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Temperatures with Depth Resulting from Frictionally Generated Heat During Metamorphism

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ABSTRACT

When deformation leads to differential movement along interfaces—for example faulting, flexural slip folding, grain-boundary slip—heat is generated by friction. Opinions differ concerning the importance of mechanically generated heat during regional dynamothermal metamorphism. Turner and Verhoogen (1960) deferred a definite conclusion until such time as quantitative data might become available.

A mathematical solution to the non-steady-state heat flow problem permits the calculation of temperature rise in response to a given rate of heat production within a bounded region in the crust. This solution may find application in specific cases, provided the variables can be evaluated reasonably precisely.

The computations presented here of temperature increases at eight depths for each of 40 models show that frictional generation of heat may result in significant local elevation of temperature. Variables used in the computations are: time and rate of heat generation, and depths of boundaries within which heat is generated. Average energy per gram converted to heat and strain rate within the deforming part of the crust are computed for each model.

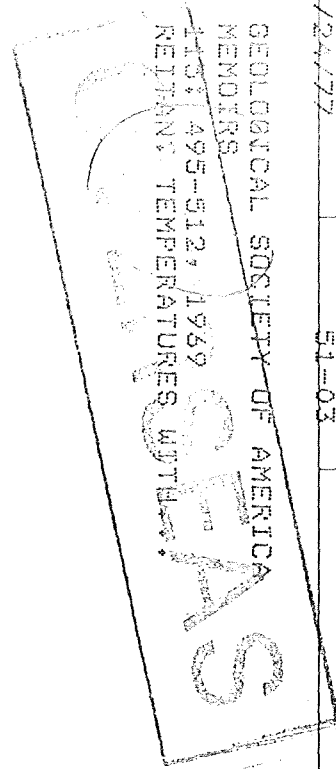
Frictional heat due to deformation is not solely responsible for the temperatures of regional metamorphism but may, when deformation is concentrated in space and time, account for local steepening of the temperature gradient, and locally for a mineral assemblage of higher metamorphic grade than would otherwise have been present.

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INTRODUCTION

Geologic observations in metamorphic terranes, coupled with laboratory studies of stability ranges of minerals and of mineralogic reactions, verify that the regional metamorphism which is typical of deformed geosynclinal accumulations reflects temperatures from about 200°C to 750°C and pressures corresponding to depths as much as 25 km. In many cases, temperatures at which partial fusion begins are reached when the pressures correspond to depths of 15 to 20 km (Hietanen, 1961, 1967). Of particular significance is the Al_2SiO_5 pressure-temperature triple point (Hietanen, 1967; Holm and Kleppa, 1966; Newton, 1966; Weill, 1966) which, though yet uncertain, suggests that the attainment of temperatures of about 600°C at depths corresponding to about 15 km must be inferred to be common.

This poses a problem. If, as has been suggested by Birch (1955, 1965), the "normal" temperature at a depth of 30 km is not likely to exceed 600°C below continents, and the "normal" temperature gradient in the upper parts of the continental crust approximates 30°C per km, the temperatures of metamorphism cannot reasonably be attributed to a "normal" heat flow from the subjacent crust through the geosynclinal accumulations. Fyfe and Verhoogen (1958) and Turner and Verhoogen (1960) have ably stated the problem.

Whence comes the enormous amount of heat energy necessary to achieve the temperatures of metamorphism at the depths at which metamorphism occurs? This problem becomes even greater when it is recognized that most of the mineralogical reactions involved are endothermic. The ultimate source of the necessary energy must be the mantle. But if the "normal" heat flow by conduction does not maintain a temperature gradient adequate to account for regional metamorphism, how is heat energy transferred to those portions of the crust undergoing metamorphism in sufficient quantities?

It is the purpose of this paper to examine the role of frictionally generated heat as one possible contributing source of heat energy in regionally metamorphosed orogenic zones. Assuming that the continental crust is deformed in response to processes in the mantle, then mechanical energy may be converted to frictional

heat energy in that volume of the crust being deformed. Consideration will be restricted here to the potential contribution of frictionally generated heat in materials that behave as brittle solids. Frictional heat generation resulting from slip between interfaces following brittle failure would correspond in effect to exothermic chemical reactions, resulting in the elevation of temperature of the mass undergoing deformation.

ACKNOWLEDGMENTS

The encouragement of T. F. W. Barth at various critical times has been deeply appreciated. Verhoogen very graciously corrected me when I had gone astray and encouraged me to try to obtain a non-steady-state solution to apply to the problem. At various times, W. R. Dickinson, J. S. King, L. H. Larsen, and B. M. Page have helped. Allen C. Robinson wrote the computer program, and computer time was paid for by a grant from the Shell Fund for Fundamental Research administered by the Department of Geology, Stanford University. H. C. Clark solved the non-steady-state heat flow problem (*see* Appendix); without his help significant progress on the problem could not have been made.

FRICTIONAL HEAT

The importance of frictional heat has been oppositely assessed. Ambrose (1936) noted a direct correlation between metamorphic grade and the degree of penetrative deformation. On the basis of field studies, he suggested that mechanically generated heat was important in controlling metamorphic temperatures. Niggli (1924) and Harker (1932) expressed the opposite opinion and believed that the role of frictional heat was minor. In the absence of any quantitative treatments of the problem, no definite conclusion can be reached.

To obtain an estimate of the heat produced along sliding interfaces, reference may be made to the work of Bowden and his co-workers (for example, Bowden and Thomas, 1954; Bowden and Persson, 1961). These investigators measured the temperatures attained at points of real contact between interfaces which slid past one another. They found, in general, that temperatures were limited upward by the melting point of materials. They also measured the temperature build-up time of these hot spots. From these data, it appears that slip of about $\frac{1}{10}$ mm or less at moderate speeds (for example, 1 m sec^{-1}) is sufficient to heat all points of real contact to temperatures approaching the melting point of the solids at the interface.

The work reported by Bowden and Tabor (1950; *see also* Orowan, 1960) on the real contact area per unit area of apparent contact shows that this ratio increases linearly with load across the interfaces. On steel flats, Bowden and Tabor measured the fraction of the macroscopic area making real contact as a function of load by relating this to the electrical resistance across the interface. The area of real contact increases because of elastic deformation of the points of contact as the load increases. As the strength of steel at room temperature is comparable to that of minerals in the metamorphic temperature range (Griggs and others, 1960), it is possible to approximate the change in real contact between interfaces with varying depth in the crust.

