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Stan Ward (& MT people)

MAGNETOTELLURIC INTERPRETATIONS IN A CRUSTAL ENVIRONMENT

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NOTICE

INTRODUCTION

This study is concerned with the effects of variable crustal electrical properties on magnetotelluric fields and the development of practical methods for modeling these effects. Our first efforts are devoted to low frequency studies where the crustal region is thin relative to the skin depth of the crust. In such situations one can model the crust as a thin layer and compute the crustal effects with two-dimensional calculations. This approach was first introduced by A.T. Price (1949), but this original analysis needs modifications to realistically treat crustal problems. Price assumed a perfect insulator underlay the surface conducting zone, which assumption restricts the solutions to the horizontal E field mode which is only inductively coupled to the mantle below. The lower crust is often a poor conductor, but not poor enough to prevent resistive coupling between the surface and the mantle and this considerably changes the resulting fields. In our earlier studies we had generalized the boundary conditions under the thin layer in order to allow a general layered media to replace the insulating region. The top layer was representative of the resistive lower crust, and the solutions showed its properties had a pronounced effect on the electric fields perpendicular to strike. It therefore is important to be able to model variable properties for this layer since its properties are probably as variable as the surface conductivity properties. This layer is also thin relative to the skin depth and relative to the dimensions of important surface features and can therefore be incorporated into a thin layer analysis. The inclusion of a conductive layer and resistive layer into a single layer makes the combined layer anisotropic and the analysis of such a layer we call a <u>generalized thin layer</u> analysis. Actually, since it is the usual situation, we maintain the construction of two thin layers, with the uppermost conductive and the lower one resistive. In the first section, we review the equations describing the thin layer effects and show examples of comparisons between thin layer and generalized thin layer calculations. These calculations were done in the k space domain and because convolution operators are involved generally require full matrices. We experimented with limiting the number of terms in the operators using σ smoothing but these results were unsatisfactory. For the one-dimensional models shown, using a full matrix is not a problem, but for realistic modeling one must go to two-dimensional solutions and then the computations would become too large to be very practical. We have, therefore, started to investigate implementing a multiple scale analysis to allow us to handle large models in a reasonable fashion. In the second section we describe some tests of these ideas on one-dimensional models.

GENERALIZED THIN LAYER ANALYSIS

From Maxwell's equations at low frequencies

$$\nabla x E = i\mu\omega H \tag{1.1}$$

$$\nabla x H = \sigma E \tag{1.2}$$

We have for a thin layer and predominately horizontal fields

$$\Delta E \cong -i\mu\omega\Delta z \hat{i}_z x H_s \tag{1.3}$$

$$\Delta H = -\sigma\Delta z \hat{i}_z x E_s \tag{1.4}$$

since $E/H \equiv \sqrt{\frac{i\mu\omega}{\sigma_{ap}}}$, ($\sigma_{ap}$ is apparent conductivity) (1.5)

the relative change of E and H across the layer are given as

$$\frac{\Delta E}{E} = \sqrt{i\mu\omega\sigma_{ap}}\ \Delta z \tag{1.6}$$

$$\frac{\Delta H}{H} = \frac{\sigma}{\sigma_{ap}} \sqrt{i\mu\omega\sigma_{ap}}\ \Delta z$$

For layers thin compared to the skin depth of the field in the mantle $\Delta E/E$ is small, but when the surface layer is quite conductive $\Delta H/H$ can still be appreciable. Thus Price set up his analysis assuming ΔE was zero. He also assumed the region below the conducting sheet was a perfect insulator allowing him to set H as the gradient of a scalar potential. This simplification leads to a scalar equation which reduces the size of the system of equations, but eliminates one electromagnetic mode. This assumption is not a necessary part of

Price's analysis and one can treat the case of a general horizontally layered media lying under the conducting sheet.

From 1.1 we have

$$\frac{\partial E_s}{\partial z} = -i\mu\omega\hat{i}_z x \vec{H}_s + \nabla_s(E_z) \tag{1.8}$$

where $\vec{E}_s = \hat{i}_x E_x + \hat{i}_y E_y$

$\vec{H}_s = \hat{i}_x H_x + \hat{i}_y H_y$

and $\nabla_s = \hat{i}_x \frac{\partial}{\partial x} + \hat{i}_y \frac{\partial}{\partial y}$

also $H_z = (\nabla_s x E_s) \cdot \hat{i}_z / i\mu\omega$ (1.9)

where $(\nabla_s x) = \hat{i}_z (\frac{\partial}{\partial x} \hat{i}_y \cdot - \frac{\partial}{\partial y} \hat{i}_x \cdot)$

From 1.2 we have

$$\frac{\partial H_s}{\partial z} = -\sigma\hat{i}_z x \vec{E}_s + \nabla_s(H_z) \tag{1.10}$$

and $E_z = (\nabla_s x H_s) \cdot \hat{i}_z / \sigma$ (1.11)

The magnetic field below the conducting sheet can be expressed in terms of the electric field knowing the H:E relationship of the underlying layered media.

$$H_s{}^L = Y^L E_s{}^L \tag{1.12}$$

At the surface all wavelengths other than the source wavelength must be outgoing and therefore again E_s and H_s have a known relationship.

