Geoexploration, 17 (1979) 33—42

EFFECT OF RELIEF IN EM METHODS WITH VERY DISTANT SOURCE

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(Received November 30, 1977; accepted May 25, 1978)

ABSTRACT

Karous, M.R., 1979. Effect of relief in EM methods with very distant source. Geoexploration, 17: 33—42.

An approximate model for the analytical calculation of the terrain relief effect in EM methods with very distant sources (VLF method, afmag method, shallow magnetotelluric sounding) is proposed in this article. The case of two-dimensional forms of relief for E-polarization of a horizontal plane electromagnetic wave is treated. An approximate method of solution is also developed so that the determination of the relief effect can be made even by means of desk calculators. The results of the method introduced when compared with model and practically measured curves show that the proposed model is sufficiently exact for determining the relief effect in EM profiling methods.

INTRODUCTION

With increasing accuracy of measuring equipment the so-called geological noise, i.e. influences of those geological bodies which are not of immediate interest, e.g. influences of inhomogeneities of overburden, effects of topographic relief etc., are a serious source of uncertainty in the interpretation of geophysical data today. Increasing the accuracy of interpreted results needs corrections to the measured data with regard to these influences and therefore appropriate attention must be given to these problems. Modern computing methods in the processing of geophysical data allow us to separate disturbing influences from useful signals and in this way to increase the accuracy of data entering the interpretation process.

One of the disturbing influences which distorts measured data in almost all electromagnetic methods is the effect of topographic relief. The emphasis of this article is laid on the solution of a model, which allows us to determine the influence of relief in the EM methods with primary source fields approximated by a plane EM, vertically incoming wave. The methods of very low frequency (VLF method), afmag and shallow (audio) magnetotelluric sounding belong to this class.

All the methods mentioned utilize artificial or natural very distant sources.

The EM field of these sources can be approximated sufficiently exactly by an EM plane wave above a conductive half-space. The determination of the effect of a known shape of relief is a direct EM problem. Today this problem cannot be solved generally and therefore it has been solved only for certain approximate conditions. Some simple calculations for the EM field in the range of very low frequencies were carried out by Tarkhov (1962) and Gordeyev (1970) made an attempt to solve this problem through simulation. Karous (1978) solved the undamped approximation analytically.

This article deals with a relief effect in the EM profiling methods, where this effect is most remarkable. In addition we shall consider only two-dimensional forms of relief perpendicular to the x-axis of the profile. The general task of three-dimensional forms is very complicated so that it can be solved only by means of computers. Two-dimensional forms parallel with the profile give constant effect along the whole profile and it is not difficult to separate them. The application of relief corrections is suitable particularly in ore prospecting with EM methods in vertically dissected terrains.

APPROXIMATE TWO-DIMENSIONAL MODEL OF RELIEF

Let the coordinate system have its horizontal x-axis in the vertical plane of the profile, the horizontal y-axis perpendicularly to the x-axis and the z-axis vertically. The origin of the coordinate system lies at the point where the effect of the relief is sought. We shall calculate the effect of a plane-polarized EM wave with primary electrical component E_0 in the y-direction. H_0 is also assumed to be horizontal.

The relief is described by a function $f(x,y) = f(x) = f$ as this function is not dependent upon y. For reasons of generality we shall consider the relief upto a distance a on both sides of the profile (Fig.1). This parameter a can be determined from a map. Further we shall suppose that the secondary EM field determining the relief effect is given by extra electrical fields at the crests and

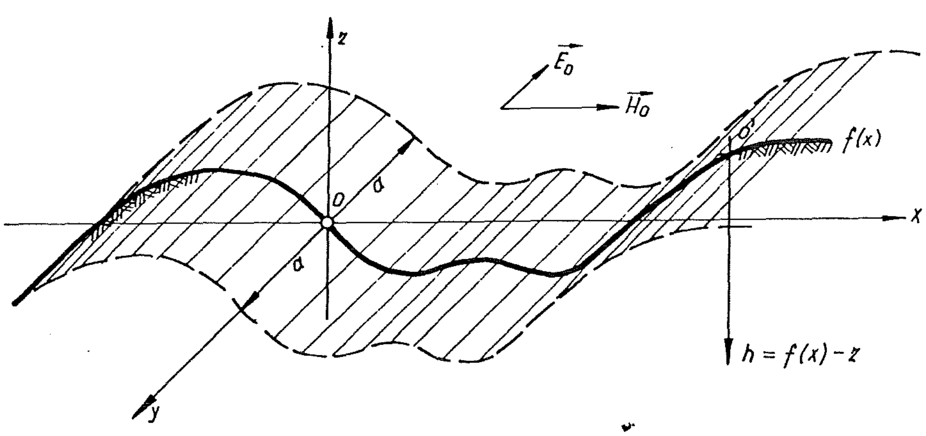

Fig. 1. Two-dimensional model of relief for calculation of its topoeffect in EM methods.