Combining the available information on the temperatures reached at points of real contact along sliding interfaces, the data on amount of real contact per unit area, the average heat capacity value for rock materials, the change of ambient temperature with depth (which limits the increase of temperature at points of contact being heated), and assuming that the depth of heating at hot spots equals the radius of the radiating area (Bowden and Thomas, 1954), one may calculate an average value of the heat produced at points of real contact per unit area of sliding interface (area of apparent contact) when slip is just sufficient to heat all the points which initially made real contact. This may be done for any depth zone if the change of the variables with depth is taken into account.¹

This method is based on average values for rock materials (heat capacity, specific gravity, strength, melting temperature) and involves several assumptions. Therefore, it can be expected to yield only approximate values. However, the general validity of the approach is verified by comparison with a calculation by Jeffreys (1942) on the heat generated along a fault at a depth of 1 km. After Jeffreys' value is adjusted for a different heat capacity value used, his value of 2×10^{-2} calories per mm of slip per cm^2 apparent contact area compares favorably with the value of 1.2×10^{-2} calories per mm of slip per cm^2 apparent contact area obtained by the method introduced here. The lower value, as derived here, was used to obtain a rate of heat production in a region undergoing deformation. By using this value, resulting error is probably in the direction of an underestimate of the heat produced, and the risk of overestimating the importance of the factor being considered is diminished.

After obtaining an estimate of the heat produced per cm^2 area of apparent contact per $\frac{1}{10}$ mm of sliding, it is necessary to convert this to a rate of heat production in a geosynclinal accumulation during orogenesis. To do this a column 1 cm^2 in cross-sectional area extending from the surface to a depth of 20 km is taken as representative. Deformation models may then be constructed by assuming slip of $\frac{1}{10}$ mm along interfaces (for simplicity of visualizing the model the interfaces may be imagined as horizontal) which are spaced at a selected average distance in any segment of the column. Slip is assumed to occur once on each interface during a given time interval. By employing this method, a rate of heat production in the deforming segment is

¹The ratio, true area to apparent area of contact, increases with load. From the results given by Bowden and Tabor (1950) for steel flats this relationship may be stated as:

$$\text{load} = C \left(\frac{\text{true contact area}}{\text{apparent contact area}} \right)$$

where load is measured in gm cm^{-2} and C is a proportionality constant and equals 10^7 gm cm^{-2} .

Load across interfaces as a function of depth is averaged for 5 km thick segments of crust using increasing specific gravity, ρ , with depth: 0 to 5 km, $\rho = 2.5$; 5 to 10 km, $\rho = 2.6$; 10 to 15 km, $\rho = 2.7$; and 15 to 20 km, $\rho = 2.8$. Then assuming that the depth of heating equals the measured radius of hot spots (Bowden and Thomas, 1954), the volume heated per interface is simply the product of the true area per cm^2 apparent area of contact times 10^{-3} cm (the assumed depth of heating).

The heat produced per interface along which slip occurs is the product of the volume heated per interface times the heat capacity times the number of degrees heated. Heat capacity is taken as a constant, $0.5 \text{ cal cm}^{-3} \text{ deg}^{-1}$; the number of degrees heated is assumed to change between each 5 km thick segment: 5 to 10 km, $\Delta T = 10^3 \text{ deg}$; 10 to 15 km, $\Delta T = 8 \times 10^2 \text{ deg}$; 15 to 20 km, $\Delta T = 5 \times 10^2 \text{ deg}$, being limited upward by the melting point of the material (Bowden and Thomas, 1954).

obtained. The strain rate in the deforming segment of the representative column and the total energy converted to heat may also be determined for each model constructed.

As the depth of the segment concerned increases, the ratio, true contact area to apparent contact area increases, and the ambient temperature also increases. The effect of the former is to increase the volume of material heated along each interface and the effect of the latter is to reduce or limit the temperature increase of that volume, owing to the fact that temperature increase at these points is limited upward by the melting point. The net effect of the two is to effectively cancel one another. A single curve then serves to relate the average spacing of slip surfaces in a given time interval to a rate of heat production in the volume regardless of depth. For example in Figure 1, the average spacing of surfaces along which slip occurs once per 1000 years is plotted against rate of heat production, Q. (It is evident that slip with average spacing of 25 cm once per 1000 years equals slip with average spacing of 2.5 cm once per 10,000 years, and so on. Obviously the strain rate, 0.1 mm slip frequency per cm length of the representative column, could be used, but the resulting number would probably be less immediately descriptive to most readers than average spacing of slip surfaces along which slip occurs once per 1000 years. (See Reitan, 1968a, for the curve relating rate of heat production to strain rate.)

STEADY-STATE SOLUTIONS

After associating a rate of heat production with a deformation model, the next step is to relate this to temperatures that would result in the crust by setting up and solving appropriate heat flow problems. Frictional heat generation results in heat

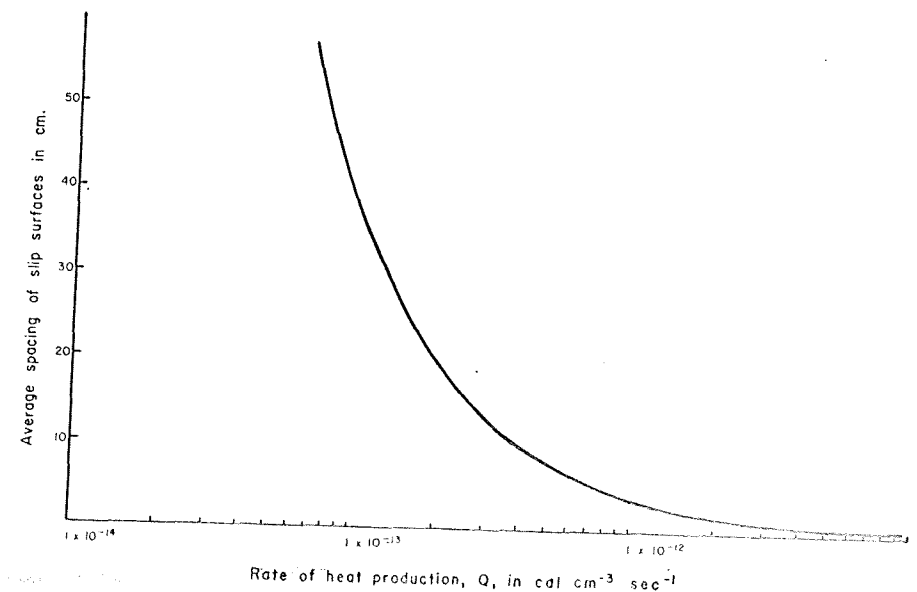


Figure 1. Rate of heat production, Q, as a function of spacing between interfaces, assuming slip on each interface once per $3 \times 10^3 \text{ sec}$ (~ 1000 years).

