increases quite rapidly. At the limit that stops the curves between $f = 0.25$ and 0.30 the dependence of 1967 pressure with depth, $\delta P67/\delta z_b$, becomes zero and beyond this P67 cannot be brought as low as 590 lb in^{-2} gauge.

At the point of intersection of the curves for P61 and P64 the computed pressures have the desired values at the 4 key dates. The history of the pressures in this case is shown in Fig. 4. There is actually little difference between the computed pressure curves for any points that lie between the curves for P61 and P64 in Fig. 3 for porosities greater than 0.05. The best-fitting pressure curve for any porosity between 0.05 and 0.30 does not deviate from the one shown in Fig. 4 by more than 0.14 bar (2 lb in^{-2}). In particular the 1958 pressure always lies between 839 and 840 lb in^{-2} gauge (57.8 and 57.9 bar).

The value of the parameter C for these best-fitting curves increases from 31 to 35 cm s^{-2} as the porosity increases from 0.05, and at the same time the depth of the "source" or bottom of the impermeable wall increases from 1.37 km to 2.7 km. Drawdown invariably begins in the last quarter of 1959, reaching a maximum of 200 m for 0.05 porosity. The maximum drawdown is only 53 m when the porosity is 0.30.

DISCUSSION

It may be remarked immediately that the divergence of the values of the parameters C and z_b just given for the time-varying solutions from those in Fig. 1 for the steady-state solution, confirms that the earlier assumption of steady-state conditions in 1955 and 1961 was not correct. Even in the case of 5% porosity when the depth $-z_b$ is least different from

Fig. 1, the steady-state pressure corresponding to the 1961 rate of mass discharge is only 38.3 bar (556 lb in^{-2} gauge), nearly 13.8 bar (200 lb in^{-2}) lower than the pressure of 51.7 bar (750 lb in^{-2} gauge) both observed and fitted by the time-varying solutions.

An assessment of the success of this single-phase drawdown model must depend on the degree of match between observed and computed pressures, and on the reasonableness of the parameter values needed to get the best match. On the first point, Fig. 4 shows that the model fits the observed pressures well between 1961 and 1967. From 1957–59, however, the computed pressures are 1.7–2 bar (25–30 lb in^{-2}) too low. In fact during 1953 to 1958 the computed pressure falls by 5.5 bar (80 lb in^{-2}), whereas the observed pressure fell only about 1.4 bar (20 lb in^{-2}). (Although only a few pressure measurements were made before 1955, they showed no significant pressure change during this period (R. S. Bolton, personal comm.).) As mentioned earlier, qualitative reasoning indicates that two-phase effects should increase the steepness with which the pressure falls when upflow is replaced by drawdown in 1959, giving a better fit. This will be considered in Part 2.

The response to the partial shutdown in 1968 is a pressure recovery of 1.38 bar (20 lb in^{-2}) when the porosity is 0.05, falling to 1.24 bar (18 lb in^{-2}) at higher porosities. The locus of solutions giving 18 lb in^{-2} recovery is shown in Fig. 3. To reduce this recovery to 0.8 bar (12 lb in^{-2}) as observed, the diffusivity κ has to be reduced to between 22 and 25 cm^{2}s^{-1}, which makes the pressures in 1961 and 1964 between 10 and 20 lb in^{-2} too high. This response to the 1968 shutdown is an aspect in which the model is only partly successful.

Reviewing the values of the parameters, we find that 3 of the 4 are in the expected range. The parameter C at 31 to 35 cm s^{-2} is positive, corresponding to downflow in the cold region, and smaller than for the steady-state solution (Fig. 1). Both the depth of the impermeable wall (1.37 to 2.7 km) and the porosity (5 to 30%) have reasonable values.

The expected values of the compressional diffusivity, however, obtained by substituting in $k_{30}/\mu f\beta$ the values of k_{30} and f for particular solutions and the values $\mu = 0.001$ poise (10^{-4} N s m^{-2}) and $\beta = 70 \times 10^{-6}$ bar $^{-1}$ used previously, make the diffusivity 7600 cm^2 s^{-1} for porosity $f = 0.05$ and 1290 cm^2 s^{-1} for $f = 0.30$. However, the actual solutions had diffusivity κ of 30 and 26 cm^2 s^{-1} respectively, smaller by factors of 250 and 50 (Fig. 3).