troughs of the relief. The current lines due to this electrical field have a finite length $2a$ in the y-direction. In a non-magnetic ($\mu = \mu_0 = 4\pi 10^{-7}\ H/\mathrm{m}$) and conductive medium the surface of the earth has no influence on distorting the primary magnetic field. The secondary magnetic field is given by a magnetic field of the extra-electrical field according to the Biot-Savart law.

Between primary electrical and magnetic components of an EM wave on the surface of a conductive half-space we have the relation (Stratton, 1941; Grant and West, 1965):

$$\hat{E}_0 = \left(\frac{\omega\mu}{\sigma}\right)^{1/2} H_0 e^{i\pi/4} \tag{1}$$

where $\hat{E}_0$ and H_0 are the components of the plane EM wave, ω is angular frequency ($\omega = 2\pi f$; f is frequency of the EM field), $\mu = \mu_0$ is the magnetic permeability, and σ is the conductivity of the medium.

The electrical field decreases exponentially with increasing depth:

$$\hat{E}(h) = \hat{E}_0\, e^{-k'h} = \left(\frac{\omega\mu}{\sigma}\right)^{1/2} H_0 e^{(i\pi/4 - k'h)} \tag{2}$$

where h is the depth ($h = f(x) - z$) and k' is the damping coefficient of the EM wave:

$$k' = (\omega\mu\sigma/2)^{1/2} \tag{3}$$

If we introduce into the Biot-Savart law (neglecting the damping of the secondary field):

$$\overrightarrow{\mathrm{d}H} = \frac{\hat{J}\cdot \mathrm{d}x \cdot \mathrm{d}z(\overrightarrow{\mathrm{d}y}\ x\ \vec{r}^{\,0})}{4\pi r^2} \tag{4}$$

the current density $J = \sigma E$ from Ohm's law, we get the effect of the current element $\mathrm{d}I = J\,\mathrm{d}x\,\mathrm{d}y$. In accordance with Fig. 2 the necessary transformation can be found as:

$$\left|(\overrightarrow{\mathrm{d}y}\ x\ \vec{r}^{\,0})\right| = \mathrm{d}y \sin\phi \qquad \sin\phi = \left(\frac{x^2 + z^2}{x^2 + y^2 + z^2}\right)^{1/2} = \frac{r'}{r}$$

$$r = (x^2 + y^2 + z^2)^{1/2} \qquad r' = (x^2 + z^2)^{1/2} \tag{5}$$

$$\mathrm{d}\hat{H}_z = \mathrm{d}\,\hat{H}\cos\psi \qquad \cos\psi = \frac{x}{(x^2 + z^2)^{1/2}} = \frac{x}{r}$$

By introducing eq. 2 into eq. 4 and using the relations of eq. 5 the vertical component of the magnetic primary field will be:

$$\hat{H}_z = \frac{k'\sqrt{2}\,H_0 \exp(i\pi/4)}{4\pi} \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{f(x)} \left[\int_{-a}^{a} \frac{x \exp[-k(f-z)]}{(x^2 + y^2 + z^2)^{3/2}}\, \mathrm{d}y \right] \mathrm{d}z \right\} \mathrm{d}x$$

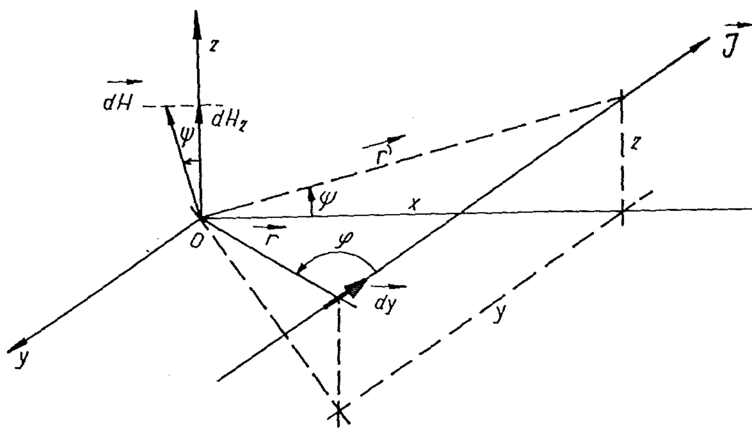

Fig. 2. Geometry for solving the terrain relief effect.