$$H_s{}^u = Y^{iu} E_s{}^u + H_s{}^o \qquad (1.13)$$

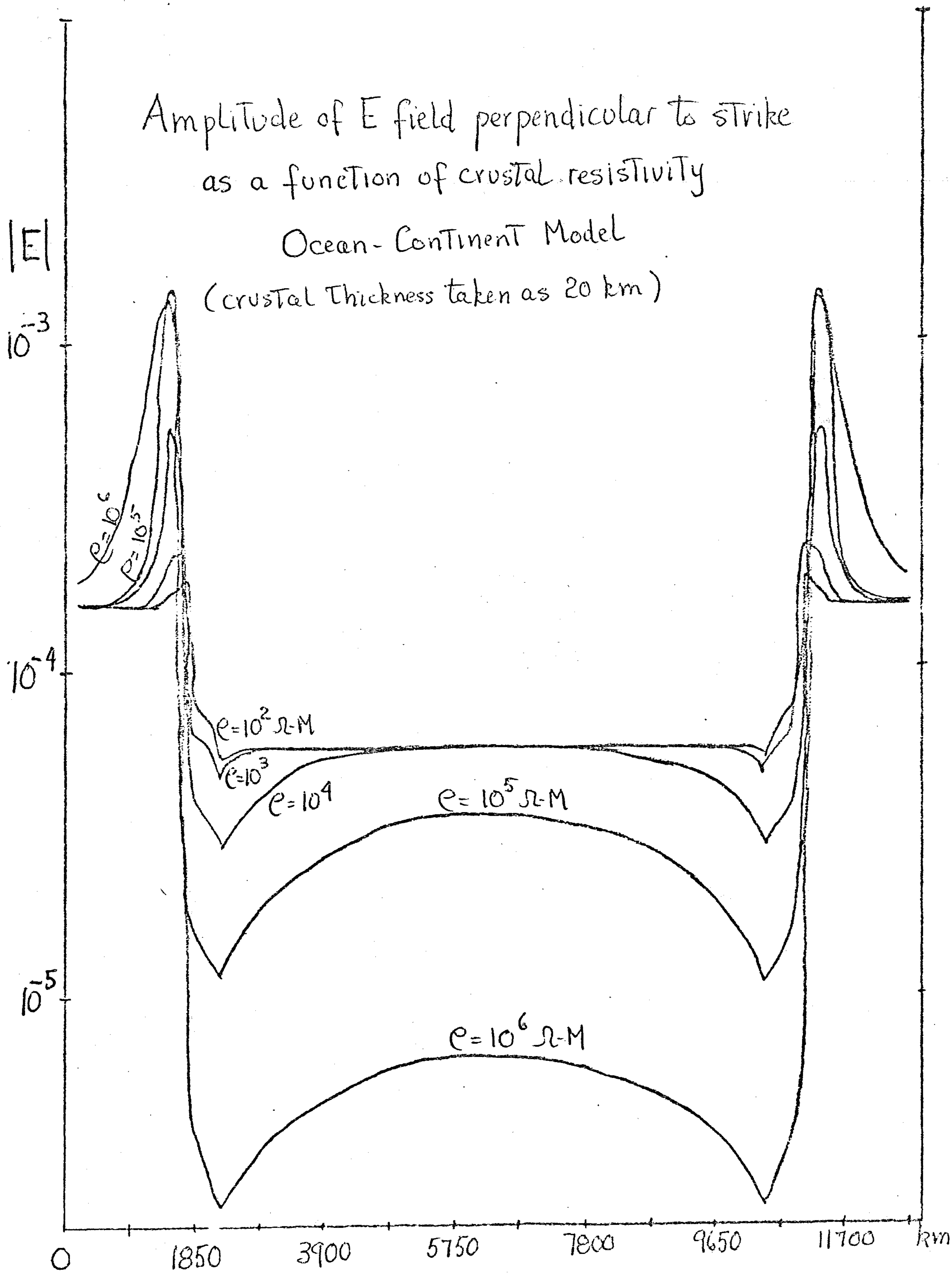
Y^{iu} is the H:E relationship for upgoing waves in the air above the conducting sheet, but exclusive of the source wavelength, and $H_s{}^o$ is the field at the source wavelength. For all practical purposes, $H_s{}^o$ is twice the incident field.

If we make the usual thin layer assumption that $E_s{}^u - E_s{}^L = 0$ we have from 1.10 using 1.9, 1.12, and 1.13.

$$(Y^L - Y^{iu})\vec{E}_s + \Delta z \sigma \hat{i}_z x \vec{E}_s - \Delta z \nabla_s((\nabla_s x \vec{E}_s) \cdot \hat{i}_z)/i\mu\omega = \vec{H}_s{}^o \qquad (1.14)$$

This system of equations involves a full matrix because in the space domain the Y operator is a convolution operator, while in the wavelength domain σ is a convolution operator. The $\nabla_s \nabla_s x$ operator, which is an induction term, is diagonal in the wavelength domain, but also relatively sparce when approximated by difference equations in the space domain.

The effect of a resistive lower crust can be studied with this model by including a resistive layer in the layered half space below the conducting sheet. Figure 1 shows such solutions for an ocean-continent boundary with the E field perpendicular to strike. Varying the resistivity of the lower crustal layer has a profound effect on the solutions and it must be recognized


Amplitude of E field perpendicular to strike
as a function of crustal resistivity
Ocean-Continent Model
(crustal thickness taken as 20 km)
|E|
10^{-3}
10^{-4}
10^{-5}
$\rho = 10^6$
$\rho = 10^5$
$\rho = 10^2$ Ω-M
$\rho = 10^3$
$\rho = 10^4$
$\rho = 10^5$ Ω-M
$\rho = 10^6$ Ω-M
0
1850
3900
5750
7800
9650
11700
km

as an important parameter in determining the telluric field. It is important therefore to be able to model it as variable in the same way that the surface conductor was modeled. Since this layer is also relatively thin, one should be able to include it in a thin layer analysis. The combination of a conductive layer on top of a resistive layer makes the combined layer anisotropic with a horizontal conductivity σ and a vertical resistivity ρ which are not reciprocals, so that

$$\rho\sigma > 1 \tag{1.15}$$

If ρ and σ are constants, we can obtain from Maxwell's equation 1.1 and 1.2 in the k domain

$$\frac{\partial^2}{\partial z^2}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} (\rho\sigma k_x^{\,2} + k_y^{\,2} - i\mu\omega\sigma) & (\rho\sigma-1)k_x k_y \\ (\rho\sigma-1)k_x k_y & (k_x^{\,2}+\rho\sigma k_y^{\,2}-i\mu\omega\sigma) \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix} \tag{1.16}$$

When $\rho\sigma \gg 1$, E_s must vary vertically much more rapidly than it does in an isotropic region as long as k_x or k_y are not identically zero. This arises because the terms involving $(\frac{\partial}{\partial x}, \frac{\partial}{\partial y})E_z$ which were dropped in 1.3 can no longer be ignored in the zones where ρ is large.

We prefer to keep our model of an anisotropic layer as two separate layers, with the conductive layer on top, as this is the usual situation in the earth. In this case, most of the change in H_s still takes place across the conductive layer while the change in E_s takes place across the resistive layer. Thus in the upper layer E_s is $E_s{}^u$ while in the lower layer H_s is $H_s{}^L$. From 1.8 and 1.11 we have

$$E^L - E^u = \Delta\vec{E}_s = i\mu\omega\Delta z_2 \hat{i}_z x H_s{}^L + \nabla_s(\rho\Delta z_2 H_s{}^L)\cdot\hat{i}_z) \qquad (1.17)$$

and from 1.10 and 1.9 we have

$$H^L - H^u = \Delta\vec{H}_s = -\sigma\Delta z_1 \hat{i}_z x\vec{E}_s{}^u + \frac{\Delta z_1}{i\mu\omega} \nabla_s((\nabla_s x E_s{}^u)\cdot\hat{i}_z) \qquad (1.18)$$

