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A Five-Component Magneto-telluric Method in Geothermal Exploration: the M.T.-5-E.X.

LOUIS MUSÉ *

ABSTRACT

The present paper describes a new method designed both at recording and processing levels for a practical solution of the overall problem of the Earth electromagnetism, in geophysics.

Up to now, the random character of the natural signals prevented any measurement of reliable values of the phase shift between the various electromagnetic components at a given place. Hence it is impossible numerically to solve the general linear relations binding these components simply by using the processes of the standard harmonic analysis.

In order to overcome this difficulty the writer designed an analysis method based on a hypothesis according to which the natural signals are considered as real exponential functions increasing with time.

The writer, thus, was able to work out systematically a magneto-telluric prospection method utilizing the five standard electromagnetic components quantitatively, and in particular the vertical magnetic component.

The application of this method — named the M.T.-5-E.X. — to geothermal prospection in Italy in the region of Travale (Tuscany) obtained various practical results of particular interest. They will be described in a paper which will be edited jointly with the researchers of the International Institute for Geothermal Research of Pisa (Italy).

Introduction

Electromagnetic methods in geothermal prospection are of considerable importance since the responses obtained by means of these methods are determined by factors which are directly linked to the presence of geothermal zones.

As it is pretty well known, geothermal zones are those zones where the electrical conductivity becomes particularly high. Now, the electromagnetic phenomena in general are the most sensitive to the presence of such conductive media.

It is not surprising therefore that the magneto-telluric method has already been tested in geothermal exploration.

Up to now, however, considerable difficulties were encountered when having to obtain by means of this method information which could be employed in the exploration and the exploitation of geothermal fields.

*G.E.M.P.,Box 6853,Santa Rosa,CA 95406

A new magneto-telluric method is now proposed which involves a new process of data recording and analysis. It can be applied to objectives of complex geometry.

This method is called the Five-Component Magneto-telluric Method with Exponential Solutions (abbreviated to M.T.-5-E.X.). It renders possible a detailed prospecting based on a very close network of recording stations.

Thus it should be possible to bring to light contrasts in resistivities whatever the depths may be and facilitate the detection and interpretation of areas which, within a field, have different significances from a geothermal point of view.

To put this to the test, the proposed method was applied in an area which is now, and will be in the future, the subject of much research, having shown itself very promising from a productive point of view.

Basic principles of the traditional magneto-telluric method

It is important to begin by reviewing the principles underlying magneto-telluric prospecting (M. T.).

Natural electric currents, called telluric currents, circulating in the ground are known to undergo continuous fluctuations. The earth's magnetic field is also known to react in the same manner.

Yet the fluctuations of the telluric currents and those of the magnetic field are not independent of one another. Generally speaking, the variations occurring in the three components of the magnetic field and in the two horizontal components of the telluric field are linked in a relatively complicated manner, but which always remains quantitative, through the laws of electromagnetism.

As these relations depend on the electric conductivity of the underground, they make it possible to determine this conductivity. The aforesaid natural electromagnetic variations do not have the characteristic of harmonic variations, but are of great complexity. It may be postulated however that they result from the superimposition of harmonic variations of different periods. More precisely it may be said that the spectrum of the magnetic variations, and also that of the corresponding telluric variations, are more or less continuous spectra, whence it is possible to extract almost any period T by means of appropriate filtering. It is felt therefore that the geophysicist may be led to use Fourier transforms to analyse the natural electromagnetic variations.

But, in fact, these natural electromagnetic variations, as mentioned above, are random phenomena, and it will be shown in the following that any analysis-tool in magneto-telluric, when based on the hypothesis that natural electromagnetic variations are harmonic, is limited in practice to the case of the horizontal layered half space.

Characteristics and limitations of the harmonic analysis method

A brief review follows of the basic concepts used in the traditional magneto-telluric method. This problem has been handled by several authors such as CAGNIARD (1953), BERDICHEWSKI (1968), THIKONOV (1950), RIKITAKE (1966). In this section the reader is referred to the publication by CAGNIARD.

Homogeneous half space

Let us assume a homogeneous half space in which the rectangular coordinates x and y are on the flat ground surface and the axis z is the downward positive vertical. Let us consider a schematic and ideal sheet of telluric current, which we shall suppose to be uniform, harmonic of period T, flowing in the electrically homogeneous medium of conductivity σ. If the harmonic sheet flows along x, the components of the Hertz vector $\Pi\ (t)$ along y, z are nil. Furthermore, $\Pi\ (t)$ depends only on z and on t.

$$\Pi\ (t) = \Pi_x e^{-i\omega t}$$

where t is time.

And therefore the electric and magnetic fields are

$$E\ (t) = E_x\ e^{-i\omega t}$$

$$H\ (t) = H_y\ e^{-i\omega t}$$

The factor $e^{-i\omega t}$ will be understood in the following rather than expressed explicitly.