This expression can be integrated with respect to y and gives

$$\hat{H}_z = \frac{a\,k'\sqrt{2}\,H_0 \exp(i\pi/4)}{2\pi} \int_{-\infty}^{\infty} \left\{ \frac{x}{\exp(k'f)} \int_{-\infty}^{f(x)} \frac{\exp(k'z)}{(x^2+z^2)(x^2+z^2+a^2)^{1/2}} \right. \mathrm{d}x \quad (6)$$

For the normalized value H_z/H_0, real $Re\,\{\hat{H}_z/H_0\}$ and imaginary part $Im\,\{\hat{H}_z/H_0\}$ are as follows:

$$H_z/H_0 = |\hat{H}_z|/H_0 = \frac{ak'\sqrt{2}}{2\pi}\,x \int_{-\infty}^{\infty} \frac{x}{\exp(k'f)} \int_{-\infty}^{f(x)} \frac{\exp(k'z)\mathrm{d}z}{(x^2+z^2)(x^2+z^2+a^2)^{1/2}}\,\mathrm{d}x$$

$$Re\{\hat{H}_z/H_0\} = Im\{\hat{H}_z/H_0\} = \frac{ak'}{2\pi}\,x \int_{-\infty}^{\infty} \frac{x}{\exp(k'f)} \int_{-\infty}^{f(x)} \frac{\exp(k'z)\mathrm{d}z}{(x^2+z^2)(x^2+z^2+a^2)^{1/2}}\,\mathrm{d}x$$

The last expression can be written as:

$$Re\,\{\hat{H}_z/H_0\} = \frac{k'}{2\pi} \int_{-\infty}^{\infty} R(x,f)\,\mathrm{d}x$$

where:

$$R(x,f) = \frac{ax}{\exp(k'f)} \int_{-\infty}^{f(x)} \frac{\exp(k'z)\,\mathrm{d}z}{(x^2+z^2)(x^2+z^2+a^2)^{1/2}} \quad (8)$$

is the so-called relief element effect. In the case of an undamped EM wave (i.e., $\exp(k'z) = 1$) the integral of eq. 8 can be evaluated analytically.

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$$R^0(x,f) = ax \int_{-\infty}^{f(x)} \frac{\mathrm{d}x}{(x^2+z^2)\,(x^2+z^2+a^2)^{1/2}} = \arctan \frac{a \cdot f(x)}{x(f^2+x^2+a^2)^{1/2}} + \frac{\pi}{2}\,\mathrm{sign}(x) \quad (9)$$

The additive member $\pm\,\pi/2$ determines the plane relief element effect $R^0(x,0)$ and it has the same sign as the variable x. In the case of integration in the symmetrical interval $x(-b,b)$ these members cancel out and therefore an undamped normalized relief element effect $R_n^0(x,f)$ is introduced:

$$R_n^0(x,f) = R^0(x,f) - R^0(x,0) = \arctan \frac{a \cdot f(x)}{x\,(f^2+x^2+a^2)^{1/2}} \quad (10)$$

Similarly the damped effect $R_n(x,f)$ is introduced:

$$R_n(x,f) = \frac{a \cdot x}{\exp(k'f)} \int_{-\infty}^{f(x)} \frac{\exp(k'z)\cdot \mathrm{d}z}{(x^2+z^2)\,(x^2+z^2+a^2)^{1/2}} - ax \int_{-\infty}^{0} \frac{\exp(k'z)\,\mathrm{d}z}{(x^2+z^2)\,(x^2+z^2+a^2)^{1/2}} \quad (11)$$

LIMITATIONS ON THE APPLICABILITY OF THE UNDAMPED MODEL

For comparing normalized damped and undamped effects the ratio

$$\kappa = \frac{R_n(x,f)}{R_n^0(x,f)} \quad (12)$$

was calculated by computer for all combinations of the following parameters: $k' = 0, 10^{-3}, 3.10^{-3}, 10^{-2}, 3.10^{-2}\,\mathrm{m}^{-1}$; $a = 200, 500, 1000$ m; $f(x) = -800$ (100) 800 m; and $x = -2000$ (100) 2000 m, i.e., for almost 14,000 points.