Using the identities

$$(\nabla_s x(\hat{i}_z x\vec{A}_s))\cdot\hat{i}_z = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y = \nabla_s\cdot\vec{A}_s \qquad (1.19)$$

$$(\nabla_s x(\nabla_s a))\cdot\hat{i}_z = 0 \qquad (1.20)$$

$$\hat{i}_z x(\hat{i}_z x\vec{A}_s) = -\vec{A}_s \qquad (1.21)$$

and the surface boundary conditions (1.13) and

$$(\nabla x H_s{}^u)\cdot\hat{i}_z = 0 \qquad (1.22)$$

which arises because $\sigma_{air} \simeq 0$ and the half space boundary condition

$$E_s{}^L = Z^L H_s{}^L \qquad (1.23)$$

and the definitions

$$\Delta z_1 \sigma = \sigma_s \tag{1.24}$$

$$\Delta z_2 \rho = \rho_s \tag{1.25}$$

one obtains from 1.17 and 1.18 by eliminating H_s^L, H_s^u, E_s^L

$$\vec{E}^u - Z^* Y^* \vec{E}^u - \nabla_s(\rho_s \nabla_s \cdot \sigma \vec{E}^u) - Z^* \frac{\Delta z_1}{i\mu\omega} \nabla_s((\nabla_s x \vec{E}^u) \cdot \hat{i}_z) = Z^* \vec{H}^o \tag{1.26}$$

where

$$Z^* \equiv (Z^L + i\mu\omega \Delta z_2 \hat{i}_z x) \tag{1.27}$$

$$Y^* \equiv (Y'^u - \sigma_s \hat{i}_z x) \tag{1.28}$$

Not all the terms are of equal importance but since a full matrix is always involved in solving 1.26, no simplification results from sorting out the smaller terms.

The effect of the resistivity thickness product is given by the third term of 1.26. For a one-dimensional model with E polarized perpendicular to strike, taken as the Y direction

$$Y' E^u = 0, \quad \nabla_s x E^u = 0 \tag{1.29}$$

Thus

$$E_x^u + Z^* i_z x \sigma_s E_x^u - \frac{\partial}{\partial x} \rho_s \frac{\partial}{\partial x} \sigma_s E_x^u = Z^* H_y^o \tag{1.30}$$

When $\sqrt{\rho_s \sigma_s}$ is much greater than the skin depth in the mantle, and ρ_s or σ_s are constant 1.30 simplifies to

$$E_x^{\,u} - \rho_s \sigma_s \frac{\partial^2}{\partial x^2} E_x^{\,u} = Z^* H_y^{\,o} \tag{1.31}$$

with solutions

$$E_x^{\,u} = E_x^{\,o} + A_x \rho^{\pm \frac{x}{\sqrt{\rho_s \sigma_s}}} \tag{1.32}$$

for a homogeneous source field.

$\sqrt{\rho_s \sigma_s}$ represents an adjustment distance for excess currents to leak out into the mantle. This behavior is clearly seen in the electric field on the ocean side in Figure 1. If $\sqrt{\rho_s \sigma_s}$ is very short, the electromagnetic adjustment distance takes over.

Figures 2 and 3 illustrate the importance of the resistivity thickness product for magnetotelluric fields when we do not have a simple layered media. In layered media, we say only the thickness of a resistive zone is important, not its resistivity. The only difference in the models shown in Figures 2 and 3 is the resistivity thickness product of the middle section, however, and yet quite large differences in the model results are clearly seen.

When the resistivity product remains constant one can model the situation with either a thin layer or a generalized thin layer and thus compare the computations. Such a comparison is shown in Figure 4. When the structure is two dimensional and the source field is uniform, one can also model the magnetotelluric response with a network which is the difference equation analogue. Figure 5 shows comparisons between such network solutions and the thin layer analysis. The network solutions are two dimensional calculations, while the thin layer analysis was one dimensional, but is limited to low frequencies when the layer is thin relative to its skin depth.