production, and is thus comparable to heat production by exothermic reactions. A steady-state solution using a constant rate of heat production may be compared directly with the treatment by Fyfe and Verhoogen (1958), who presented the results of computations of temperatures at two depths (10 km and 20 km), taking into account endothermic mineral reactions occurring in the crust above each depth for two different constant rates of heat consumption.

The equation of heat conduction for the steady-state exothermic case is:

$$\frac{\partial T}{\partial t} = h^2 \left(\frac{\partial^2 T}{\partial z^2} \right) + E = 0$$

where T is temperature, t is time, z is depth, E is the rate of heat production divided by specific heat (c , taken as $0.2 \text{ cal gr}^{-1} \text{ deg}^{-1}$), and $h = \frac{k}{\rho c}$ (where k is conductivity, taken as $4 \times 10^3 \text{ cal cm}^{-1} \text{ sec}^{-1} \text{ deg}^{-1}$, and ρ is density).

If there is a constant heat flow from below sufficient to maintain a steady-state gradient of 30° per km in the absence of local heat effects, then after integration and evaluation of the constants of integration, the temperature at the base of the unit within which heat is being produced is given by:

$$T = -\frac{E}{2h^2} z^2 + Kz$$

where K , a constant of integration, equals $1.1 \times 10^{-3} \text{ deg cm}^{-1}$.

For different rates of heat production, Q , within the uppermost 10, 15, and 20 km, the temperature at depths 10, 15, and 20 km is shown in Figure 2.

Examination of Figure 2 suggests that if the rate of heat production exceeds about $1 \times 10^{-13} \text{ cal gm}^{-1} \text{ sec}^{-1}$, the elevation of temperature above any prior thermal

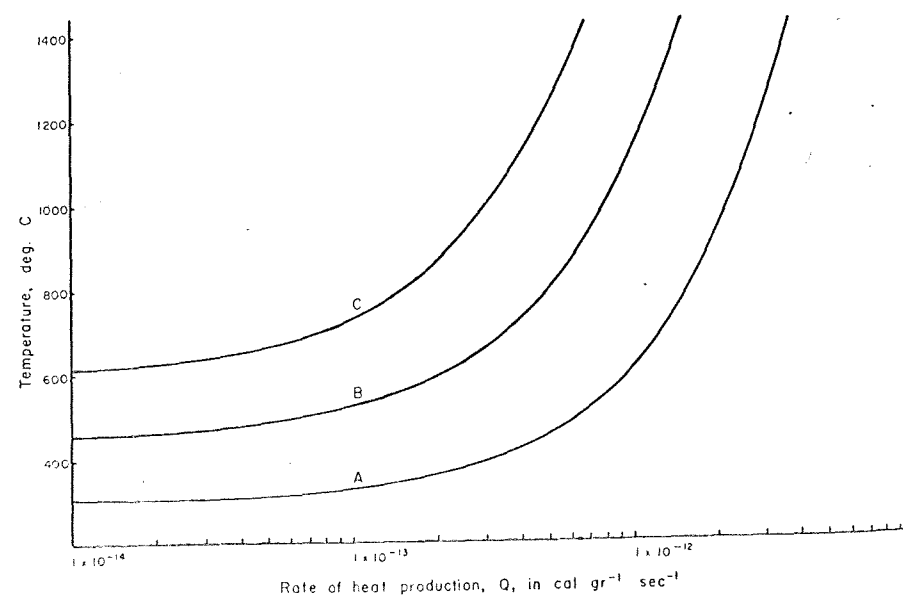


Figure 2. Steady-state temperatures at 10 km (curve A), 15 km (curve B), and 20 km (curve C), when heat is produced at the rate, Q , uniformly throughout the top 10, 15, and 20 km respectively. In all cases heat flow from below is assumed just sufficient to maintain a gradient of 30° per km in the absence of heat effects in the crust.

gradient would be significant. Comparison with Figure 1 indicates that slip on interfaces spaced about 10 cm apart occurring once per 1000 years—a strain rate of about $3 \times 10^{-14} \text{ sec}^{-1}$ —would result in a rate of heat production this great (note: Q is given in $\text{cal cm}^{-3} \text{ sec}^{-1}$ in Figure 1 and in $\text{cal gm}^{-1} \text{ sec}^{-1}$ in Figure 2, thus a conversion factor equal to the density must be used to obtain equivalence of scales).

NON-STEADY-STATE SOLUTIONS

Time is an important variable in relating frictional heat generation to the temperatures produced in deforming bodies. Recognizing that a high local rate of heat production for a relatively short time may be realistic, whereas that same rate of heat production continued for infinite time is not, and feeling that a uniform style of deformation above a given depth also fails to provide a good basis for comparison with real situations, it seemed desirable to obtain a non-steady-state solution to a heat flow problem in which a depth region of deformation could be selected. Solution to this problem allows the construction of a temperature versus depth curve for any selected time after deformation had begun. The deformation is restricted to a defined region with upper and lower boundaries.