Referring to the quite long periods T generally used in M.T., Maxwell's equations are satisfied if

$$\nabla^2 \Pi_x + 4\pi\sigma\omega i \Pi_x = 0 \qquad (1)$$

In general, the electric field $\mathscr{E}(t)$ and magnetic field $\mathscr{H}(t)$ are expressed by:

$$\left.\begin{aligned} \mathscr{E}(t) &= \text{grad div } \Pi\ (t) - \nabla^2\Pi\ (t) \\ \mathscr{H}(t) &= 4\pi\sigma \text{ curl } \Pi\ (t) \end{aligned}\right\} \qquad (2)$$

and in the present case by

$$\left.\begin{aligned} E_x &= 4\pi\sigma\omega i \Pi_x & E_y &= E_z = 0 \\ H_y &= 4\pi\sigma \frac{\partial \Pi_x}{\partial z} & H_x &= H_z = 0 \end{aligned}\right\} \qquad (3)$$

As in this case E_x is proportional to Π_x, we can choose E_x as the Hertz vector, so that

$$\left.\begin{aligned} \frac{\delta^2 E_x}{\delta z^2} + 4\pi\sigma\omega\ i\ E_x &= 0 \\ H_y = -\frac{i}{\omega} \frac{\delta E_x}{\delta z} \end{aligned}\right\} \qquad (4)$$

Owing to (4), E_x and H_y must be in the form

$$\left.\begin{aligned} E_x &= Ae^{a\sqrt{\sigma}z} + Be^{-a\sqrt{\sigma}z} \\ H_y &= e^{i\frac{\pi}{4}} \sqrt{2\sigma T} \left[-Ae^{a\sqrt{\sigma}z} + Be^{-a\sqrt{\sigma}z}\right] \end{aligned}\right\} \qquad (5)$$

A and B representing two arbitrary constants and a being defined as

$$a = 2 \frac{\pi}{\sqrt{T}} (1 - i)$$

In the case of a half space the terms with positive exponents equal zero, so we have

$$\left.\begin{aligned} E_x &= Be^{-a\sqrt{\sigma}z} \\ H_y &= Be^{i\frac{\pi}{4}} \sqrt{2\sigma T}\ e^{-a\sqrt{\sigma}z} \end{aligned}\right\} \qquad (6)$$

At ground level ($z = 0$)

$$\left.\begin{aligned} E_x &= B \\ H_y &= Be^{i\frac{\pi}{4}} \sqrt{2\sigma T} \end{aligned}\right\} \qquad (7)$$

each of these axes. The first three columns indicate recording and process characteristics.

The values of Θ obtained are generally very good. In any survey, a map of the longitudinal directions may be obtained from them. In areas with geothermal anomalies, the computed longitudinal directions vary very little as a function of τ for any given station.

There is little dispersion of the values of ROALON. The numerical value of the longitudinal conductance may be computed from ROALON for the station considered.

Thus, it is possible to map the longitudinal conductance prevailing in the surveyed region.

Numerical values listed in the ROATRA column may sometimes show a wider dispersion in the case of perfect cylinder geometry. This is due to the fact that, when having to deal with a perfectly cylindrical structure, telluric currents tend to flow along the main trends, for obvious reasons of symmetry.

Nevertheless, the numerical value of the transversal conductance for each station may be computed from ROATRA as well.

The numerical difference between the longitudinal conductance and the transversal conductance may be very large and particularly significant in geothermal exploration.

It should be noted that the apparent resistivities ROX and ROY may show large differences for any station.

The values of ROATRARE and of ROALORES may show some dispersion. They are used only to indicate, by their order of magnitude, whether or not there is a case of cylindrical symmetry. Sometimes, the values of ROAVERT are also somewhat dispersed.

Only their order of magnitude is used. Small numerical values of ROAVERT indicate the proximity of a geological anomaly of some amplitude.

Conclusions

The M.T.-5-E.X. method applied to geothermal exploration defines quantitatively the directional characteristics of electromagnetic phenomena, and thus detects and locates abrupt anomalies in the subsurface. In a paper to be published in *Geothermics*, results will be shown of an M.T.-5-E.X. survey with 84 recording stations, over about 30 km² in the region of Travale (Italy).

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We should also remember a term which is used constantly, that is the « depth of penetration » p, at which the fields' amplitudes are restricted to the fraction $1/e$ of the ground level value. When using the practical units below

$$p = \frac{1}{2\pi} \sqrt{10 \varrho T}$$

Layered half space

The half space now has to be considered as being divided up into homogeneous layers by planes $z_j = \sum_{i=1}^{j} h_i$ (where $z = 0$ is the ground level), h_j is the thickness of the j-th layer, while the thickness of the deepest layer is assumed to be ∞. By σ (j = 1, 2, ... n) we denote the conductivity of the media forming the layer $\left(\sigma_j = \frac{1}{\varrho_j}\right)$.