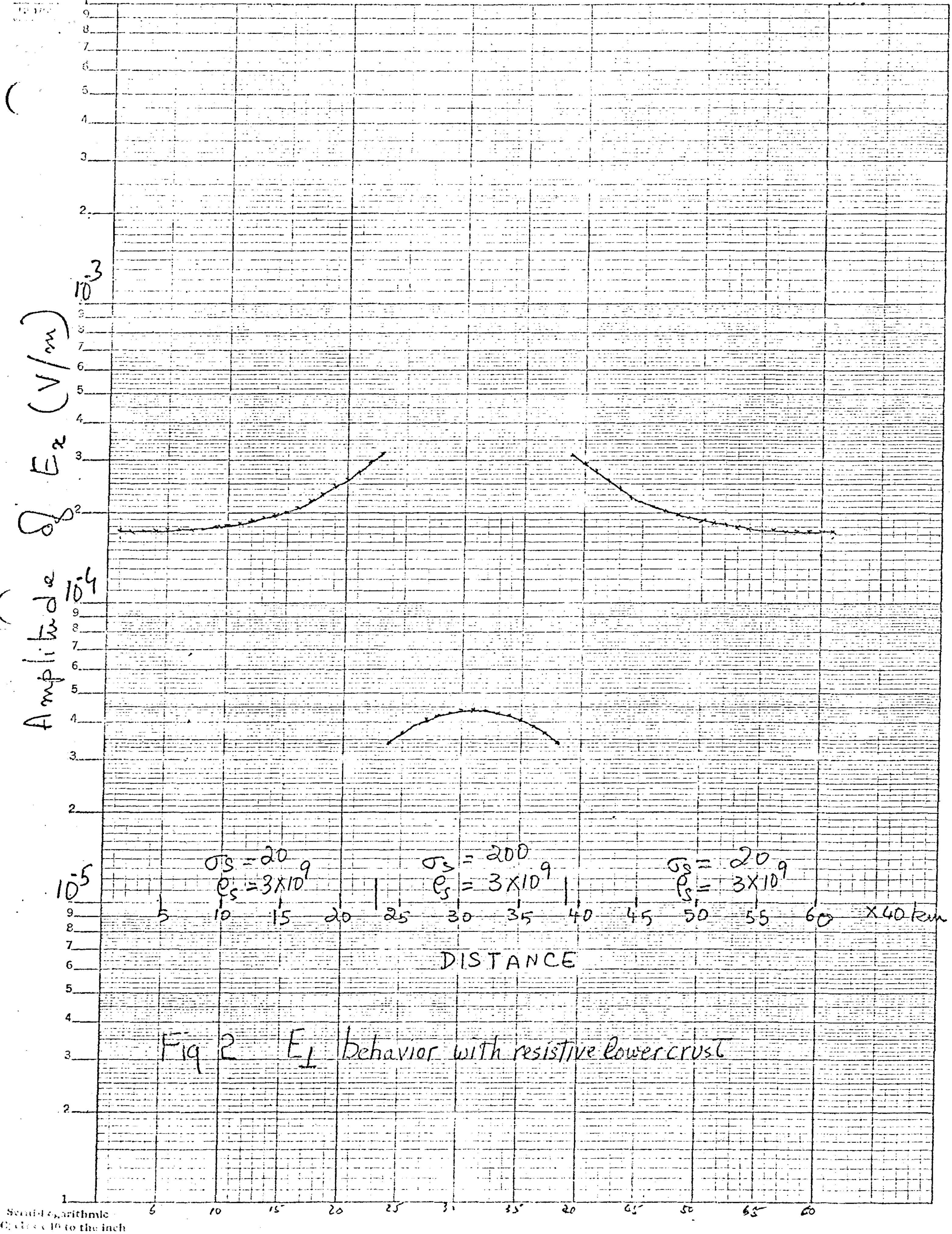


Fig 2 $E_\perp$ behavior with resistive lower crust

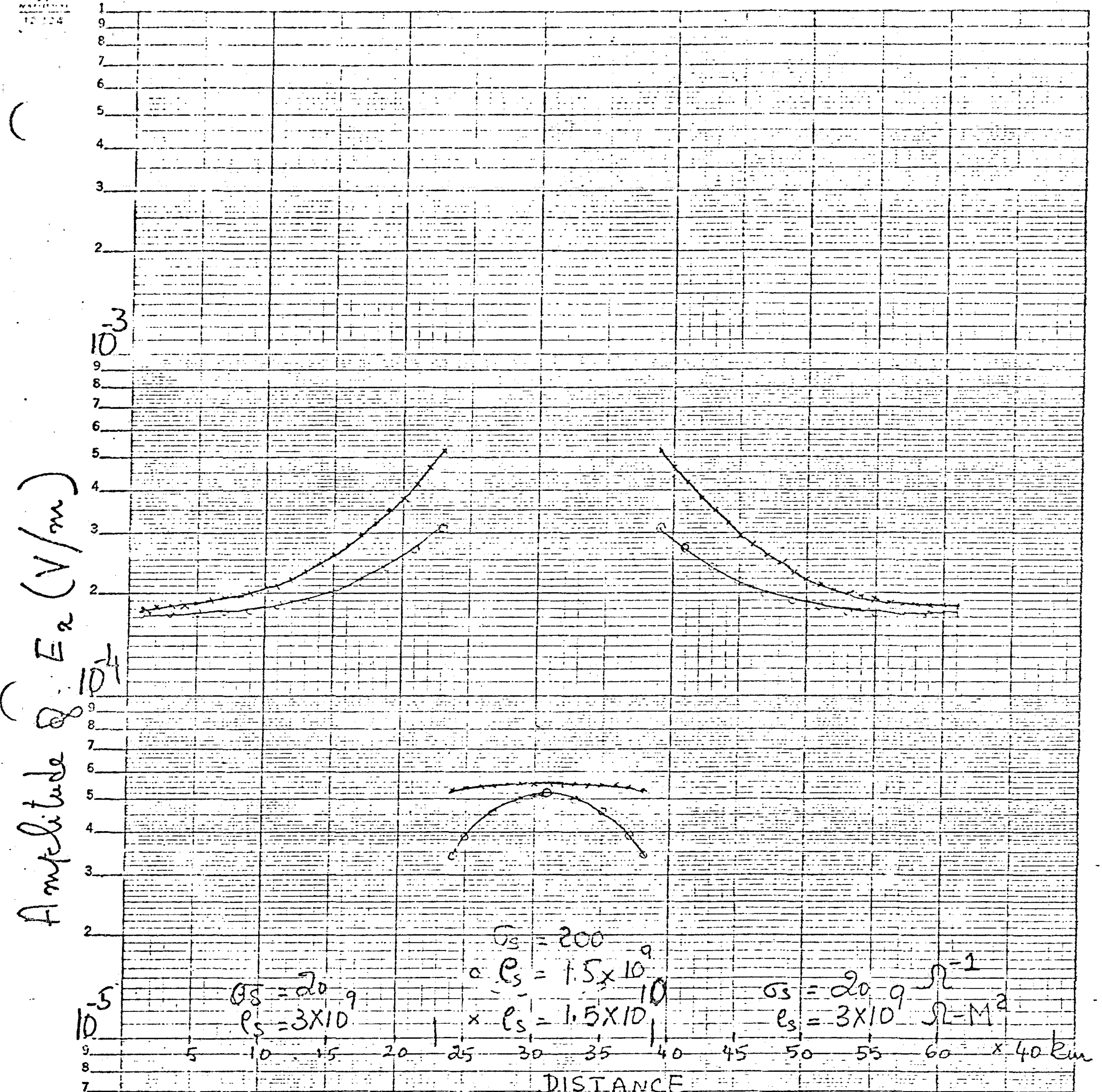


Fig 3 Effect of resistivity-thickness product on Telluric Fields for $E_\perp$

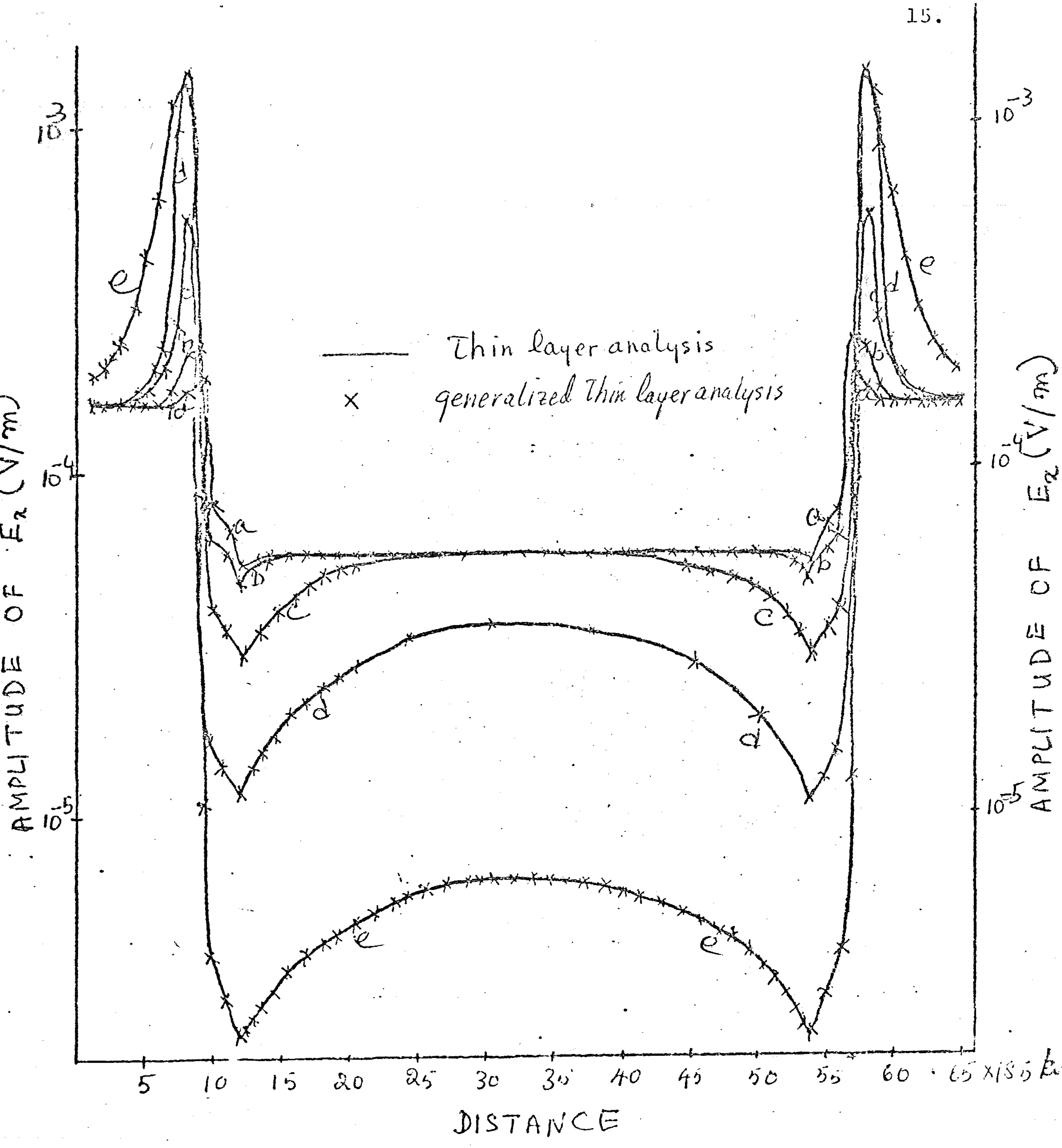


Fig 4 Comparison of results obtained from thin layer analysis with those of generalized thin layer analysis.

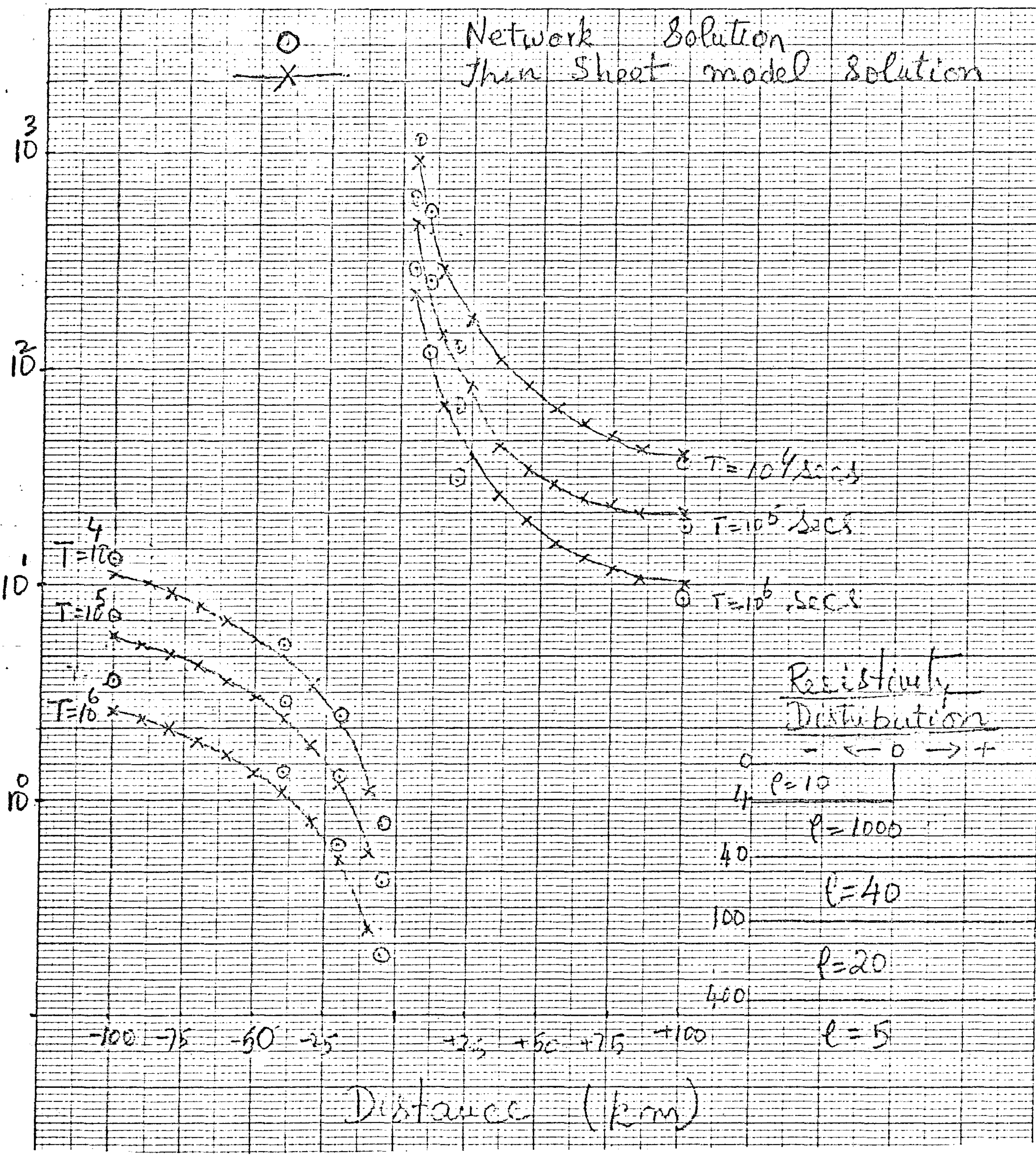


Fig 5 Comparison of one dimensional Thin layer analysis with Two dimensional network solution (Kasameyer, 1974)

MULTIPLE SCALE APPROXIMATIONS

With typical crustal resistivities of 10^5 Ω-M, one has adjustment distances of hundreds of kilometers. This means that distant regions can influence the local telluric fields and to model the situation correctly one must include these regions. For one-dimensional models this is not a severe limitation, but two-dimensional models will be impractical unless reasonable approximate methods can be developed. We at first experimented with reducing the number of wavelength terms in the solution, but this always produced Gibbs phenomena around boundaries. Some improvement was made with sigma smoothing, but good results were only obtained when the full set of wavelengths was used.

One needs to include distant regions in the models because these regions help determine the local current levels, but it is not necessary to know the solution in these regions in great detail as long as the correct average fields are known. This seems then like an ideal situation for developing a multiple scale analysis. The approach we are experimenting with is to solve equation 1.26 on a large scale, having determined the appropriate average properties by local small scale calculations, and then to use the outer region solutions as knowns in a new calculation of 1.26 at a smaller scale. This process can be cascaded through several scale changes.

It is important to recognize that the average σ_s property of a composite region will, in general, be anisotropic, so that local small scale solutions must be made at two different polarizations to assess the tensor nature of $<\sigma_s>$.