The problem to be solved may be stated as follows: determine the temperature, T , at any depth, z , at the time, t , when heat is produced within the crust between the depths a and b at a constant rate, Q , for various values of Q . The boundary conditions are: (1) surface temperature, T , equals 0 at all times, t ; (2) when $t > 0$ heat is generated at a rate, Q , from depths $z = a$ to $z = b$; (3) there are no discontinuities in the temperature versus depth curve, that is:

$$\left. \begin{array}{l} T_{+a} = T_{-a} \\ T_{+b} = T_{-b} \end{array} \right\} t \geq 0$$

$$\frac{\partial T}{\partial z_{+a}} = \frac{\partial T}{\partial z_{-a}} \text{ and } \frac{\partial T}{\partial z_{+b}} = \frac{\partial T}{\partial z_{-b}}$$

The applicable differential equations are:

$$\text{for } z < a \quad \frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa} \frac{\partial T}{\partial t} = 0$$

$$\text{for } a < z < b \quad \frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa} \frac{\partial T}{\partial t} = -\frac{Q}{k}$$

$$\text{for } b < z \quad \frac{\partial^2 T}{\partial z^2} - \frac{1}{\kappa} \frac{\partial T}{\partial t} = 0$$

where k is conductivity (taken as $4 \times 10^3 \text{ cal cm}^{-1} \text{ deg}^{-1}$), κ is thermal diffusivity (taken as $8 \times 10^{-3} \text{ cm}^2 \text{ sec}^{-1}$), z , the depth (in cm), is positive downward, and heat flows linearly upward.

The solution to the problem was not found in the reference book by Carslaw and Jaeger (1959), but was worked out by H. C. Clark of Rice University who has supplied the explanation found in the Appendix.

For temperature increase at a specific depth at a specific time the solution has the form:

$$T_{(z,t)} = \frac{Q\kappa}{2k} \left\{ \left[\left(t + \frac{(a-z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(a-z)}{2\sqrt{\kappa t}} \right) - (a-z) \left(\sqrt{\frac{t}{\pi\kappa}} \right) e^{-\frac{(a-z)^2}{4\kappa t}} \right] - [B] - [C] + [D] \right\}$$

where [B], [C], and [D] are terms similar to the first term in brackets (*see* Appendix for all these terms stated in full for each depth zone), $T_{(z,t)}$ is the number of degrees C by which the temperature has been raised above any prior gradient at the depth, z , at the time, t . k and κ are conductivity and diffusivity, and erfc is $1 - \operatorname{erf}$ where erf is the error function, values for which are tabulated in several sources (for example, National Bureau of Standards, 1954).

Arithmetic computation of values for $T_{(z,t)}$ are extremely tedious and time consuming. Thus, to define temperature versus depth curves for several models, a computer program was written by Allen C. Robinson of Stanford University. The total Burroughs 5500 computer time required for evaluation of $T_{(z,t)}$ for eight depths for each of 40 models was only a few minutes.

The models for which solutions have been computed were constructed from various combinations of specific values for the variables. They are: time, $t = 3 \times 10^{13}$ sec (~ 1 million years) and 3×10^{14} sec.; upper, a_1 , and lower, b , boundaries of the region of heat generation, $a_1 = 10$ km and $b_1 = 15$ km, $a_2 = 15$ km and $b_2 = 20$ km, $a_3 = 10$ km and $b_3 = 20$ km, and $a_4 = 5$ km and $b_4 = 20$ km; and rate of heat production, Q , in $\text{cal cm}^{-3} \text{sec}^{-1}$, = 3.0×10^{-12} , 1.5×10^{-12} , 1.0×10^{-12} , 3.0×10^{-13} , and 1.5×10^{-13} .

Eight values of $T_{(z,t)}$ are given in Table I for each of 40 models, the numbers in the body of the table for each model signify the temperature increases at the depth indicated. Twenty of the models are illustrated in Figure 3a, b, c, and d. The figure shows the temperature versus depth curve resulting if, at $t = 0$, there is an 30° per km temperature gradient and the heat flow from below 20 km remains constant at a value just sufficient to maintain this gradient in the absence of local heat effects. The selection of models for illustration was based on an arbitrary exclusion of any model for which the energy converted to heat averaged over the upper 20 km of the crust exceeded 27 cal gm^{-1} or was less than 4 cal gm^{-1} .

The temperature increases indicated by these models arise from deformation of material which fails by fracture, that is, behaves as a brittle material, with subsequent sliding between the interfaces produced. The deformation model and computations based on it cannot be extended to material which yields plastically.

The strain rates of the deformation proposed vary from about $2 \times 10^{-13} \text{ sec}^{-1}$ to $1 \times 10^{-14} \text{ sec}^{-1}$; this range compares with the strain rate reported by Gilluly (1949) at Cajon Pass of $6 \times 10^{-14} \text{ sec}^{-1}$, and that inferred by Crittenden (1967) of the order of $10^{-14} \text{ sec}^{-1}$ from studies of Lake Bonneville.

GEOLOGICAL IMPLICATIONS AND CONCLUSIONS

When brittle failure is followed by differential movement along the interface or whenever stress leads to slip along existing interfaces, frictional heat generation is unavoidably concomitant (with the possible exception of, for example, movement immediately following a phase change which releases fluid in such quantity as to reduce the stress transmitted across grain boundaries or other solid-solid interfaces sensibly to zero). During faulting, slip with generation of heat along interfaces is

obvious. Both large faults and the movements responsible for microearthquakes, which in seismic areas occur at rates as high as several hundred per day (Oliver and others, 1965), imply fracture and slip. Microseismic activity suggests that the rock is cracking on a small scale or that surfaces within the rock are sliding on one another (Press and Brace, 1966). Watanabe (1963), in laboratory experiments, was able to associate small shocks with local brittle fractures in rocks subjected to uniaxial compression. Griggs and Handin (1960) report a single quartz crystal broke in compression at 5000 bars confining pressure, 500°C , with explosive release of strain energy, but the fault along which the failure occurred rehealed. Griggs and Blacic (1965) observed a lowering of strength of silicate minerals in the presence of water; sudden shearing failure locally may occur in rocks under natural conditions weakened because of hydrolysis of silicon-oxygen bonds. However, it may be that such rocks would deform more readily by intra-crystalline slip or recrystallization—that is, “plastically”—than would otherwise comparable “dry” rocks. Flexural slip folding (Paterson and Weiss, 1966), movement along cleavage surfaces, intergranular slip or grain-boundary slip and other shearing movements—at least in the upper part of the crust—also involve slip along interfaces.

An important difference between calcite and/or metal crystals, and silicate crystals is that few glide systems have been recognized in silicates and some of these are “hard” (Flinn, 1965; McLean, 1965). Metals deform easily by translational gliding; silicates do not. Retention of coherence may be impossible unless there are enough glide systems to allow strain by translational gliding in polycrystalline aggregates. Consequently, some of the experimental work, for example on marble, may not be completely directly applicable to the deformation of silicate assemblages without modification.