The ratio κ was plotted on logarithmic coordinate paper for values of $f(x) = -800, -400, 400$ and 800 m (Fig. 3). The function $(k'x)$ seems not to be dependent on the parameter a, but only on the parameter $k'x$, which verifies the principle of similarity of quasi-static electromagnetic models.

For small values of $k'x \leqq 0.1$ the ratio $\kappa = 1$, i.e., the fields of the damped and undamped models do not differ. The zone:

$$k'x = (\omega\mu\sigma/2)^{1/2} x \leqq 0.1 \quad (13)$$

is thus the zone of validity of the undamped model. Therefore the relief effect can be found from the relation

$$Re\{\hat{H}_z/H_0\} = \frac{k'}{2\pi} \int_{-\infty}^{\infty} R_n^0(x,f)\mathrm{d}x = \frac{k'}{2\pi} \int_{-\infty}^{\infty} \arctan\left[\frac{a \cdot f(x)}{x(f^2+x^2+a^2)^{1/2}}\right] \mathrm{d}x \quad (14)$$

in this zone.

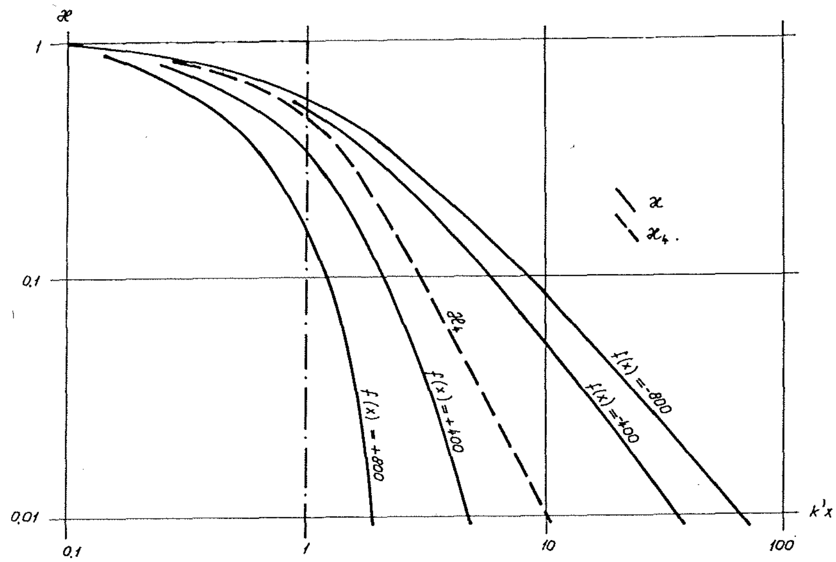

Fig. 3. Dependence of function $\kappa(k'x)$ upon parameter $k'x$ for different values of $f(x)$.

CALCULATION OF RELIEF EFFECT IN UNDAMPED MODEL

The integral of eq. 14 is calculated in practical cases numerically:

$$Re\{\hat{H}_z/H_0\} \doteq \frac{k'}{2\pi} \sum_{i=-n}^{n} \text{arc tan}\left[\frac{a\cdot f(x_i)}{x_i(f^2(x_i)+x_i^2+a^2)^{1/2}}\right]\Delta x \qquad (15)$$

where Δx is the chosen step of summation (e.g., 50 m). The relief effect can be calculated either by means of a computer or by means of nomograms (Karous, 1978). The length of the calculation interval $I < -n.x, n.x >$ is selected with respect to the field frequency f, the size of cross-section S of a distant relief shape, which has to be taken into account, conductivity of rocks σ and so on. The relation for estimation of interval I in which the relief forms giving effects less than 2% are neglected is given by Karous (1978) as:

$$I = 0.004\, S\, (f\sigma)^{1/2} \qquad (16)$$

So, for example, for a hill with a triangular cross-section, with a height of 100 m, a base of 400 m ($S = 2\cdot 10^4$ m^2) and with rock conductivity $\sigma = 10^{-3}\, S\, \mathrm{m}^{-1}$ ($\Omega^{-1}\, \mathrm{m}^{-1}$) at the frequency $f = 20$ kHz (VLF method), the length of interval is $I = 360$ m. It means that the topoeffect of this hill can be neglected, if it lies outside of the interval I. But distant reliefs cause effects changing very slowly and monotonically, so that the distortion of measured data has more or less a stationary character and therefore it is not difficult to separate it. Therefore smaller intervals I than those given by eq. 16 can be used in practical cases.