Errors always occur at boundaries where scale changes take place so that a buffer region is needed between these boundaries and the local region of interest. Figure 6 shows the scaling of a one-dimensional model. The results of a multiple scale calculation are shown in Figures 7, 8, 9 and 10 in comparison to a full calculation using small spacings across the entire model. Note in these results the distinct variations in E_y which is the polarization parallel to strike. This results from the inductive coupling term which is dropped from the usual thin layer analysis, and is emphasized by the non-uniform nature of the source field.

These results are encouraging, but much more experience is needed to develop optimum strategies for such calculations. The great saving in computational time will arise when two-dimensional models are attacked, but new complications are also bound to appear.

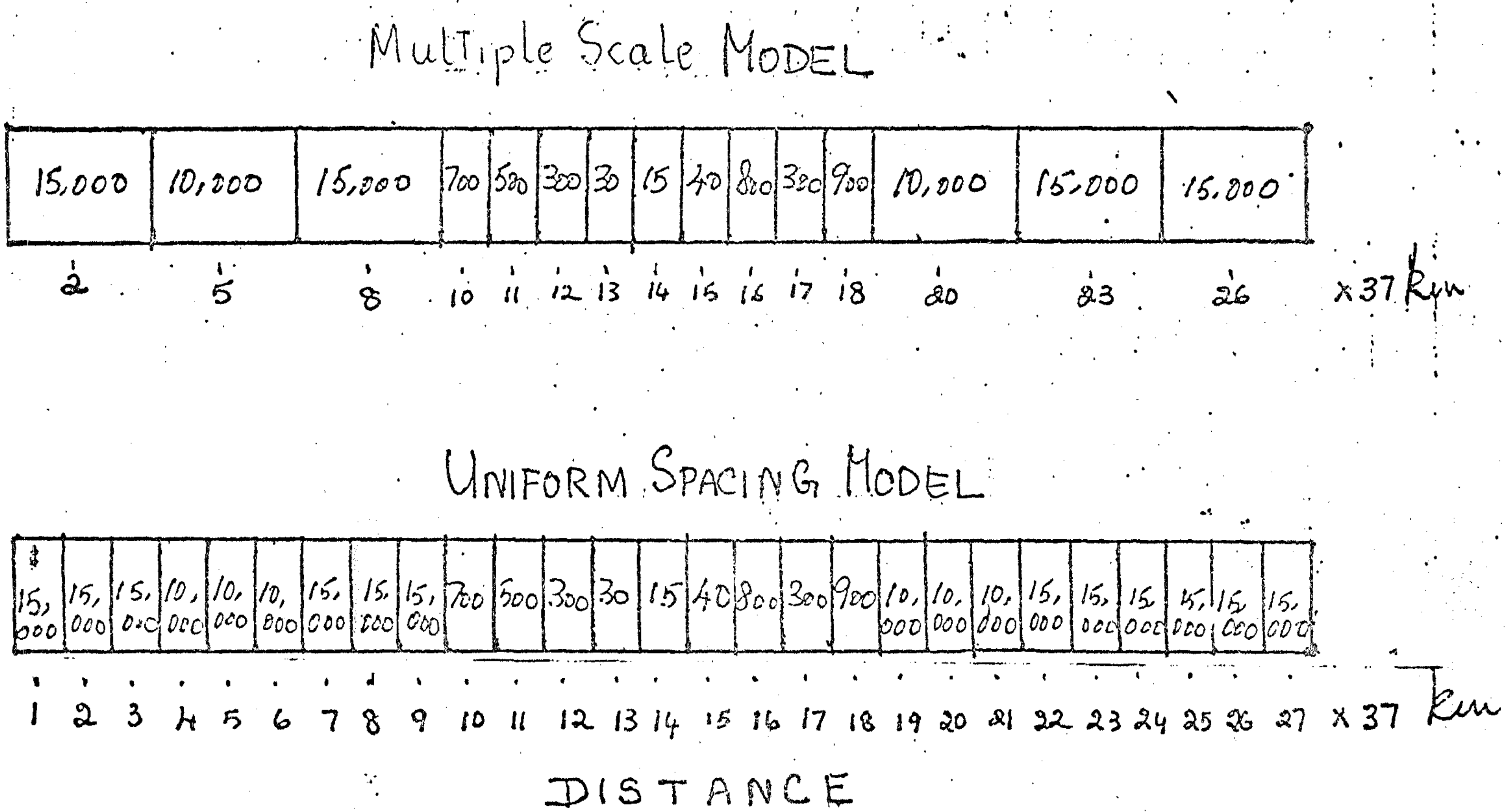


Fig 6 CONDUCTANCE OF THE THIN SHEET

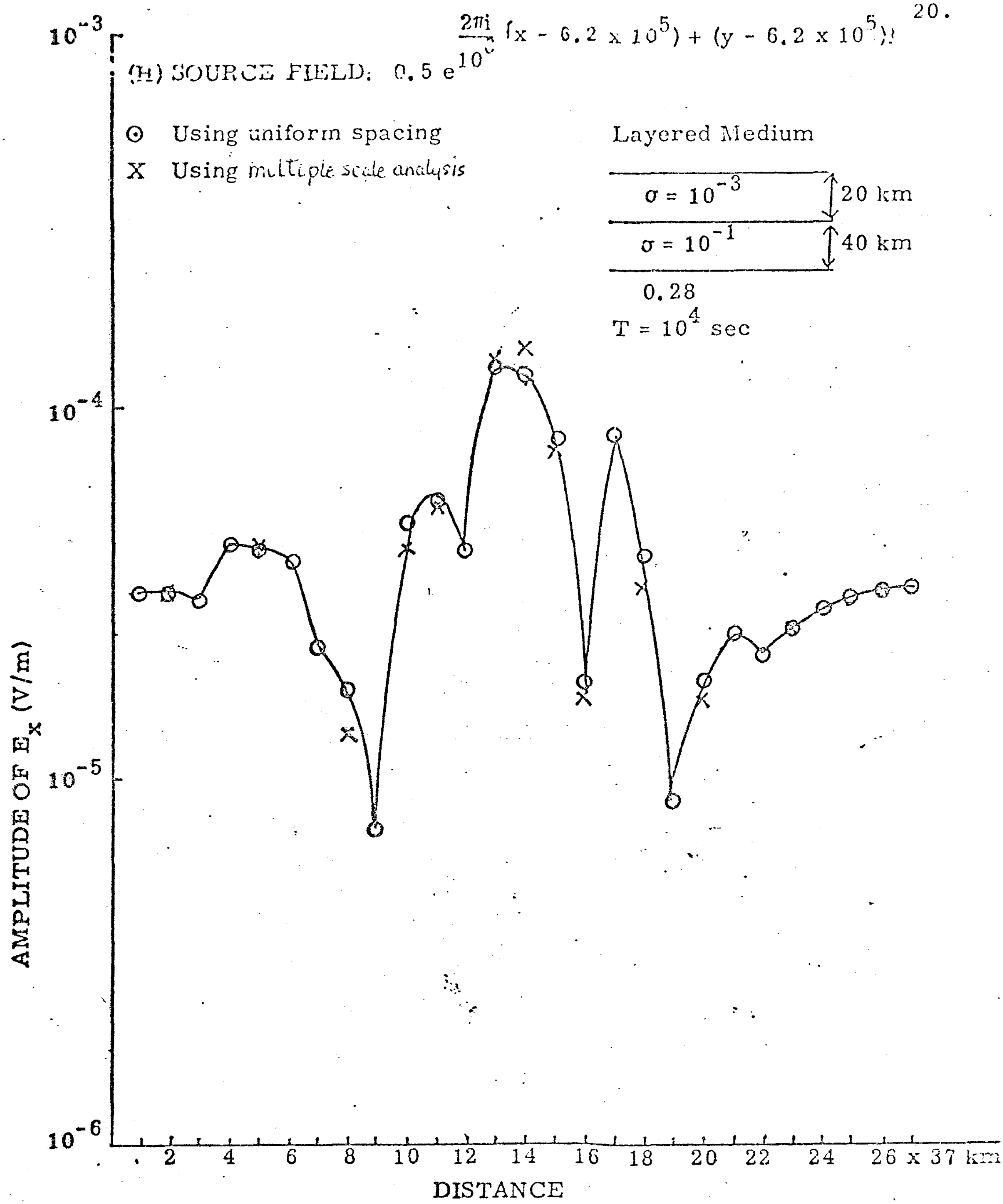


Fig 7 Comparison of fine scale computations with those using multiple scale analysis for $E_\perp$