As we must assure the continuity of the electric and the magnetic fields at the different interfaces, we obtain recurrent relations yielding the values E_x and H_y at ground level

$$E_x = M e^{-i\varphi}$$

$$H_y = \sqrt{2\sigma T}\; N e^{i\left(\frac{\pi}{4} - \psi\right)}$$

where M, N values and angles φ, ψ are suitably evaluated.

Note that the phase shift of H_y with respect to E_x is $\left(\frac{\pi}{4} + \varphi - \psi\right)$.

The well-known relation is obtained for the apparent resistivity

$$\varrho_a = \varrho_1 \left[\frac{M}{N}\right]^2 \quad (8)$$

However, we can agree that the modulus of the ratio $\frac{E_x}{H_y}$ is equal to $\frac{1}{\sqrt{2\sigma_a T}}$ in which σ_a would be the conductivity of a homogeneous formation which would give the same modulus of the ratio between fields whose value has been observed experimentally. The quantity $\sigma_a = \frac{1}{\varrho_a}$ is, by definition, the apparent conductivity, and

$$\varrho_a = 0.2\, T \left|\frac{E_x}{H_y}\right|^2 \quad (9)$$

The pratical units are as follows

$\rho = \frac{1}{\sigma}$ is expressed in ohm · meter (Ω m);

T, is expressed in seconds (s);

E_x, is espressed in millivolt/kilometer (mV/km);

H_y, is expressed in gamma (γ).

In other words, after having carried out an M. T. recording in a region characterized by a tabular resistivity - depth distribution, and after having proceeded with a harmonic analysis of the electric- and magnetotelluric signals, the modulus of the ratio $\frac{E_x}{H_y}$ and the phase shift with respect to T can be obtained. This is achieved by analogical or mathematical filtering or directly by Fourier transforms.

In theory, the knowledge of the modulus of the ratio $\frac{E_x}{H_y}$ for a given period T is sufficient to obtain the value of the corresponding phase shift. Unfortunately, as experience shows, the precision offered by any harmonic analysis for obtaining moduli of the ratio $\frac{E_x}{H_y}$ is quite insufficient to calculate the aforesaid phase shift with the required accuracy. For the same reason, it is all the more difficult to measure reliable and repetitive phase shifts from the M. T. recordings. As mentioned earlier, this is due to the random character of these phenomena. In application, we can only obtain experimentally the modulus of ratio $\frac{E_x}{H_y}$ with an unsatisfactory accuracy, particularly when noise affects recordings.

General case

In the general case, if we accept that M. T. signals in the subsurface come from plane waves whatever the structure and resistivities in the subsurface may be, then all magnetic and electric components in any given location are related by a set of linear equations with constant coefficients (independent of t but function of T). At ground level we assume

$$E_z = 0$$

Then, the following relations may be written,

$$\begin{aligned} H_x &= aE_x + bE_y \\ H_y &= cE_x + dE_y \\ H_z &= gE_x + hE_y \end{aligned} \quad (10)$$

where the components are complex espressions, and so are the six constants a, b, c, d, g, h. To know these constants, the phase shift between the different components of the electric and magnetic fields has to be computed. The moduli of the ratios are no longer sufficient as in the case of the horizontal layer. The dispersion of the phase shift values obtained after any analysis prevents practical use of the phase differences. It becomes thoroughly impossible to solve equation systems of the type (10), and consequently it is impossible to use the vertical magnetic component quantitatively.

Theoretical concept of M.T.-5-E.X.

When it is understood that in practice it is impossible to measure the phase differences between the various electromagnetic natural components, it is then necessary to find another method without using these phase differences.

To this end, a solution to Maxwell's equations in the form of real exponentials will be considered.

To solve (2) it is enough to introduce solutions of the type

$$\Pi(t) = \Pi e^{\frac{t}{\tau}}, \quad H(t) = He^{\frac{t}{\tau}}, \quad E(t) = Ee^{\frac{t}{\tau}}$$

in which τ is real and positive. The time constant τ in this method is comparable to the period T in harmonic analysis.

It should be noted that *the solutions under exponential form* are not justified as a true representation of the natural electromagnetic variations, but *are a mathematical tool*, which avoids the difficulty of obtaining the phase difference.

Homogeneous half space

In this section, information will be given on the consequences related to apparent resistivity and depth of penetration for the homogeneous half space, according to the assumed hypotheses.

