PREDICTION OF THERMAL CONDUCTIVITY IN ROCKS FROM OTHER PHYSICAL PARAMETERS AND FROM STANDARD GEOPHYSICAL WELL LOGS

By

Ronald Goss*, Jim Combs*, Aytekin Timur+
University of Texas at Dallas, +Chevron Oil Field Research Co.

ABSTRACT

Measurements on each of 52 samples were made for thermal conductivity, bulk density, porosity, electrical resistivity and conductivity, compressional velocity, and for core samples, permeability, shear velocity, and free fluid index. These physical properties were analyzed to obtain empirical prediction equations for thermal conductivity. An empirical relationship is developed for the Imperial Valley of Southern California. The standard deviation for this regression is ± 0.4 mcal/cm-sec-°C, which implies a reliability of approximately 10% for the predicted thermal conductivity. Application of the derived relationship to a thoroughly investigated borehole section, indicates that the technique may be more reliable than cell measurements for determination of the thermal conductivity of unconsolidated sedimentary sequences.

INTRODUCTION

Knowledge of thermal conductivity is an absolute necessity in heat flow studies. It is an important parameter in the detection and development of a geothermal field, and also has importance for secondary recovery techniques in the oil industry. The measurement of this property is, however, relatively time consuming and expensive. Thus, even though it is desirable, it is often not economically feasible data to collect. Alternative approaches to obtaining thermal conductivity are needed.

In many circumstances, downhole and laboratory methods for measuring thermal conductivity are unsatisfactory. A conductivity logging tool would be ideal, but no tool exists. Many of the properties which are regularly measured during geophysical logging relate to the same physical phenomena that control thermal conductivity, therefore it should be possible to derive thermal conductivity from a relationship to other physical properties.

Theoretical relationships between properties like thermal conductivity and velocity, have been derived for specific media (Debye, 1914; Kittel, 1971).

These theoretical relationships apply reasonably well only to ideal materials, which rocks are not. Thus theoretical work is not likely to serve for prediction of thermal conductivities in the earth.

Empirical studies have had some success (Karl, 1965; Tikhomirov, 1968; Anand, et al., 1973). Most empirical studies have been concerned with either correlation of only one physical property at a time, or with suites of rocks from many different environments. Since thermal conductivity can seldom be related closely to rock type, the results of these studies have not been generally applicable.

In this study an attempt was made to find an empirical relationship between thermal conductivity and several physical properties in combination. Each set of physical properties including thermal conductivity was measured on the same sample (Table 1). Details pertaining to the samples, experimental equipment, and measurement techniques are presented in Goss (1974).

A suite of samples, petrologically similar and obtained from the same geological environment, was chosen. An investigation was then made of possible extensions and limitations over a wider range of rock types. Finally, some use of, and application to, borehole logging data was included.

EXPERIMENTAL DATA

Samples used in this investigation have been collected and analyzed as four successively more general groups. The first group consists of 25 core samples taken from two exploratory holes at the Dunes Geothermal Anomaly in the Imperial Valley (see Figure 1). These Dunes samples vary from siltstones through graywackes to pebble sandstones, all having been hydrothermally altered and cemented.

The second group adds a Mesa core sample and drill cuttings from boreholes drilled in the Mesa Geothermal Field. A set of these values represents averages over zones where the logs and measurements changed conformably with each other. Five sets are from the deep zone (1670 to 1740 meters) of one hole, and eight sets of values are from the shallow zone (200 to 800 meters) of another hole. These Mesa samples are representative of the unconsolidated sediments of the Imperial Valley. Taken with the Dunes samples, the combined suite covers most of the range of the Imperial Valley subsurface sedimentary section.

To provide some indication of the potential for generalization, a third suite of samples of Berea, Boise, Navajo (quartzite) and Raven Ridge Sandstones was added. These samples, like most from areas other than the Imperial Valley, were taken from quarried blocks. They represent rock types or environments to which a sedimentary basin relationship might be expected to apply.