The measured VLF-curve $Re\{H_z/H_0\}$ and the computed effect of relief on a profile across the High Jeseník Mts. ridge (Czechoslovakia) for two different intervals I = 500 and 1000 m are given in Fig. 4. It is evident that theoretical curves for the two values of I do not differ. The differences between measured and computed curves are caused by local conductive bodies and they are the object of geophysical investigation.

The condition of validity (eq. 13) of the undamped model is very rigorous and it is very difficult to satisfy it in practice. For example, the undamped model for resistivities $\rho = 10^3\ \Omega$m and for frequencies f = 10 kHz (VLF method) is satisfied only to distances of about 20 m; for frequencies f = 100 Hz (afmag method, shallow magnetotelluric sounding) only to distances of about 200 m. Therefore it is necessary to make a simple determination of the topoeffect in the damped model.

CALCULATION OF RELIEF EFFECT IN THE DAMPED MODEL

The influence of relief in the case of a damped EM wave can be found from the formula

$$Re\{\hat{H}_z/H_0\} = \frac{k'}{2\pi} \int_{-\infty}^{\infty} R_n(x,f)\mathrm{d}x \qquad (17)$$

where $R_n(x,f)$ is given by eq. 11. But this solution of topoeffect according to eq. 17 is unnecessarily difficult and requires a long computer time. An approximate, but in practice sufficiently exact calculation can start from the undamped model and introduce a correction by means of the function $\kappa(k'x)$. This function does not depend on the parameter a but depends on the value $f(x)$. The comparison of the function $\kappa(k'x)$ with simple mathematical func-

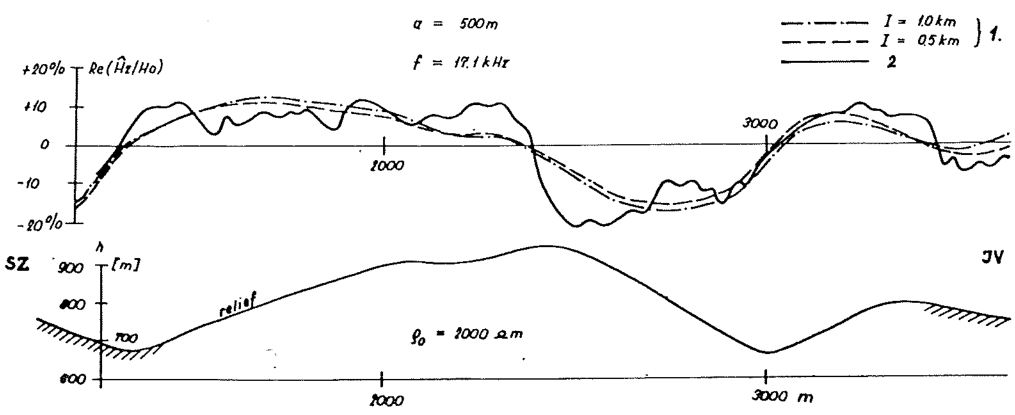

Fig. 4. The curves $Re\ \{H_z/H_0\}$ along a regional profile in the High Jeseníky Mts. (Czechoslovakia). *1* = curve computed according to eq. 15; *2* = measured curve (UMS — 17.1 kHz).

tions shows that the function κ can be approximated sufficiently exactly by the function

$$\kappa_n = \left[\sum_{m=0}^{n} (k'x)^m \right]^{-1/2} \tag{18}$$

For a mean value of $f(x) = 0$ the function $(k'x)$ is nearly identical with the function

$$\kappa_4 = \left[\sum_{m=0}^{4} (k'x)^m \right]^{-1/2} \tag{19}$$

which can be used as a correction function of the damped model. Therefore for relief effect in the damped model the following formula is valid:

$$Re\{\hat{H}_z/H_0\} = \frac{k'}{2\pi} \sum_{i=-n}^{n} \kappa_4 (k'x_i) \arctan \left\{ \frac{a \cdot f(x_i)}{x_i[f^2(x_i) + x_i^2 + a^2]^{1/2}} \right\} \Delta x \tag{20}$$

The curves of topoeffect $H_z/H_0 = \sqrt{2}\, Re\{H_z/H_0\}$ computed according to eq. 15 for the undamped model and according to eq. 20 for the damped model across symmetrical and asymmetrical models of a ridge that is infinitely long ($a = \infty$) are given in Fig. 5. Gordeyev carried out his modelling measurements above the same forms and with the same parameters (Gordeyev, 1970). A good agreement between the measured and computed curves shows the ability of the proposed methods of calculating the topoeffect. This ability and sufficient accuracy were proved by computing of topoeffects of different relief forms for the undamped model (Karous, 1978).