The relations previously computed on the basis of a harmonic hypothesis, will now be recomputed with the exponential solutions. All the other features of the telluric current sheet remain. The equation (1) becomes

$$\nabla^2 \Pi_x - \frac{4\pi\sigma}{\tau} \Pi_x = 0 \qquad (11)$$

and

$$\left.\begin{aligned} H_y &= 4\pi\sigma \frac{\partial \Pi_x}{\partial z} \qquad & H_x &= H_z = 0 \\ E_x &= -\frac{4\pi\sigma}{\tau}\Pi_x \qquad & E_y &= E_z = 0 \end{aligned}\right\} \qquad (12)$$

Owing to our exponential solution scheme we obtain

$$E_x = Ae^{a\sqrt{\sigma}\,z} + Be^{-a\sqrt{\sigma}\,z}$$

$$H_y = 2\sqrt{\pi\sigma\tau}\left[-Ae^{a\sqrt{\sigma}\,z} + Be^{-a\sqrt{\sigma}\,z}\right]$$

with

$$a = 2\sqrt{\frac{\pi}{\tau}}$$

The term with an increasing exponential has no physical meaning in the case of a medium with infinite thickness. Therefore, dropping the B constant, one has

$$E_x = e^{-2\sqrt{\frac{\pi\sigma}{\tau}}\,z}$$

and

$$H_y = 2\sqrt{\pi\sigma\tau}\; e^{-2\sqrt{\frac{\pi\sigma}{\tau}}\,z}$$

We find the resistivity in practical units

$$\varrho = 0.4\,\pi\tau\left[\frac{E_x}{H_y}\right]^2$$

while for the depth of penetration *p* one has

$$p = \frac{1}{2\sqrt{\pi}}\sqrt{10\,\varrho\tau}$$

Layered half space

In the case of several horizontal layers, each having a different conductivity, the apparent resistivity is introduced exactly the same as in the harmonic case, with a recurrent expression for each interface. These expressions contain only real numbers and consequently the computations are half as long as in the harmonic case. The following expression for apparent resistivity is obtained

$$\varrho_a = 0.4\,\pi\tau\left[\frac{E_x}{H_y}\right]^2$$

General case

In the general case and when the source is considered at the infinite, whatever the structure of the underground may be, the five electromagnetic components at ground level are related by constant parameters independent of time *t* but function of the time constant τ. The same expressions are obtained (10) as in the harmonic hypothesis, but E_x, E_y, H_x, H_y, H_z, and *a*, *b*, *c*, *d*, *g*, *h*, are now real expressions. One is then able to compute the apparent resistivity in any direction after a rotation of the axis. In other words, it is possible to consider the case of cylindrical or near-cylindrical structures by using simple computation.

Let us assume that the coordinate system used is defined by our pick-up devices on the ground: that is, we are recording the several electric and magnetic components along the axes OX, OY, OZ. Let Θ be the angle at the surface between the parallel OX' to the

SOUNDING 62 - C.N.R. - TRAVALE - ITALY - 1.7.1973

NRAC	NE	DELAF	RATO	NTH		THETA	SIGTH	ROALON	ROATRA	ROALORES	ROATRARE	ROAVERT	ROX	ROY
228.	6041.	11111	8.000	—1	0	56.9	0.	453.1	489.5	107918.3	37573.7	1976.2	710.9	315.3
228.	6041.	11111	7.071	—1	0	56.4	0.	382.4	405.4	52967.3	38051.5	1710.4	569.3	284.0
228.	6041.	11111	6.000	—1	0	55.4	0.	299.1	314.7	56171.6	36962.5	1367.7	432.6	246.7
250.	6411.	11111	5.000	1	1	56.7	1.	277.0	284.6	88086.7	53106.4	1126.4	356.9	220.6
249.	6401.	11111	4.000	9	6	56.7	1.	204.9	211.3	31009.2	11293.0	820.0	249.2	175.7
249.	6401.	11111	3.162	35	23	54.8	1.	160.0	164.7	16801.3	20508.0	689.0	183.9	143.8
250.	6691.	11111	2.646	36	23	57.7	1.	143.9	152.4	8802.7	52317.4	500.6	204.5	122.9
231.	6681.	11111	2.000	89	46	55.6	1.	125.6	131.1	11894.9	6034.3	460.1	185.8	119.1

Provided that enough functions of the type *U* (*t*) have been computed, it becomes possible at the outlet, after some calculations, to obtain a function $\Omega(t)$ for each component from a linear *K*-order combination.

We can then write

$$\Omega(t) = 0 \text{ for } t \leq t_{0_1+K-1}$$

the numerical value of *K* being chosen so that the three conditions (16) be satisfied.

For each component, the required Maxwell's exponential solution is represented by an expression of the type

$$e^{\frac{t}{\tau}}\int_0^{\infty} e^{-\frac{t}{\tau}}\,\Omega(t)\,dt$$

The following in practice may substitute the expression of the preceding type

$$e^{\frac{t}{\tau}}\int_0^{t=5\tau} e^{-\frac{t}{\tau}}\,\Omega(t)\,dt$$

After having constructed enough sets of five solutions of this type it becomes possible to operate several statistical calculations. Threeforе, we obtain the longitudinal direction defined by the angle value « Θ » (13) and, after rotation of the axis, the various apparent resistivities versus the time constant τ which are expressed under the form of the two examples given by soundings 83 and 62. No more details are to be given of the process outlined. Particularly it presents a few practical problems which concern only the specialist and that would exceedingly overload the present text.