A final miscellaneous group of rock cores included four pieces of two limestones, a dolomite, a shale, and two pieces of a manufactured porcelain. The purpose for these final samples was to determine whether the predictive equations

TABLE 1 - MEASURED DATA

LABORATORY MEASUREMENTS ON CORE

IDENT.	K $\left(\frac{\text{mcal}}{\text{cm-sec-°C}}\right)$	D_B $\left(\frac{\text{g}}{\text{cm}^3}\right)$	ϕ (%)	k (mdarc)	R_o (ohm-m)	→ F $\left(\frac{R_o}{0.05 \text{ ohm-m}}\right)$	σ $\left(\frac{\text{mmhos}}{\text{m}}\right)$	V_P $\left(\frac{\text{km}}{\text{sec}}\right)$	V_S $\left(\frac{\text{km}}{\text{sec}}\right)$	FFI (%)
DUNES DWR 1 CORE DATA, FROM 100-300 METERS										
UCR-1	7.54	2.47	9.56	0.76	16.7	334	59.9	5.09	3.17	8.33
UCR-2	7.51	2.49	8.62	0.50	23.6	472	42.4	5.07	3.23	6.52
UCR-3	6.14	2.35	12.9	350	3.25	65.0	308	4.63	2.60	11.1
UCR-4	7.50	2.46	10.4	15.0	8.04	161	124	5.03	3.13	8.61
UCR-5	6.65	2.40	13.1	470	3.24	64.8	309	4.91	2.87	10.0
UCR-6	7.36	2.44	11.3	7.8	5.32	106	188	4.86	3.01	8.68
UCR-7	6.98	2.43	12.4	1.3	5.88	118	170	4.33	2.57	9.12
UCR-8	6.39	2.35	18.5	62.0	2.83	56.6	353	4.01	2.24	11.7
UCR-9	6.45	2.36	16.3	14.0	2.77	55.4	361	3.98	2.22	4.11
UCR-10	6.46	2.36	16.5	39.0	2.72	54.4	368	3.88	2.15	11.2
UCR-11	7.55	2.56	3.93	0.01	193	3860	5.18	5.54	3.49	2.91
UCR-13	7.58	2.47	9.66	0.56	15.8	316	63.3	4.94	3.03	7.45
UCR-14	7.50	2.56	4.16	0.08	497	9940	2.01	5.49	3.52	3.53
UCR-15	7.68	2.55	3.14	0.02	445	8900	2.25	5.54	3.56	2.78
UCR-16	7.92	2.54	5.32	0.02	154	3080	6.49	5.34	3.49	4.21
UCR-17	7.51	2.53	5.90	0.02	58.2	1160	17.2	5.34	3.30	4.77
UCR-18	7.69	2.55	4.64	0.01	90.1	1800	11.1	5.38	3.36	3.23
UCR-19	7.52	2.51	7.64	0.16	23.8	476	42.0	5.14	3.17	5.97
UCR-20	7.77	2.53	6.74	0.02	29.7	594	33.7	5.10	3.17	4.60
UCR-21	7.58	2.53	6.44	0.38	31.7	634	31.5	5.35	3.32	5.07
UCR-22	7.86	2.53	6.35	0.08	41.8	836	23.9	5.29	3.31	4.92
UCR 115 CORE DATA, FEW METERS FROM DUNES HOLE										
115-A	7.22	2.50	8.09	0.59	21.2	424	47.2	5.09	3.50	6.53
115-B	7.66	2.53	3.99	0.01	584	11700	1.71	5.25	3.52	3.21
115-C	7.32	2.53	4.13	0.03	402	8040	2.49	5.31	3.73	3.37
115-D	7.58	2.60	3.86	0.02	336	6720	2.98	5.31	3.56	2.87
CORE VARIETIES, MOSTLY QUARRIED FROM RESERVOIR ROCKS										
Sandstones										
BER	7.56	2.37	17.3	79	1.34	26.8	746	3.79	2.05	15.6
BOI-2	5.00	2.11	30.0	1700	1.06	21.2	943	3.27	1.64	26.8