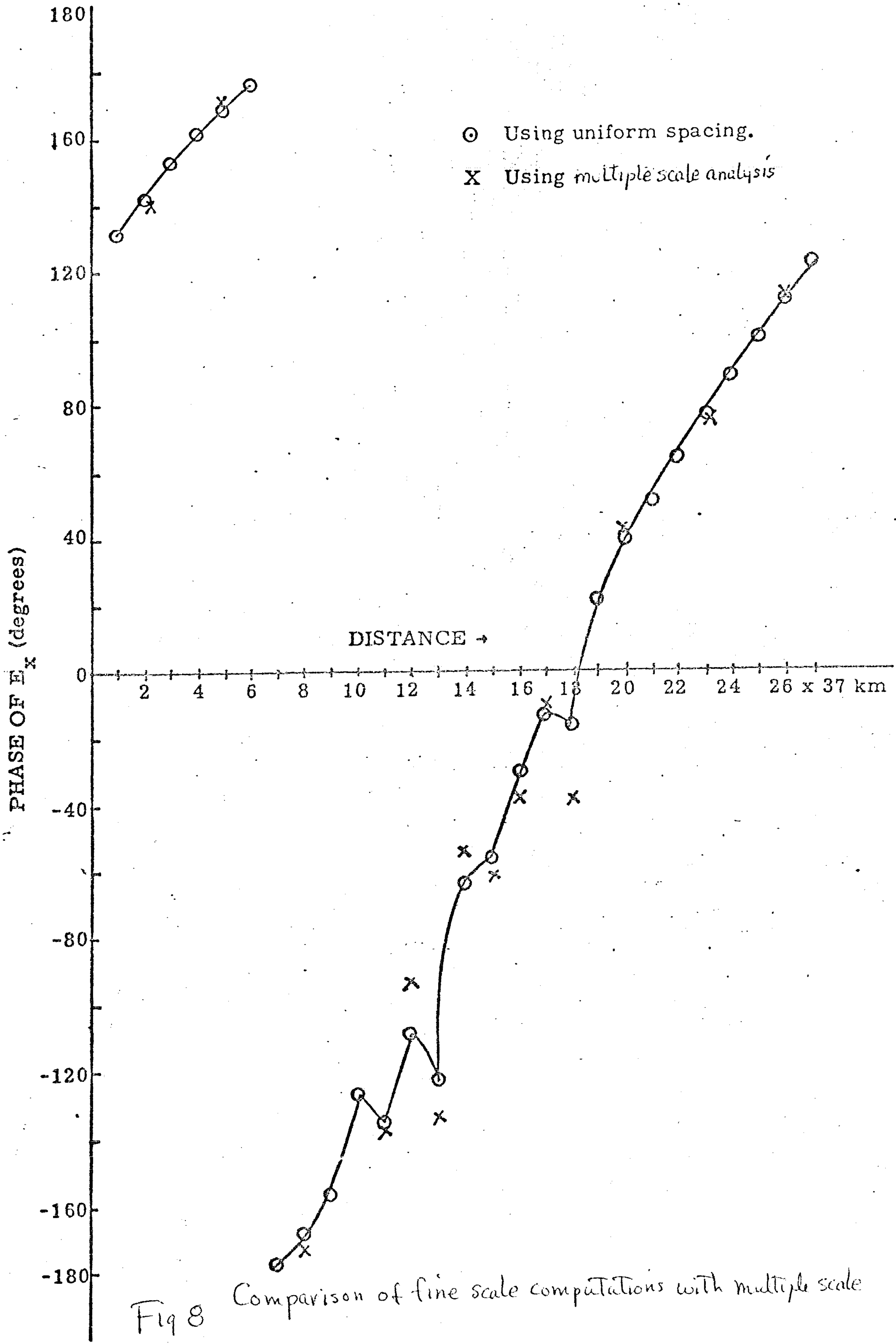


Fig 8 Comparison of fine scale computations with multiple scale

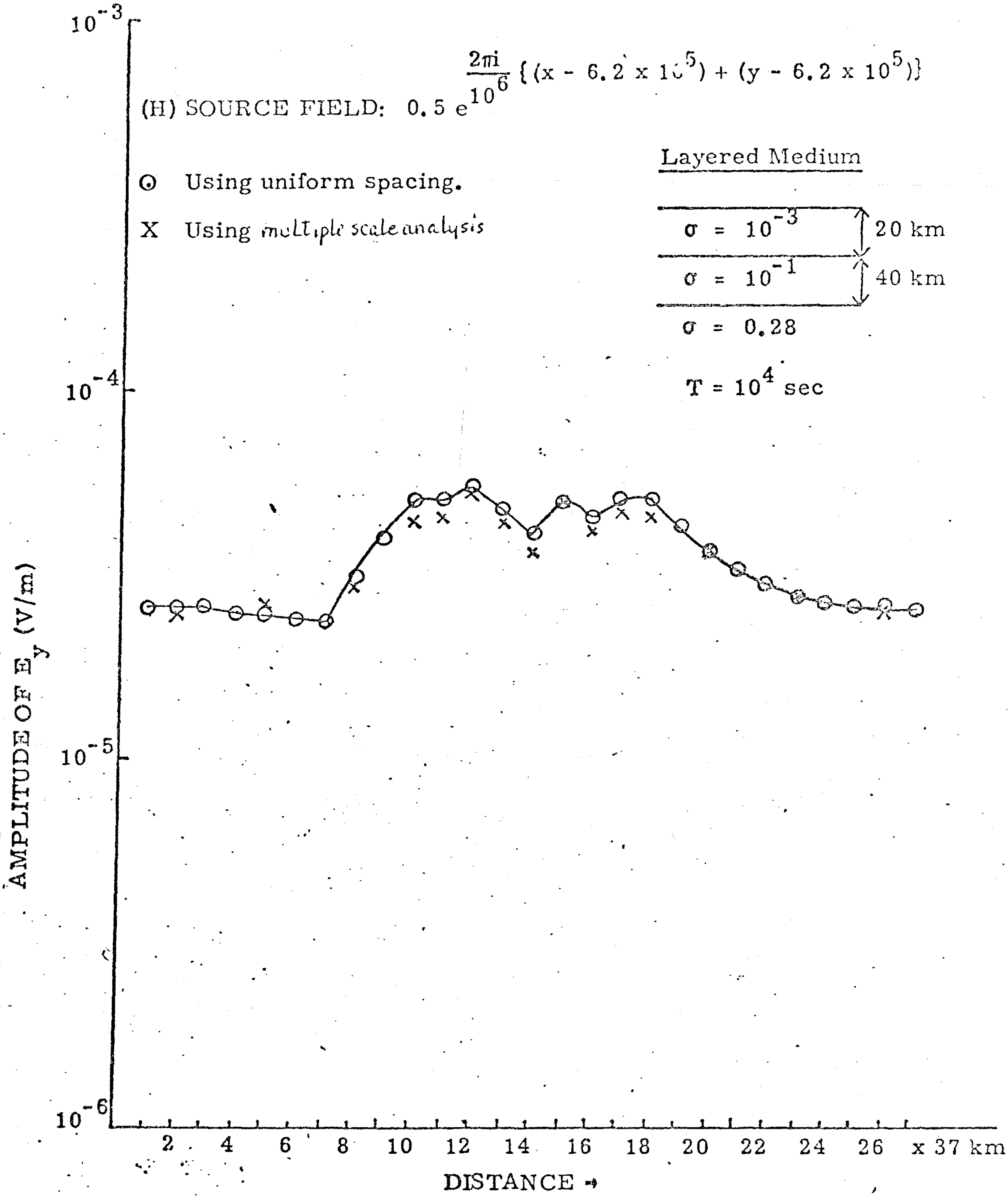


Fig 9 Comparison of fine scale computations with