Presentation of results

The process described above requires a large memory capacity computer in order to handle five-component recordings. For instance, the data above were processed on a CDC - 7600 computer.

The best way to discuss the type of results obtained is to examine a print-out from the computer after processing with a programme using the exponential solutions.

The two examples mentioned above are from recordings made respectively at two stations in the area of Travale (Italy). The duration of each recording was 1 h 30 m and the sampling rate was 2 scans/sec. The results are shown on a fourteen column chart which will be described summarily.

The fourth column « RATO » shows the values of the square root of τ arranged in decreasing order from the first row, and in function of which are explicited all the other computed values.

The column « THETA » indicates the values of the angle Θ between the longitudinal direction and the OX axis in the coordinate system described above.

The column « SIGTH » shows the sign of THETA, with respect to the OX axis: « 1 » indicates a positive value; « —1 » indicates a negative value; « 0 » indicates indetermination of sign. The values of Θ and their sign are computed independently of one another. The column « NTH » indicates a possible weighting concerning the computed sign.

The columns « ROALON, ROATRA, ROALORES, ROATRARE, ROAVERT » give respectively apparent longitudinal resistivities, apparent transversal resistivities, apparent residual longitudinal resistivities, apparent residual transversal resistivities, and apparent vertical resistivities, which are defined on page 5.

The columns ROX, ROY, indicate the apparent resistivities which would have been obtained along the OX and OY axes respectively, had only one telluric component and one magnetic component been used for scribed above. It is also assumed that the recordings which are thus obtained are digitalized on magnetic tape, and that the time constant τ considered is large enough with respect to the time spacing defined by the sampling rate.

On the records six arbitrary instants t_0, t_1, t_2, t_3, t_4, t_5, are selected: in practice it may be advantageous to tie up the order of magnitude of the time intervals determined by the instant t_i, to the considered τ. On each component of the record, each instant t_i corresponds to a sample labelled with an index n_i. Five arbitrary constants are then determined a_1, a_2, a_3, a_4, a_5, so that on each component $X(t)$ a relation of the following type is verified.

$$X(t_0) + a_1 X(t_1) + a_2 X(t_2) + a_3 X(t_3) + a_4 X(t_4) + a_5 X(t_5) = 0 \qquad (17)$$

By writing this relation for each component, we define a linear system of five equations. So the numerical value of the five constants mentioned above are calculated.

For each considered component one constructs a function of the type $U(t)$ by taking instant t_0 as the initial time, that is

$$U(t) = X(t) + a_1 X(t+t_1) + a_2 X(t+t_2) + a_3 X(t+t_3) + a_4 X(t+t_4) + a_5 X(t+t_5) \qquad (18)$$

the function $U(t) = 0$ for $t \leq t_0$

Let us assume now that we have constructed six functions of the type (18) for each component. Every one of these six functions takes zero value until an origin instant that we shall call respectively

$$t_{0_1},\ t_{0_2},\ t_{0_3},\ t_{0_4},\ t_{0_5},\ t_{0_6}$$

corresponding respectively to the samples expressed above

$$n_1 \quad n_2 \quad n_3 \quad n_4 \quad n_5 \quad n_6$$

and are expressed by taking instant t_{0_1} as initial time

$$U_1(t),\ U_2(t+t_{0_2}),\ U_3(t+t_{0_3}),\ U_4(t+t_{0_4}),\ U_5(t+t_{0_5}),\ U_6(t+t_{0_6}),$$

Let us now consider the instant set defined on each component by the samples with the indexes

$$n_{1+1} \quad n_{2+1} \quad n_{3+1} \quad n_{4+1} \quad n_{5+1} \quad n_{6+1},$$

corresponding to instants that are respectively expressed by

$$t_{0_{1+1}},\ t_{0_{2+1}},\ t_{0_{3+1}},\ t_{0_{4+1}},\ t_{0_{5+1}},\ t_{0_{6+1}}.$$

The five constants α, β, γ, δ, η are now introduced in order to satisfy the following condition for each component

$$U_1(t_{0_{1+1}}) + \alpha U_2(t_{0_{2+1}}) + \beta U_3(t_{0_{3+1}}) + \gamma U_4(t_{0_{4+1}}) + \delta U_5(t_{0_{5+1}}) + \eta U_6(t_{0_{6+1}}) = 0$$

Now, by taking instant t_{0_1} as the initial time, we construct a function of the following type for each component

$$\Psi(t) = U_1(t) + \alpha U_2(t+t_{0_2}) + \beta U_3(t+t_{0_3}) + \gamma U_4(t+t_{0_4}) + \delta U_5(t+t_{0_5}) + \eta U_6(t+t_{0_6}) \qquad (19)$$

Functions of type (19) equal zero for $t \leq t_{0_{1+1}}$. We shall call them « linear second order combination ».