The models for which temperature increases due to frictionally generated heat have been computed are too generalized to be related to any specific metamorphic terrane. However, examination of the curves in Figure 3a to d shows that it is possible to account for a significant local increase in temperature due to frictionally generated heat. If deformation is concentrated in space and time, the thermal gradient will be steepened significantly in the vicinity of the deformed region.

The mathematical solution of the non-steady-state temperature rise in response to a given rate of heat production may be applied to specific cases, provided the variables can be evaluated reasonably precisely. It may be possible to determine to what extent a local perturbation of the “normal” temperature gradient can be attributed to frictionally generated heat. There can, of course, be no doubt that local temperature increases are associated with deformation (Dillon, 1962; Erdmann and Jahoda, 1963; Grunfest, 1963; Grunfest and Becker, 1965). But there are also several other heat sources that contribute to the total heat budget at any point in the crust during metamorphism. The “normal” heat flow from the mantle, possible surges of heat into the crust due to transfer of hot material within the mantle, juvenile water, crustal thickening, and the heat from radioactivity all have importance, but all are large-scale phenomena which would induce large-scale variations in the thermal gradient. Local steep gradients associated with evidence of intense deformation may more properly be attributed to mechanically generated heat, particularly in the absence of evidence of the existence and distribution of igneous bodies or of sources of chemical energy (assemblages which could lead to exothermic reactions).

The widespread, broadly synchronous association of regional metamorphism and moderate to intense deformation is hardly simply fortuitous. However, it cannot be

TABLE 1. VALUES FOR TEMPERATURE INCREASES AT EACH OF 8 DEPTHS FOR EACH OF 40 MODELS

Model No.	Q in cal cm ⁻³ sec ⁻¹	Av. strain rate in deforming region in sec ⁻¹	Time in sec	Approx. average energy converted to heat in cal g ⁻¹	Upper boundary a, in km	Lower boundary b, in km	Temperature increase in deg. C over any pre-existing gradient at depth z, in km.							
							1	2	4	6	9	12.5	17.5	21
1A	3.0x10 ⁻¹²	2.2x10 ⁻¹³	3x10 ¹³	8.2	10	15	3	6	14	25	53	82	38	15
(10A)			3x10 ¹⁴	82			(21)	(43)	(86)	(129)	(196)	(253)	(211)	(170)
1B	1.5x10 ⁻¹²	1.1x10 ⁻¹³	3x10 ¹³	4.1			2	3	7	13	26	41	19	7
(10B)			3x10 ¹⁴	41			(11)	(21)	(43)	(65)	(98)	(126)	(105)	(85)
1C	1.0x10 ⁻¹²	7.2x10 ⁻¹⁴	3x10 ¹³	2.7			1	2	5	8	18	27	13	5
10C			3x10 ¹⁴	27			7	14	29	43	65	84	70	57
1D	3.0x10 ⁻¹³	2.2x10 ⁻¹⁴	3x10 ¹³	0.8			0	1	1	3	5	8	4	1
10D			3x10 ¹⁴	8.2			2	4	9	13	20	25	21	17
1E	1.5x10 ⁻¹³	1.1x10 ⁻¹⁴	3x10 ¹³	0.4			0	0	1	1	3	4	2	1
10E			3x10 ¹⁴	4.1			1	2	4	6	10	13	11	8
2A	3.0x10 ⁻¹²	2.5x10 ⁻¹³	3x10 ¹³	8.2	15	20	1	1	2	6	15	38	82	53
(20A)			3x10 ¹⁴	82			(16)	(32)	(64)	(97)	(148)	(211)	(286)	(254)
2B	1.5x10 ⁻¹²	1.2x10 ⁻¹³	3x10 ¹³	4.1			0	1	1	3	7	19	41	26
(20B)			3x10 ¹⁴	41			(8)	(16)	(32)	(49)	(74)	(105)	(143)	(127)
2C	1.0x10 ⁻¹²	8.3x10 ⁻¹⁴	3x10 ¹³	2.7			0	0	1	2	5	13	27	18
20C			3x10 ¹⁴	27			5	11	21	32	49	70	95	85
2D	3.0x10 ⁻¹³	2.5x10 ⁻¹⁴	3x10 ¹³	0.8			0	0	0	1	1	4	8	5
20D			3x10 ¹⁴	8.2			2	3	6	10	15	21	29	25
2E	1.5x10 ⁻¹³	1.2x10 ⁻¹⁴	3x10 ¹³	0.4			0	0	0	0	1	2	4	3
20E			3x10 ¹⁴	4.1			1	2	3	5	7	11	14	13
3A	3.0x10 ⁻¹²	2.4x10 ⁻¹³	3x10 ¹³	16.4	10	20	3	7	17	31	68	120	120	68
(30A)			3x10 ¹⁴	164			(37)	(75)	(150)	(226)	(343)	(463)	(498)	(424)
3B	1.5x10 ⁻¹²	1.2x10 ⁻¹³	3x10 ¹³	8.2			2	4	8	16	34	60	60	34
(30B)			3x10 ¹⁴	82			(19)	(37)	(75)	(113)	(172)	(232)	(248)	(212)
3C	1.0x10 ⁻¹²	8.0x10 ⁻¹⁴	3x10 ¹³	5.5			1	2	6	10	23	40	40	23
(30C)			3x10 ¹⁴	55			(12)	(25)	(50)	(75)	(114)	(154)	(166)	(141)
3D	3.0x10 ⁻¹³	2.4x10 ⁻¹⁴	3x10 ¹³	1.6			0	1	2	3	7	12	12	7
30D			3x10 ¹⁴	16.4			4	7	15	23	34	46	50	42
3E	1.5x10 ⁻¹³	1.2x10 ⁻¹⁴	3x10 ¹³	0.8			0	0	1	2	3	6	6	4
30E			3x10 ¹⁴	8.2			2	4	7	11	17	23	25	21
4A	3.0x10 ⁻¹²	2.6x10 ⁻¹³	3x10 ¹³	24.6	5	20	15	29	74	105	143	157	129	71
(40A)			3x10 ¹⁴	246			(65)	(130)	(260)	(388)	(533)	(626)	(619)	(523)
4B	1.5x10 ⁻¹²	1.3x10 ⁻¹³	3x10 ¹³	12.3			7	15	37	52	71	79	65	35
(40B)			3x10 ¹⁴	123			(32)	(65)	(130)	(194)	(267)	(313)	(309)	(261)
4C	1.0x10 ⁻¹²	8.5x10 ⁻¹⁴	3x10 ¹³	8.2			5	10	25	35	48	52	43	24
(40C)			3x10 ¹⁴	82			(22)	(43)	(87)	(129)	(178)	(209)	(206)	(174)
4D	3.0x10 ⁻¹³	2.6x10 ⁻¹⁴	3x10 ¹³	2.5			1	3	7	10	14	16	13	7
40D			3x10 ¹⁴	24.6			6	13	26	39	53	63	62	52
4E	1.5x10 ⁻¹³	1.3x10 ⁻¹⁴	3x10 ¹³	1.2			1	1	4	5	7	8	6	4
40E			3x10 ¹⁴	12.3			3	6	13	19	27	31	31	26