TABLE 1 - CONTINUED

IDENT.	K	D_B	ϕ	k	R_o	$\rightarrow$ F	σ	V_P	V_S	FFI
MESA-2	6.15	2.51	8.39	0.07	4.41	88.2	227	3.94	2.08	6.5
NAV-1	8.88	2.44	12.1	83	2.55	51.0	392	4.65	2.64	11.0
NAV-2	8.85	2.44	11.9	100	2.55	51.0	392	4.66	2.65	10.8
RAV	7.39	2.44	13.5	42	1.89	37.8	529	4.77	2.22	11.3
				Other						
ALH-1	6.30	2.28	21.0	55	0.979	19.6	1020	3.68	1.99	19.0
ALH-2	7.13	2.28	21.0	460	1.19	23.8	840	3.81	2.03	18.9
IND-1	5.00	2.47	13.5	0.55	4.81	96.2	208	4.75	2.42	11.5
IND-2	4.99	2.47	13.7	0.36	3.94	78.8	254	4.72	2.40	12.0
DOL	8.00	2.84	0.10	0.02	1940	38800	0.516	6.98	3.76	0.06
SHA	6.42	2.65	1.20	0.05	54.2	1080	18.4	5.10	2.84	0.52
POR-1	14.6	3.06	28.1	18.0	1.14	22.8	877	6.92	4.11	28.1
POR-2	14.7	3.06	28.2	19.0	1.65	33.0	606	6.87	4.07	28.2

DRILL CUTTINGS AND BOREHOLE LOGGED MEASUREMENTS*

IDENT. Mesa borehole-depth (meters)	K (cell measure)	D_B	ϕ	k (est.by type)	(deep rdg.)	$\rightarrow$ F (5-R_w est. 6-Humble Eq.)	σ	V_P	Temp. (°C)	Salin. (ppm)
			MESA 5-1 LOG DATA (DEEP), AVG OF 3 FOR 10 METER INTERVALS							
5-1680	4.2	2.25	24.0	100	6.4	7.1	160	3.11	150	1400
5-1690	3.8	2.30	21.0	100	7.1	12	140	3.30	150	2300
5-1700	4.4	2.25	19.6	100	4.0	8.9	290	3.16	150	3000
5-1710	4.0	2.23	22.6	100	4.4	6.8	240	2.83	150	2000
5-1730	4.2	2.30	20.3	100	8.0	6.7	170	3.40	150	1000
			MESA 6-1 LOG DATA (SHALLOW), AVG OF 3 FOR 30 METER INTERVALS							
6-220	3.6	2.05	22.5	200	3.5	15	350	1.85	65	
6-470	4.6	2.10	21.6	200	1.7	17	600	2.27	80	
6-500	4.1	2.17	18.0	200	1.1	25	960	2.15	90	
6-560	4.2	2.15	20.0	200	1.7	20	630	2.28	100	
6-650	3.8	2.20	23.7	200	1.1	14	940	2.17	105	
6-680	3.8	2.17	24.3	200	1.3	14	770	2.32	110	
6-710	4.0	2.20	25.0	200	1.2	12	860	2.29	115	
6-740	3.6	2.20	22.5	200	1.2	15	840	2.26	120	

*D_B obtained from FDC log; ϕ from FDC, CNL, and BHC logs; ρ and σ from DIL-8 log; F calculated from ϕ and ρ; V_P from BHC log.