SOUNDING 83 - C.N.R. - TRAVALE - ITALY - 3.7.1973

NRAC	NE	DELAF	RATO	NTH		THETA	SIGTH	ROALON	ROATRA	ROALORES	ROATRARE	ROAVERT	ROX	ROY
116.	6370.	11111	8.000	—1	0	32.7	0.	105.7	156.4	10505.7	1421.1	329.9	57.8	282.6
116.	6370.	11111	7.071	—1	0	33.6	0.	90.4	135.1	18703.2	1341.5	296.5	48.6	247.0
116.	6370.	11111	6.000	0	0	33.4	0.	73.7	108.4	8448.4	1276.7	238.1	39.2	197.9
238.	9550.	11111	5.000	0	0	34.6	0.	65.6	95.9	3931.5	3463.6	281.0	38.5	181.7
238.	9550.	11111	4.000	11	9	33.9	—1.	51.6	67.6	2645.3	1277.3	181.1	29.0	126.6
235.	9550.	11111	3.162	28	16	34.5	—1.	40.8	48.9	2320.5	2585.8	157.8	22.1	92.0
250.	9820.	11111	2.646	41	21	32.7	—1.	38.2	49.5	1192.3	2263.4	109.0	20.7	87.2

cylindrical or near-cylindrical structure axis and the OX axis (Figure 1).

After rotation Θ, we can write

$$E_x = E_{x'} \cos\Theta - E_{y'} \sin\Theta$$

$$E_y = E_{x'} \sin\Theta + E_{y'} \cos\Theta$$

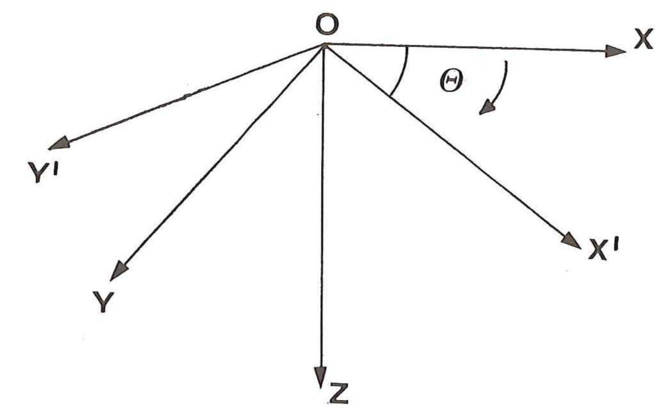

FIG. 1

and, by remembering (10), we obtain H_z

$$H_z = E_{x'} [g\cos\Theta + h\sin\Theta] + E_{y'} [h\cos\Theta - g\sin\Theta]$$

If the telluric sheet is parallel to the axis of the cylinder, H_z generally does not equal zero (except for symmetrical cases which do not happen in nature) but the contribution of E_y' equals zero. Consequently we obtain

$$g\sin\Theta = h\cos\Theta$$

In other words, the direction of axis OX' is given by

$$\tan\Theta = \frac{h}{g} \qquad (13)$$

Knowing Θ, as mentioned above, it is easy to compute the various following apparent resistivities after rotation of the axis. The direction given by Θ will be called « longitudinal direction » or in more geological terms « main trend ».

Other results may then be obtained

apparent longitudinal resistivity (along the axis OX')

$$= 0.4\,\pi\tau \left[\frac{E_{x'}}{H_{y'}}\right]^2 = 0.4\,\pi\tau \left[\frac{g^2+h^2}{cg^2 - bh^2 + (d-a)\,gh}\right]^2$$

apparent transversal resistivity (along the axis OY')

$$= 0.4\,\pi\tau \left[\frac{E_{y'}}{H_{x'}}\right]^2 = 0.4\,\pi\tau \left[\frac{g^2+h^2}{bg^2 - ch^2 + (d-a)\,gh}\right]^2$$

apparent longitudinal residual resistivity

$$= 0.4\,\pi\tau \left[\frac{E_{x'}}{H_{x'}}\right]^2 = 0.4\,\pi\tau \left[\frac{g^2+h^2}{ag^2 + dh^2 + (b+c)\,gh}\right]^2$$

apparent transversal residual resistivity

$$= 0.4\,\pi\tau \left[\frac{E_{y'}}{H_{y'}}\right]^2 = 0.4\,\pi\tau \left[\frac{g^2+h^2}{ag^2 + dh^2 - (b+c)\,gh}\right]^2$$

apparent vertical resistivity

$$= 0.4\,\pi\tau \left[\frac{E_x}{H_z}\right]^2 = \frac{0.4\,\pi\tau}{g^2+h^2}$$

Thus, the use of the quantitative variations of the vertical magnetic component is very important since it enables to determine the longitudinal direction Θ and all the set of resistivities shown above. The physical meaning of these quantities and their application in geothermal prospection will be shown later on. However, it should be noted that, as long as there is a high enough value of H_z a direction will always be found, whether the considered case is cylindrical or not. The criteria for the cylindrical case are given by the « apparent residual longitudinal resistivity » or by the « apparent residual transversal resistivity », which in this case must be infinite or at least very large.