Specific values for temperature increase over any pre-existing temperature for each of eight depths for each of 40 models. All symbols are explained in the Appendix. Numbers in the body of the table are temperature increases in degrees C. When the average energy converted to heat in cal g⁻¹ exceeds 27, the temperature values and model number are in brackets.

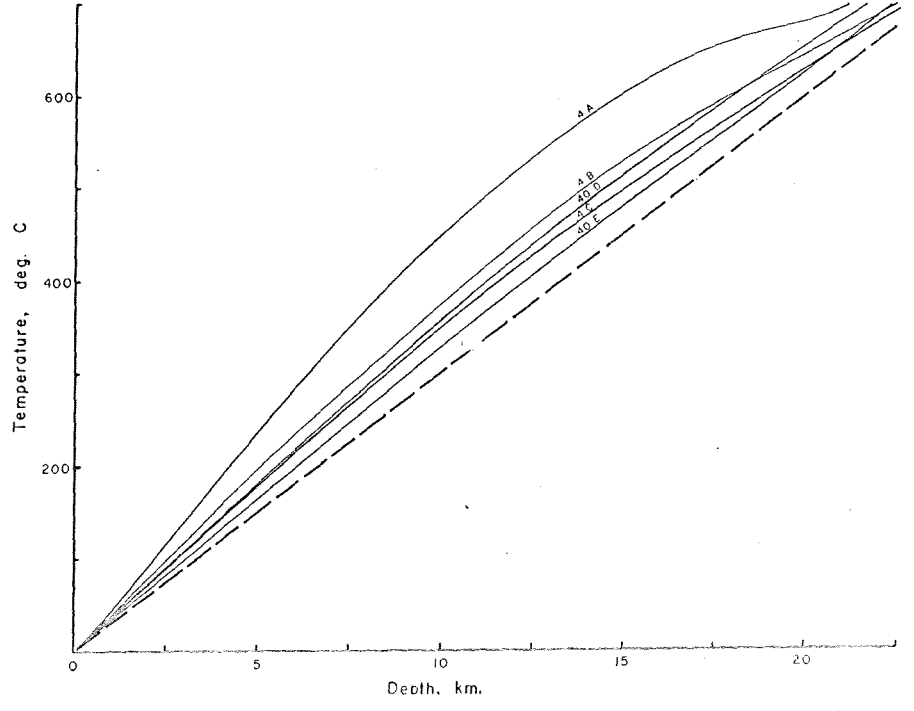
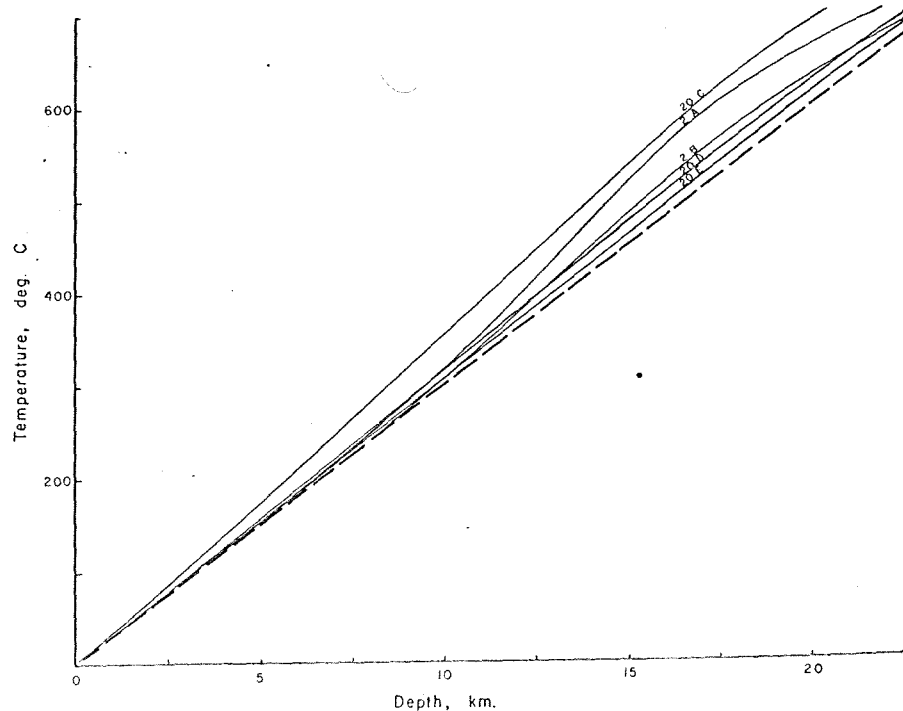
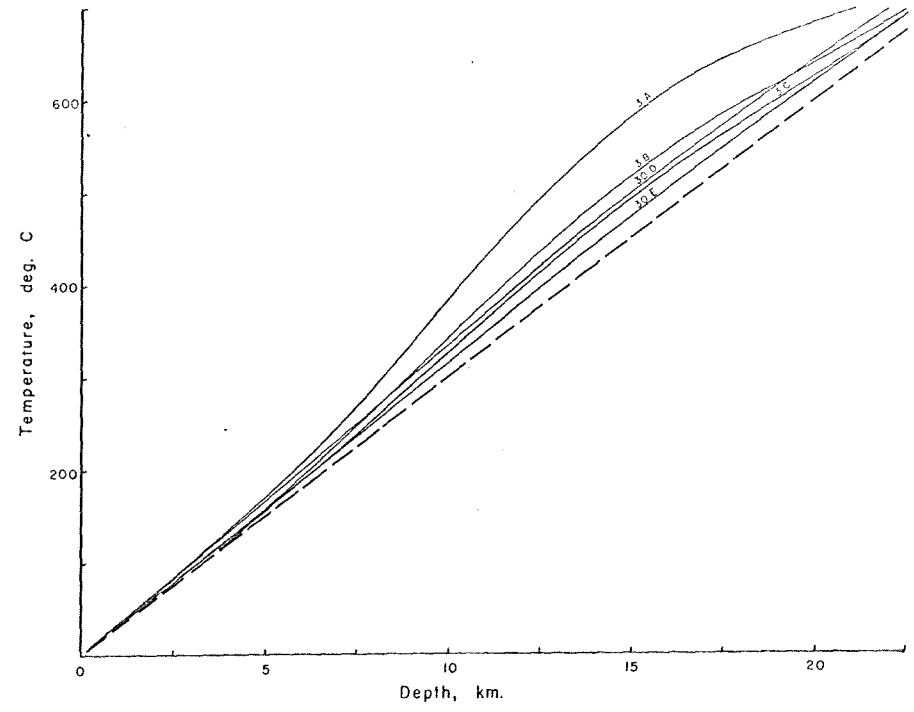
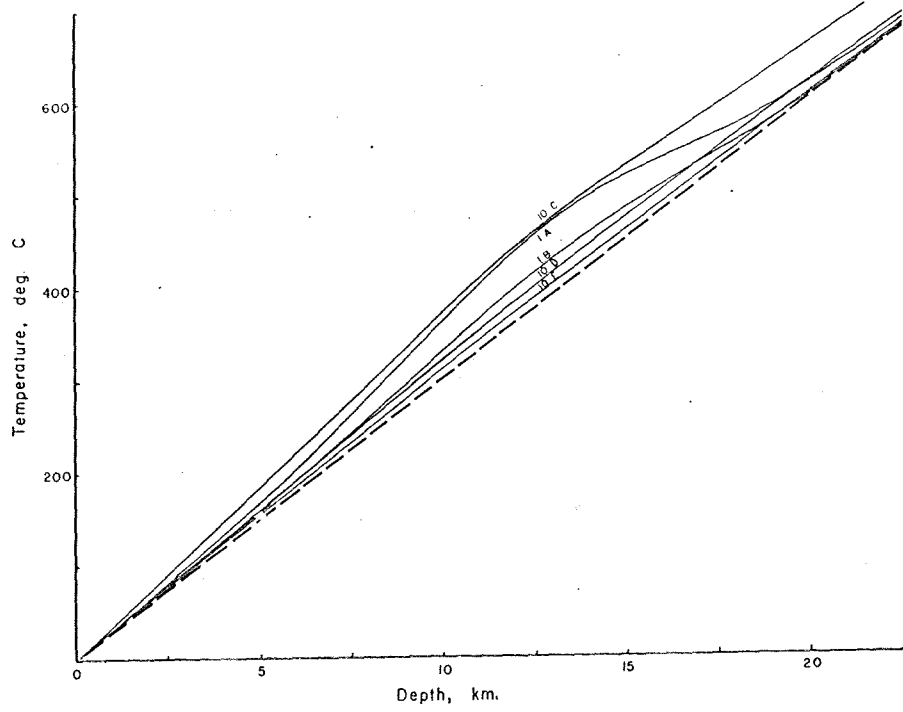