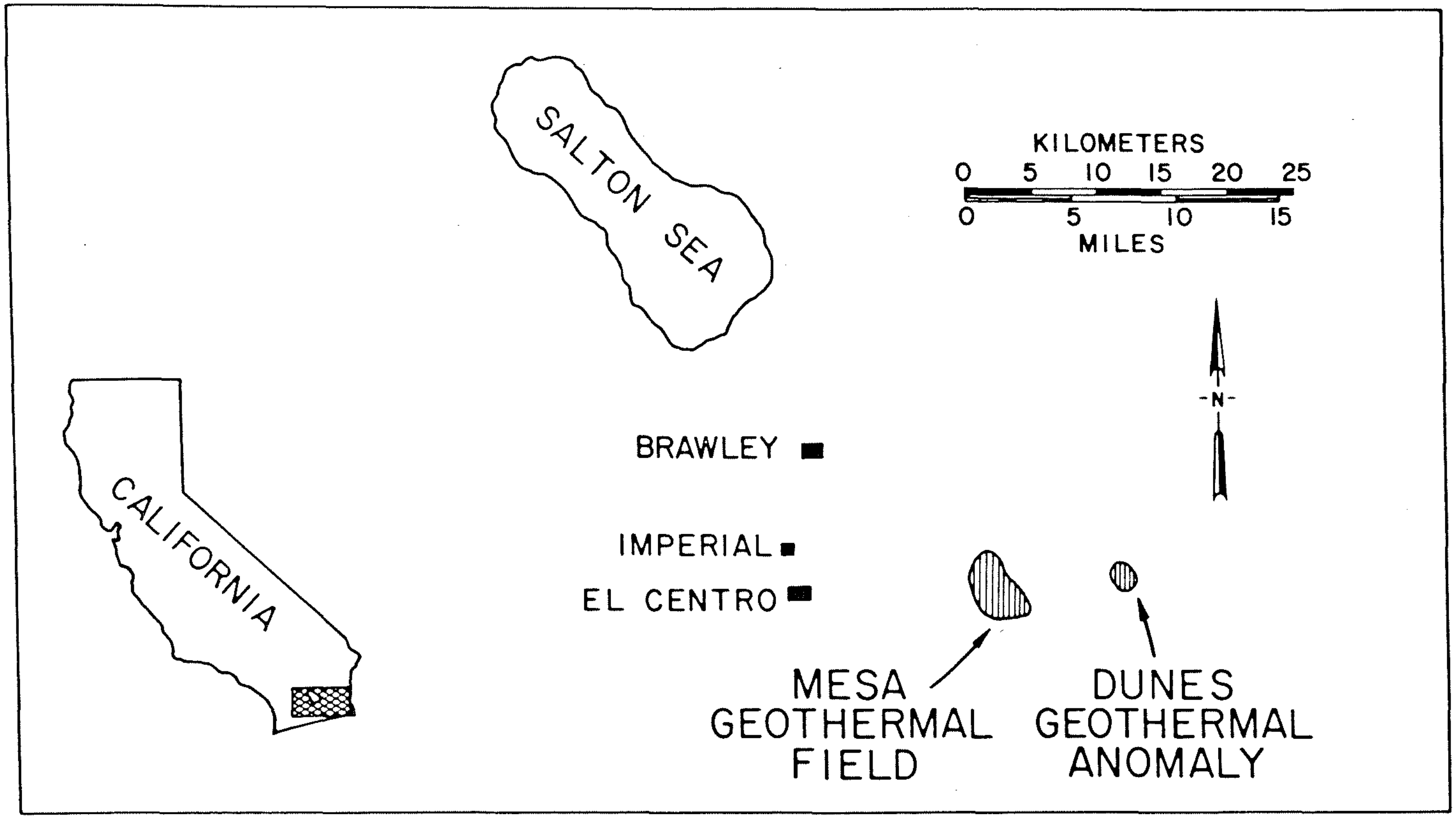

Figure 1. Imperial Valley location map, showing the Mesa Geothermal Field and the Dunes Geothermal Anomaly.

derived for sands and sandstones could be used for other rock types, or to indicate differences which might exist.

Physical properties which were measured include thermal conductivity (K), bulk density (D_B), porosity (ϕ), permeability (k) - for Mesa drill cuttings, this was estimated by using mean sediment type and other comparative information, saturated electrical resistivity (R_o) and electrical conductivity (σ), and compressional velocity (V_P). Shear velocity (V_S) and free fluid index (FFI) were also determined on the core samples. The conditions of investigation were 100% saturation with a sodium chloride brine of 161,334 ppm (fluid resistivity R_w=0.05Ωm). Temperatures near room condition (24°C, 75°F) were used, with uniaxial pressures of 200 bars (2900 psi).

DATA ANALYSIS AND RESULTS

In any empirical approach, many models are possible. The meager guidance of previous theory and investigations discussed below does not suggest or justify the use of anything beyond linear models. The statistically modest quantity of data indicates that sophisticated analytical methods would not be worthwhile. For these reasons, a straightforward multiple linear regression was considered adequate. For purposes of comparing different combinations of variables, goodness of fit, correlation coefficients, standardized partial regression coefficients, and F-tests were used (Davis, 1973). Several of the most significant correlations are presented in Table 2 for six subsets of samples from Table 1.

A plot of predicted thermal conductivities against measured thermal conductivities for the core and porcelain samples is presented in Figure 2. The predictive equation for the regression estimates was derived using seven independent variables. While the porcelains do not deviate much from the linear relationship for other samples, they fall quite far outside the present range of variables. Since results should not depend significantly on one type of sample, the porcelains have been eliminated from further multiple regressions. The forced fit can be noted by comparing the goodness of fit for Group I, 7 (0.858) which contains the two porcelains, with that for Group II, 7 (0.405) without the porcelains. It is noted, however, that the tendency for this manufactured material to follow the linear relationship of the rock samples is gratifying.