Since the viscosity μ is a function of temperature which is known, and the permeability k_{30} and the porosity are already determined, the explanation of this discrepancy must be sought in terms of the compressibility, β. When the compressibility of the porous medium itself is taken into account when deriving the differential equation, as well as the liquid compressibility, it is found that the 2 compressibilities must simply be added, with appropriate geometrical weighting factors (De Wiest 1969). This, however, does not seem likely to add a factor of more than 2 or 4. On the other hand, if an (insoluble) gas is present in the pores with the liquid, Verruijt (1969) finds that β must be replaced by $\beta' = \beta + (1 - S_r)/p$, where S_r is the degree of saturation of the pore space with liquid and p is the pressure. Taking $p = 40$ bar, the saturated vapour pressure of water at 250°C, and $S_r = 0.3$ makes $\beta' = 250\beta$ and κ therefore 250 times smaller. Although the situation in two-phase flow is more complicated than this, it does seem likely that the presence of steam may be sufficient to explain the discrepancy of κ values at shallow depths.

At deeper levels, where the greater pressure allows only the liquid phase to exist, some other explanation must be found. It was surmised at this stage that the lateral inflow hypothesis might provide an answer. The resistance to fluid flow in the side passages should act like the side chambers in an acoustic filter such as a motor car muffler or like a low-pass resistance-capacitance filter in an electrical network. It will be shown in Part 3 that lateral inflow can indeed provide the increased damping required.

It is of interest to consider the implications for long-term power production of this single-phase drawdown model. When the values for the parameters C and z_b for the best-fitting pressure curves are inserted in the steady-state solution, one finds that for greater porosities and in later years the situation is increasingly far from equilibrium until the drawdown exceeds about 380 m, when the solutions become meaningless because the water level is encroaching into the sink region (Fig. 2). The sink strength that would keep the water level steady at −380 m was therefore calculated. It was found that this sink strength ranged from 1.619 tonne/sec for 5% porosity down to 0.923 tonne/sec for 30% porosity, the respective pressures being 32.3 and 28.1 bar (469 and 408 lb in^{-2} gauge).

This range of steady-state sink strengths results from the influence of the porosity on the relative proportions in the total mass discharged of the removal of water in place on the one hand and induced upward flow from greater depths on the other. The total mass discharged during the period shown in Fig. 4 was 645 × 10^6 tonne. The field being assumed circular with a radius of 2 km, and the density of the hot water 0.8 g cm^{-3}, this mass represents a depth of 64.13 m. With 5% porosity the drawdown in 1968 at 200 m corresponds to 10 m of water, or 15.6% of the total. With 30% porosity, however, although the 1968 drawdown is only 53 m the water that is "mined" out represents 24.8% of the total.

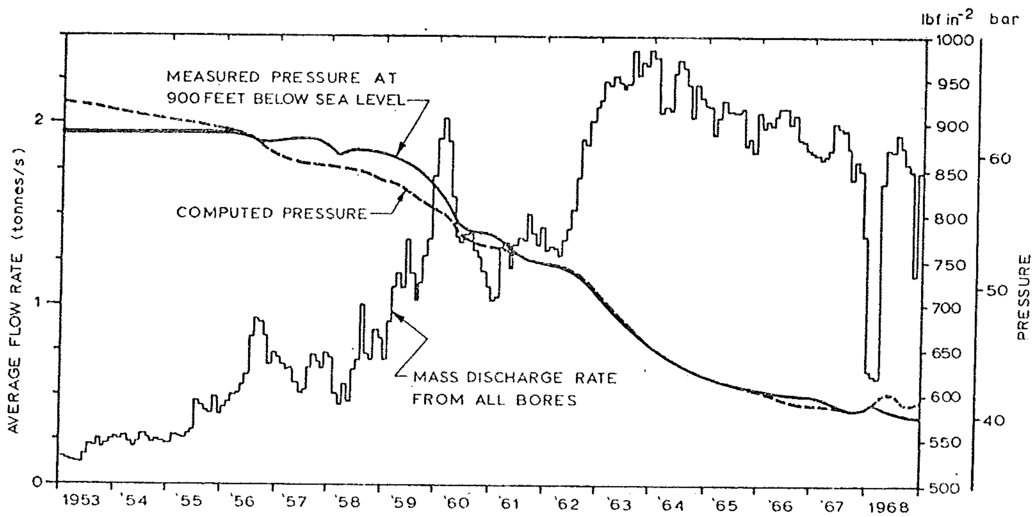


FIG. 4—The best-fitting computed pressure history (dashed curve) compared with the measured pressures and mass discharge rates (Bolton 1970).