Figure 3a-b. Temperatures with depth for models: variables and numerical values of temperatures at specific depths for each model are given in Table I.

Figure 3c-d. Temperatures with depth for models: variables and numerical values of temperatures at specific depths for each model are given in Table I.

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suggested that frictionally generated heat due to deformation is solely responsible for the temperatures of metamorphism. Frictional heat may account for local steepening of the temperature gradient; the conversion of mechanical energy to heat may be responsible for a mineral assemblage of somewhat higher metamorphic grade than would otherwise have been present. The association of metamorphism with deformation is also unquestionably partly attributable to a "catalytic" affect of deformation on the processes of recrystallization and reconstitution by which metamorphism is recognized (Carter and others, 1964; DeVore, 1965; Reitan, 1960, 1965).

What may have been intuitively obvious has been demonstrated here quantitatively. If the work of deformation is uniformly distributed throughout a large volume of the crust and the rate of mechanical energy input is low, any temperature increase due to the deformation will be trivial. A significant temperature increase due to frictionally generated heat can only be expected when the strain rate is relatively high and the total work of deformation is concentrated in a portion of the whole system. More detailed evaluations of the effects of concentration of heat generation in time and in space may be found in Reitan (1968a, b).

The hypothesis that mechanical energy converted to heat may result in significant local temperature increases in the crust seems to be tenable on theoretical grounds.

Of special importance in further evaluation of the hypothesis are: (1) recognizing the critical importance of rate of heat generation, precise definition of the time duration of tectonic events (so far most dating establishes a maximum time interval during which events took place, not the actual time duration of the events); and (2) examining field evidence with the hypothesis in mind, to determine the extent to which intensive deformation and abnormally steep temperature gradients correlate, and to attempt to determine the extent to which essentially brittle behavior of rocks corresponds with the initiation of significant elevation of temperature in the deforming region.

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APPENDIX

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The heat conduction problem of this paper has been solved using a Laplace transform method (Carslaw and Jaeger, 1959). The transform of a function is obtained by multiplying the function by e^{-st} and integrating with respect to t from zero to infinity, that is:

$$L \{ T_{(z,t)} \} = \int_0^{\infty} e^{-st} T_{(z,t)} dt.$$

The transformed differential equations become:

$$\frac{d^2 \bar{T}_{(z,s)}}{dz^2} - \frac{s}{\kappa} \bar{T}_{(z,s)} = 0 \quad \text{for } z < a, z > b$$

$$\frac{d^2 \bar{T}_{(z,s)}}{dz^2} - \frac{s}{\kappa} \bar{T}_{(z,s)} = -\frac{Q}{ks} \quad \text{for } a < z < b.$$