Some applied aspects of M.T.-5-E.X.

As mentioned above, natural electromagnetic variations are no more harmonic than exponential. In either case, it is necessary to construct solutions corresponding to the type of analysis chosen. The following will show what the various conditions are, under which the exponential type solution may be used, and how exponential type solutions solving Maxwell's equations may be easily determined from field recordings.

Exponential type solution to Maxwell's equations, constructed from the output signal of a magnetic sensor

Some words will be given to an illustration of a number of topics related to the magnetic sensor. Such a sensor consists of an air or magnetic cored coil with a convenient number of turns.

This winding may be represented by the simple circuit of Figure 2, regardless of its shape or its complexity. One emphasises that the capacity C is either the distributed capacity of the coil, or the distributed capacity plus an external capacity added in order to create resonance effects favourable to an increased sensitivity in certain regions of the spectrum.

Let this circuit (Figure 3) be placed at the input of an amplifier and recorder. The input impedance of the amplifier being ξ, this may be assumed to be a pure resistance to simplify the discussion. If the coil is energized by the magnetic field H (x, y, z, t), in abbreviation H (t), the current i flowing through ξ is given by

$$L\xi C\frac{d^2 i}{dt^2} + (\xi RC + L)\frac{di}{dt} + (R+\xi)\, i = -\chi\frac{\partial H(t)}{\partial t} \qquad (14)$$

with

$$\chi = \mu_a N S$$

μ_a = apparent permeability of the core;

N = number of turns of the coil;

S = surface of the mean turn of the coil.

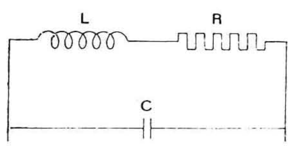

FIG. 2

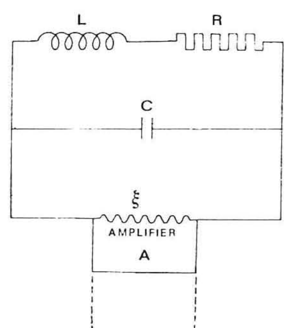

FIG. 3

This relationship, which involves a second order differential equation, only concerns the spectrum of usual magnetic variations utilized in geothermal prospection and characterized by slow variations. Under this restriction, the output tension of the amplifier is defined as:

$$U = \xi i$$

the constant K of amplification being understood.

Instant t being taken as time origin until which i equals zero, we write

$$\xi I \subset \xi i$$

ξ I is the Laplace transform of the tension ξ i, function of time t. Now, I is expressed by

$$I = \left[\frac{i_{(t=0)}\left[\frac{L\xi C}{\tau} + (\xi RC + L)\right] + L\xi C\frac{di}{dt}_{(t=0)}}{\Xi} - \frac{\chi}{\Xi}\int_0^\infty e^{-\frac{t}{\tau}}\frac{\partial H(t)}{\partial t}\,dt\right]$$

by taking

$$\Xi = \frac{L\xi C}{\tau^2} + \frac{(\xi RC + L)}{\tau} + (R+\xi)$$

It is obvious that when

$$i_{(t=0)} = 0 \text{ and } \frac{di}{dt}_{(t=0)} = 0$$

one has

$$I = -\frac{\chi}{\Xi}\int_0^\infty e^{-\frac{t}{\tau}}\frac{\partial H(t)}{\partial t}\,dt$$

Now, we go back to Maxwell's equations to find out the conditions under which expression $Ie^{\frac{t}{\tau}}$ is a Maxwell's solution. Being any electromagnetic magnitude, X should verify the equation

$$\nabla^2 X - \frac{4\pi}{\varrho}\frac{\partial X}{\partial t} = 0 \qquad (15)$$

being understood that X = (x, y, z, t).