Multiple regression techniques require uncorrelated variables, thus in the stepwise elimination of nonessential physical properties from the regression equations, special attention was given to the cancelling effect of obviously related variables. For example, neither compressional nor shear velocity was highly significant when both were included since each offsets the effect of the other. Each velocity, when taken one at a time, was one of the most significant variables. During the present investigation, electrical resistivity R_o, log R_o, electrical conductivity σ, formation factor F, $^1/F$, and log F were all examined. The most significant functional forms were commonly R_o, σ and F. Therefore they are used in the present regression analysis. Free fluid index was one of the least significant variables. Final eliminations almost always resulted in porosity, permeability, electrical conductivity and compressional velocity as the meaningful variables.

TABLE 2 - CORRELATIONS OF THERMAL CONDUCTIVITY VERSUS OTHER PHYSICAL PARAMETERS

Groups	Constant	D_B	ϕ	k	R_o	F	σ	V_P	V_S	FFI	Goodness of Fit	Regression Coefficient	F-Test
GROUP I = ALL CORE WITH PORCELAINS													
I, 2	-1.62E+1	9.08E+0	9.68E-2								0.780	0.884	64.0
I, 2	-1.47E+1	8.52E+0								1.02E-1	0.786	0.886	66.1
I, 2	-2.68E+0						4.73E-3		3.06E+0		0.775	0.880	61.8
I, 3	-1.15E+1	5.37E+0	1.38E-1						1.38E+0		0.835	0.914	59.1
I, 3	-1.10E+1	5.04E+0					3.77E-3		1.71E+0		0.841	0.917	61.8
I, 3	-1.50E+1	8.65E+0					3.46E-4			8.94E-2	0.786	0.887	43.0
I, 4	-1.23E+1	5.57E+0			-1.20E-3		3.48E-3		1.78E+0		0.877	0.937	60.8
I, 4	-1.23E+1	5.57E+0				-5.98E-5	3.48E-3		1.78E+0		0.877	0.937	60.7
I, 4	-1.41E+1	7.54E+0					3.73E-3	-1.06E+0	2.42E+0		0.854	0.924	49.9
I, 7	-1.03E+1	5.44E+0	7.27E-2	-9.21E-4		-1.23E-5		-4.75E-1	1.74E+0	8.34E-2	0.858	0.926	26.7
GROUP II = ALL CORE WITHOUT PORCELAINS													
II, 2	5.00E+0			-5.76E-4					7.63E-1		0.367	0.606	9.8
II, 2	2.39E+0						1.56E-3		1.53E+0		0.400	0.632	11.3
II, 2	4.78E+0							-1.26E-1	1.03E+0		0.341	0.584	8.8
II, 3	3.95E+0		-7.55E-2				2.46E-3		1.18E+0		0.441	0.664	8.7
II, 3	2.09E+0			-1.10E-3			2.37E-3		1.61E+0		0.485	0.697	10.4
II, 3	3.08E+0						2.87E-3		1.42E+0	-8.34E-2	0.445	0.667	8.8
II, 4	2.83E+0		-3.38E-2	-9.51E-4			2.66E-3		1.44E+0		0.492	0.701	7.7
II, 4	1.71E+0			-1.10E-3	-2.74E-4		2.50E-3		1.74E+0		0.493	0.702	7.8
II, 4	1.71E+0			-1.10E-3		-1.37E-5	2.51E-3		1.74E+0		0.493	0.702	7.8
II, 7	1.42E+1	-3.68E+0	-9.10E-2	-1.15E-3		2.03E-5		-2.83E-4	8.34E-1	7.68E-2	0.405	0.636	2.8
GROUP III - SANDSTONE CORE													
III, 2	-1.54E+1	9.10E+0					1.80E-3				0.499	0.706	13.9
III, 2	-1.79E+1	9.91E+0								1.02E-1	0.501	0.708	14.0
III, 2	4.77E+0			-9.56E-4				5.43E-1			0.508	0.713	14.4
III, 3	4.82E-1			-1.62E-3			2.87E-3	1.33E+0			0.662	0.814	17.6
III, 3	2.00E+0			-1.95E-3			3.77E-3		1.63E+0		0.641	0.801	16.1
III, 3	1.25E+0			-2.02E-3				1.09E+0		1.30E-1	0.637	0.798	15.8
III, 4	-4.35E+0	2.26E+0		-1.36E-3			3.09E-3	1.16E+0			0.671	0.819	13.2
III, 4	4.61E-1			-1.80E-3			3.60E-3	8.55E-1	7.48E-1		0.681	0.825	13.9
III, 4	-1.10E-1			-1.98E-3			2.02E-3	1.38E+0		6.88E-2	0.685	0.828	14.1