those using multiple scale analysis for E∥

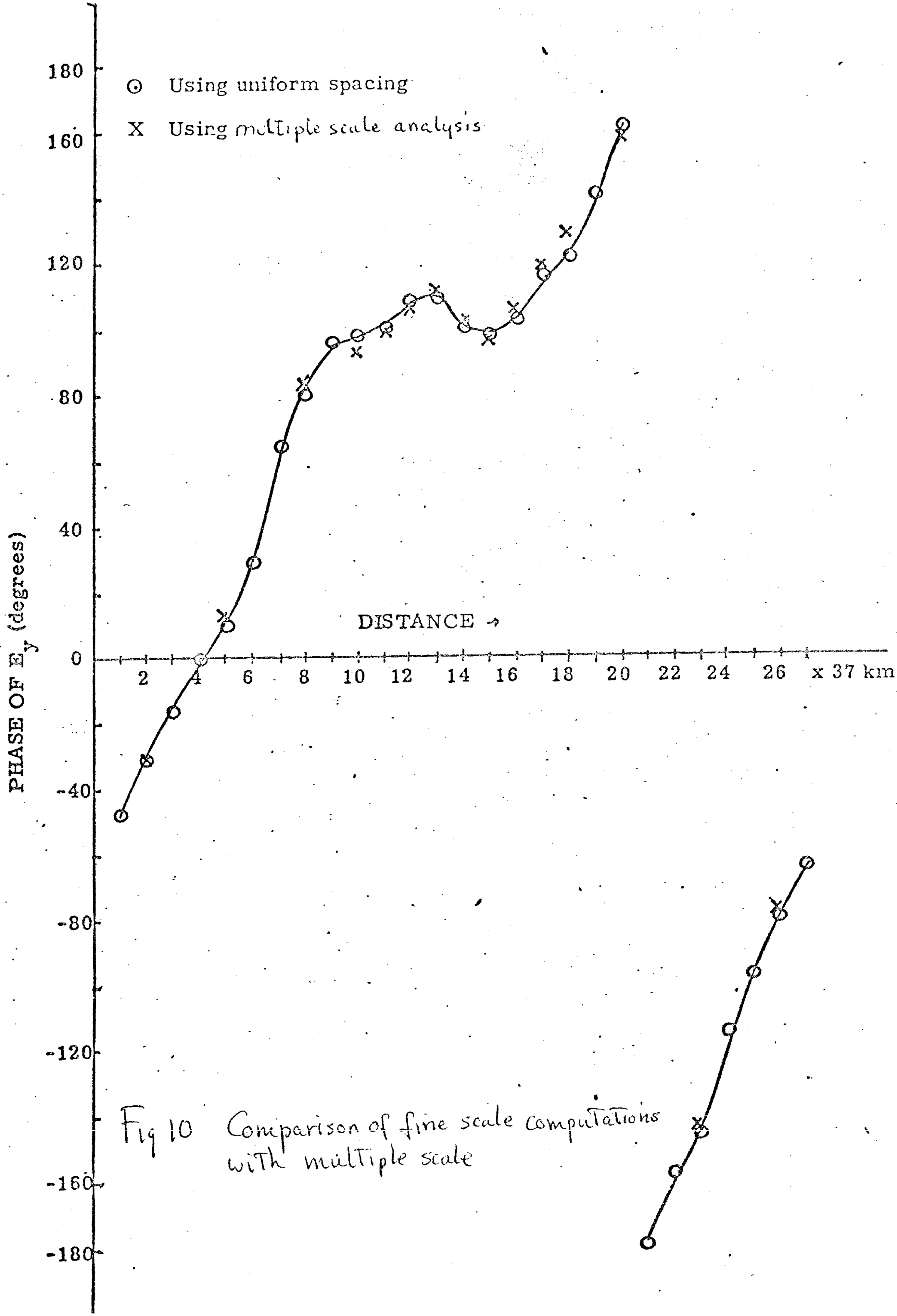


Fig 10 Comparison of fine scale computations with multiple scale

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