It must be emphasised that this analysis applies only to the model. The extent to which it can be transferred to the actual field at Wairakei can be judged by the degree of match between the theoretical and observed pressures in Fig. 4, which is not good in the early years. It appears that a reliable prediction of the future of the field depends in part on getting deeper insight into the relative importance of the mining of water in place and the inducing of greater flow from beyond the immediate vicinity of the bores.

To sum up, these time-varying solutions of the single-phase drawdown model are encouraging in that a considerable part of the pressure history of the Wairakei field can be closely matched. At the same time the small values of the compressional diffusivity required for this match are inconsistent with the model and point to the possibility of significant two-phase effects and lateral inflow.

APPENDIX 1

Numerical methods

Three different methods were used in obtaining the results in this paper, because of changing computer facilities and program requirements. The first method used fourth-order Runge–Kutta integration formulae in conjunction with a "shooting" method to solve the ordinary differential equation at each time step. In this the potential gradient at the upper boundary is iterated until the value is found that gives the correct potential at the lower boundary. This method which was developed on the Wang 370 calculator has the advantage of allowing a comparatively large depth step and generally requiring rather small storage. It was found in the Wang trials that a step length of 50 m gave 4-figure accuracy (i.e. less than 1 lb in^{-2} pressure). It also has the advantage of giving the potential gradient explicitly at each depth node. This gradient is required to obtain the additional drawdown at each time step, and numerical differentiation is notoriously inaccurate. The gradient at the water level is obtained by interpolation using the gradients at the top 4 nodes, the water level itself being obtained by iteration.

Translated into Fortran this program was run using the 16 year discharge history on an IBM 1130 computer, and the general outline of Fig. 3 was obtained.

To solve the nonlinear differential equation in Part 2 required yet another level of iteration, and since these are time-consuming on the computer the shooting method was replaced by the line inversion method (Wachspress 1960) used in Marshall (1970). This method retains the advantage of giving an explicit value for the potential gradient at each step. In what was felt to be an improvement, 2 depth steps at the upper boundary were adjusted to make the upper boundary of the line inversion coincide with the water level at each time step. This did not actually remove the need to interpolate since the function values at the new nodes had to be obtained by interpolation.

From previous experience a depth step of 20 m was considered adequate for the accuracy of the line inversion, yet in the test case using 2% porosity the 1967 pressures given by the 2 methods differed by 1.0 bar (14 lb in^{-2}). A number of factors made it clear that the discrepancy resided in the calculation of the drawdown, and since this second method uses a shorter step length it was judged more accurate, and work was continued with it.

With the recent acquisition of a PDP 11/40 minicomputer, however, which can be used "hands on" to make many quick program changes and short runs, the relative accuracy of the two methods has been made clear. When the step length in the second method was halved to 10 m the discrepancy was reduced from 1.0 to 0.3 bar (14 to 4 lb in^{-2}). The second (line inversion) program was then rewritten with the original method of interpolating for drawdown. In the test case this differed by only 0.06 bar (0.8 lb in^{-2}) from the solution using the shooting method, and reducing the step length to 10 m made only 0.023 bar (0.34 lb in^{-2}) difference. Evidently the successive interpolations involved in changing the step lengths at the upper boundary at each time step degraded the accuracy more rapidly than one would expect.

The discharge history supplied by the Ministry of Works is given in monthly averages, and the first computations used a time step of one month. When the compressional diffusivity was reduced to about 50 cm^2 sec^{-1} the smoothness of the pressure curve indicated that a time step of 3 months would be quite adequate, reducing the computing time by two-thirds. At the conclusion of the work this judgment has been checked by repeating a few runs with a time step of one month. With 25% porosity the difference in pressures fluctuated with a general rising trend up to 0.1 bar (1.4 lb in^{-2}) difference in 1967. With 5% porosity the pattern of fluctuations was similar but slightly smaller in amplitude. At the point of intersection of the curves for P61 and P64 in Fig. 3, C had to be increased by 0.23 cm s^{-2} and the wall deepened by 0.013 km to make P55 and P67 fit again. This left P61 0.034 bar (0.5 lb in^{-2}) higher and P64 0.014 bar (0.2 lb in^{-2}) higher. The general effect of reducing the time step from 3 months to one month is to shift the P61 curve in Fig. 3 to slightly higher diffusivity values at the lower porosities with a similar but smaller effect for the P64 curve. The point of intersection shifts from $(\kappa, f) = (28.0\ cm^2\ s^{-1}, 0.07)$ to $(27.6\ cm^2\ s^{-1}, 0.08)$. The nature of the diagram is not changed, nor are the conclusions discussed in this paper.

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