The analytical formulas for the calculation of the topoeffect allow us also to find a dependence upon different parameters. There is a significant dependence on the conductivity of the medium. At the beginning, for small parameters $k'x < 0.1$, the relief effect increases with increasing conductivity and increasing frequency. In the zone of large parameters $k'x > 1$ saturation occurs and with increasing conductivity or frequency the effect does not change any more, because of the limiting function $k'\,\kappa_4$ for $\sigma f \to \infty$.

The value of conductivity or resistivity appearing in the formulas can be determined from resistivity profile measurements and it is assumed to be either constant or variable for the computed profile.

CONCLUSION

It is possible to utilize eq. 15 if the relation of eq. 13 is valid, or eq. 20 in more general cases for computing the relief effect in the EM profile methods, which measure EM fields of very distant sources, especially in the VLF method. The zone of validity of eq. 13 is very narrow and implies very high resistivities

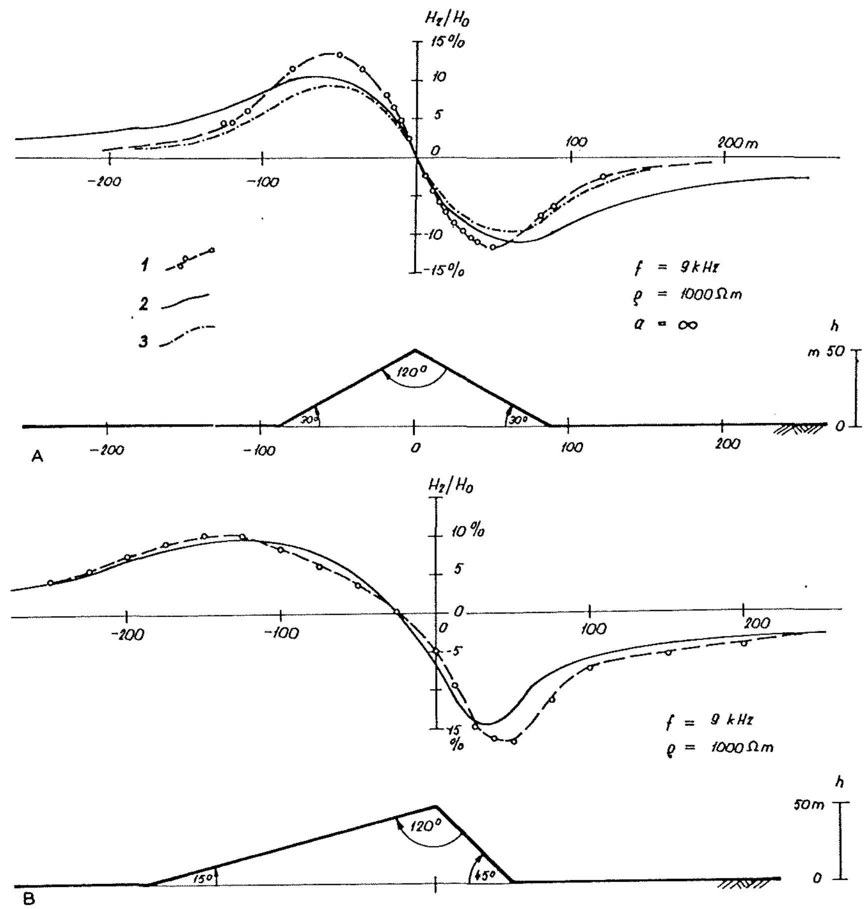

Fig. 5. Model and computed curves of relief effect H_z/H_0. A. Symmetrical model of hill; B. asymmetrical model. *1* = model curves (adapted from Gordeyev, 1970); *2* = computed curves for undamped model according to eq. 15; *3* = computed curves for damped model according to eq. 20.

of rocks or very low frequencies and possibly neglecting the influence of distant reliefs. The calculation according to the simple formula of eq. 20 is more precise. It is appropriate to compute the relief effect at least on one profile in very dissected terrains. Then it is possible to separate the influences of relief from those of conductive local bodies also on other profiles, at least qualitatively. But the effect of relief has always smaller horizontal gradients than an anomaly above local conductivity inhomogeneities.

ACKNOWLEDGEMENTS

The author would like to express his acknowledgements to Prof. S.E. Hjelt for very helpful discussions on the manuscript of this work at the time of the author's study stay at the University of Oulu.

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Geoexploration, 17 (1979)

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ABSTRACT

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INTRODUCTION

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