The Laplace transform solutions are then:

$$\bar{T}_{(z,s)} = Be^{\sqrt{\frac{s}{\kappa}}z} - Be^{-\sqrt{\frac{s}{\kappa}}z} \quad \text{for } 0 < z < a$$

$$\bar{T}_{(z,s)} = Ce^{\sqrt{\frac{s}{\kappa}}z} + De^{-\sqrt{\frac{s}{\kappa}}z} + \frac{\kappa Q}{ks^2} \quad \text{for } a < z < b$$

$$\bar{T}_{(z,s)} = Ee^{-\sqrt{\frac{s}{\kappa}}z} \quad \text{for } b < z < \infty.$$

The constants are determined by the boundary conditions and the Laplace transforms solutions become:

$$\bar{T}_{(z,s)} = \frac{Q\kappa}{2ks^2} \left(e^{-(a+z)\sqrt{\frac{s}{\kappa}}} - e^{-(a+z)\sqrt{\frac{s}{\kappa}}} - e^{-(b+z)\sqrt{\frac{s}{\kappa}}} + e^{-(b+z)\sqrt{\frac{s}{\kappa}}} \right) \quad \text{for } 0 < z < a$$

$$\bar{T}_{(z,s)} = -\frac{Q\kappa}{2ks^2} \left(e^{-(b+z)\sqrt{\frac{s}{\kappa}}} - e^{-(b+z)\sqrt{\frac{s}{\kappa}}} + e^{(a-z)\sqrt{\frac{s}{\kappa}}} + e^{-(a+z)\sqrt{\frac{s}{\kappa}}} - 2 \right) \quad \text{for } a < z < b$$

$$\bar{T}_{(z,s)} = -\frac{Q\kappa}{2ks^2} \left(-e^{-(b+z)\sqrt{\frac{s}{\kappa}}} - e^{-(b+z)\sqrt{\frac{s}{\kappa}}} + e^{-(a+z)\sqrt{\frac{s}{\kappa}}} + e^{-(a+z)\sqrt{\frac{s}{\kappa}}} \right). \quad \text{for } b < z < \infty$$

The final solutions, obtained from a table of Laplace transforms (Roberts and Kaufman, 1966), are:

for $0 < z < a$

$$T_{(z,t)} = \frac{Q\kappa}{2k} \left\{ \left[\left(t + \frac{(a-z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(a-z)}{2\sqrt{\kappa t}} \right) - (a-z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(a-z)^2/4\kappa t} \right] \right. \\ \left. - \left[\left(t + \frac{(a+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(a+z)}{2\sqrt{\kappa t}} \right) - (a+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(a+z)^2/4\kappa t} \right] \right. \\ \left. - \left[\left(t + \frac{(b-z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(b-z)}{2\sqrt{\kappa t}} \right) - (b-z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(b-z)^2/4\kappa t} \right] \right. \\ \left. + \left[\left(t + \frac{(b+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(b+z)}{2\sqrt{\kappa t}} \right) - (b+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(b+z)^2/4\kappa t} \right] \right\}$$

for $a < z < b$

$$T_{(z,t)} = -\frac{Q\kappa}{2k} \left\{ \left[\left(t + \frac{(b-z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(b-z)}{2\sqrt{\kappa t}} \right) - (b-z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(b-z)^2/4\kappa t} \right] \right. \\ \left. - \left[\left(t + \frac{(b+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(b+z)}{2\sqrt{\kappa t}} \right) - (b+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(b+z)^2/4\kappa t} \right] \right. \\ \left. + \left[\left(t + \frac{(a+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(a+z)}{2\sqrt{\kappa t}} \right) - (a+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(a+z)^2/4\kappa t} \right] \right. \\ \left. + \left[\left(t + \frac{(z-a)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(z-a)}{2\sqrt{\kappa t}} \right) - (z-a) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(z-a)^2/4\kappa t} \right] - 2t \right\}$$

for $b < z < \infty$

$$T_{(z,t)} = -\frac{Q\kappa}{2k} \left\{ \left[\left(t + \frac{(z-a)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(z-a)}{2\sqrt{\kappa t}} \right) - (z-a) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(z-a)^2/4\kappa t} \right] \right. \\ \left. + \left[\left(t + \frac{(a+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(a+z)}{2\sqrt{\kappa t}} \right) - (a+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(a+z)^2/4\kappa t} \right] \right. \\ \left. - \left[\left(t + \frac{(z-b)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(z-b)}{2\sqrt{\kappa t}} \right) - (z-b) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(z-b)^2/4\kappa t} \right] \right. \\ \left. - \left[\left(t + \frac{(b+z)^2}{2\kappa} \right) \operatorname{erfc} \left(\frac{(b+z)}{2\sqrt{\kappa t}} \right) - (b+z) \left(\frac{t}{\pi\kappa} \right)^{1/2} e^{-(b+z)^2/4\kappa t} \right] \right\}$$

Notation and symbols:

 T is temperature \bar{T} is a transformed solution t is time z is depth (positive downwards) a is the depth of the upper boundary of the region within which heat is produced b is the depth of the lower boundary of the region within which heat is produced Q is the rate of heat production k is thermal conductivity κ is thermal diffusivity $\text{erfc}(x)$ is a tabulated function (*see*, for example, National Bureau of Standards, 1954)

The Hydrous Vapor Phase in Equilibrium with Granite and Granite Magmas

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ABSTRACT

The composition of the hydrous vapor in equilibrium with synthetic granite and water-saturated synthetic granite magmas has been determined at pressures up to 10,000 bars at temperatures above and below the solidus. At temperatures above the solidus, the silicate constituents of the vapor phase are depleted in potassium feldspar relative to "normal" granites. The anhydrous composition of the vapor phase in equilibrium with alkali feldspar and quartz at 2 kb is, in contrast, very rich in silica; at 10 kb the anhydrous composition of the vapor phase is approximately the same as that of the liquid phase at the beginning of melting.

The addition of sodium disilicate to a synthetic granite composition at 5 kb, 650°C was found to greatly enhance the solubility of water in the granitic liquid. The solubility of silicates in the aqueous vapor phase was also increased considerably. This relationship suggests that peralkaline granitic magmas may show continuous solubility with respect to water. That is, a continuous transition from magma to a hydrothermal solution.

These experiments bear on the possible metasomatic changes to be expected in the immediate vicinity of a granite pluton, as well as on the process of forming "granite looking" rocks. The continuous transition from quartz veins to pegmatite veins, as the granite contact is approached, is to be expected on the basis of the experimental study. Certain features of the concentrically zoned pegmatites, notably the quartz cores, may be explained in terms of the data presented here.

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