Let's consider the expression

$$Y = e^{\frac{t}{\tau}}\int_0^\infty e^{-\frac{t}{\tau}}\frac{\partial X}{\partial t}\,dt$$

The requirements for Y to be a solution of (15) are now considered from

$$Y = e^{\frac{t}{\tau}}\left[\frac{1}{\tau}\int_0^\infty e^{-\frac{t}{\tau}} X\,dt - X_{(t=0)}\right]$$

Thus

$$\nabla^2 Y = e^{\frac{t}{\tau}}\left[\frac{1}{\tau}\int_0^\infty e^{-\frac{t}{\tau}}\nabla^2 X\,dt - \nabla X_{(t=0)}\right]$$

On the other hand

$$\frac{\partial Y}{\partial t} = \frac{e^{\frac{t}{\tau}}}{\tau}\int_0^\infty e^{-\frac{t}{\tau}}\frac{\partial X}{\partial t}\,dt$$

If one brings $\nabla^2 Y$ and $\frac{\partial Y}{\partial t}$ into (15), the latter becomes

$$e^{\frac{t}{\tau}}\left[\frac{1}{\tau}\int_0^\infty e^{-\frac{t}{\tau}}\nabla^2 X\,dt - \nabla^2 X_{(t=0)} - \frac{4\pi}{\varrho}\frac{1}{\tau}\int_0^\infty e^{-\frac{t}{\tau}}\frac{\partial X}{\partial t}\,dt\right] = 0$$

or briefly

$$e^{\frac{t}{\tau}}\left[\frac{1}{\tau}\int_0^\infty e^{-\frac{t}{\tau}}\left(\nabla^2 X - \frac{4\pi}{\varrho}\frac{\partial X}{\partial t}\right)dt - \nabla^2 X_{(t=0)}\right] = 0$$

The term $\nabla^2 X - \frac{4\pi}{\varrho}\frac{\partial X}{\partial t}$ which is present under the integral symbol, equals zero by definition, since X is an electromagnetic magnitude. In this particular case it becomes necessary that

$$\nabla^2 X_{(t=0)} = 0$$

or otherwise, since (15) must be verified, that

$$\frac{\partial X}{\partial t}_{(t=0)} = 0$$

In other words, the expression ξ $Ie^{\frac{t}{\tau}}$ will be a Maxwell's solution, provided that $\frac{\partial X}{\partial t}_{(t=0)}$ is equal to zero.

Besides, equation (14) gives the relation

$$(R+\xi)\, i_{(t=0)} + \frac{di}{dt}_{(t=0)}(\xi RC + L) + L\xi C\frac{d^2 i}{dt^2}_{(t=0)} = -\chi\frac{\partial H(t)}{\partial t}_{(t=0)}$$

Otherwise, if to the two conditions already expressed above one adds

$$\left.\begin{aligned} i_{(t=0)} &= 0 \qquad \frac{di}{dt}_{(t=0)} = 0 \\ \frac{d^2 i}{dt^2}_{(t=0)} &= 0 \end{aligned}\right\} \qquad (16)$$

all the requested conditions are met and $Ie^{\frac{t}{\tau}}$ actually is a solution of Maxwell's equation (15) since then $\frac{\partial H(t)}{\partial t}_{(t=0)}$ equals zero according to the condition $\frac{\partial X}{\partial t}_{(t=0)}$

Construction of exponential type solutions from actual recordings

It is always possible to find a characteristic time as defined by the three conditions (16) in a recording from a single magnetic sensor along any axis. However, there is some difficulty in finding these three conditions satisfied simultaneously on the recordings from the three magnetic sensors energized by $H_x(t)$, $H_y(t)$, $H_z(t)$, respectively. Besides, the telluric recordings [$E_x(t)$, and $E_y(t)$] yielded by traditional telluric lines are a priori not justified in satisfying the conditions (16). Then, various adaptations of the recording device have to be made before exponential type solutions, as described above, may be constructed.

Adaptation of the recording device

Some adaptations of the recording device are briefly described in the following paragraph.

Let us assume that the three magnetic recording channels are of the above defined type and characterized by the response (14). By introducing a « distortion generator » into each telluric recording channel we obtain, between the output terminals of each aforesaid telluric recording channel, an output signal of the ξi type. ξ is input impedance of the amplifiers used in the telluric recording chains, and current i is linked to the variations of the electric component, for instance $E_x(t)$, by the following relation, with an approach at least equal to 10^{-2} in the used spectrum

$$L\xi C\frac{d^2 i}{dt^2} + (\xi RC + L)\frac{di}{dt} + (R+\xi)\, i = -K\frac{\partial E_x(t)}{\partial t}$$

with L, C, and R having the same value as those of physical components of the magnetic recorders. The five-component recordings then have to be processed all incoming from recording channels characterized by identical transfer functions, and identical transient responses to the energizing signals. The recording device so designed otherwise offers many practical advantages, and in particular eliminates all troubles of electrods-polarization on the telluric lines. It has been subject to various patents pending in several countries.

Now let's examine how it is possible, from records made by devices designed as described above, to elaborate exponential solutions in accordance with the conditions (16). Among other possible methods, experience proved that the simplest procedure is that which we shall call « the linear combination method », which the next paragraph will deal with.

Linear combination method

Assumption is made that the five-electromagnetic components are recorded according to the device de-