TABLE 2 - CONTINUED

Groups	Constant	D_B	ϕ	k	R_o	F	σ	V_P	V_S	FFI	Goodness of Fit	Regression Coefficient	F-Test
GROUP IV = SANDSTONE CORE AND CHIPS													
IV, 2	-7.61E-1	9.63E-1						1.15E+0			0.839	0.916	106.9
IV, 2	1.65E+0		-1.62E-2					1.18E+0			0.839	0.916	107.1
IV, 2	1.15E+0			-2.33E-4				1.25E+0			0.840	0.916	107.3
IV, 2	8.53E-1				-1.16E-3			1.33E+0			0.845	0.920	113.2
IV, 2	8.56E-1					5.60E-5		1.33E+0			0.846	0.920	112.8
IV, 2	-8.65E-1						1.69E-3	1.61E+0			0.875	0.935	143.4
IV, 3	6.26E-1		-4.67E-2				1.90E-3	1.39E+0			0.882	0.939	99.8
IV, 3	-1.16E+0			-1.06E-3			2.36E-3	1.66E+0			0.894	0.946	113.5
IV, 3	1.61E+0				-2.21E-1	1.10E-2		1.19E+0			0.898	0.948	117.5
IV, 4	-1.38E+0	1.58E+0			-2.21E-1	1.10E-2		1.01E+0			0.900	0.949	87.5
IV, 4	1.86E+0			-5.16E-4	-2.32E-1	1.15E-2		1.15E+0			0.903	0.950	91.0
IV, 4	1.00E+0				-1.88E-1	9.34E-3	4.48E-4				0.899	0.948	87.1
GROUP V = DUNES CORE													
V, 1	-7.74E+1	6.06E+0									0.789	0.888	86.0
V, 1	8.13E+0		-9.50E-2								0.699	0.836	53.4
V, 1	7.68E+0						-3.54E-3				0.853	0.923	133.0
V, 1	3.02E+0							8.58E-1			0.681	0.825	49.0
V, 1	4.44E+0								9.19E-1		0.680	0.824	48.8
V, 2	6.11E+0	6.20E-1					-3.21E-3				0.853	0.924	63.8
V, 2	7.49E+0		3.72E-2				-4.72E-3				0.864	0.930	70.0
V, 2	7.68E+0			-5.61E-4			-3.27E-3				0.864	0.929	69.8
V, 2	3.61E+0			-1.92E-3				7.55E-1			0.861	0.928	68.3
V, 2	7.98E+0						-3.72E-3	-5.52E-2			0.853	0.924	63.9
V, 2	7.66E+0						-3.63E-3			5.53E-3	0.853	0.924	63.9
V, 3	-1.97E+0	3.65E+0	7.19E-2				-3.87E-3				0.878	0.937	50.4
V, 3	7.75E+0			-5.17E-4	-3.78E-4		-3.55E-3				0.878	0.937	50.2
V, 3	3.27E+0			-2.00E-3	-4.41E-4			8.35E-1			0.880	0.938	51.1
V, 3	7.75E+0			-5.17E-4		-1.88E-5	-3.55E-3				0.878	0.937	50.1
V, 3	3.27E+0			-2.00E-3		-2.20E-5		8.35E-1			0.880	0.938	51.1
V, 4	-4.26E+0	3.97E+0	9.76E-2				-3.68E-3	2.50E-1			0.884	0.940	38.0
V, 4	-4.13E-1	1.95E+0		-1.61E-3		-2.61E-5		6.03E-1			0.885	0.941	38.4
V, 4	3.96E+0		5.73E-2	-1.22E-3			-2.65E-3	6.36E-1			0.889	0.943	40.0
V, 4	5.37E+0			-1.26E-3	-4.38E-4		-1.75E-3	4.46E-1			0.891	0.944	40.7
V, 4	5.37E+0			-1.26E-3		-2.18E-5	-1.75E-3	4.45E-1			0.891	0.944	40.7
V, 4	5.92E+0			-1.14E-3			-2.43E-3	6.56E-1	-5.08E-1		0.885	0.941	38.5

TABLE 2 - CONTINUED

Groups	Constant	D_B	ϕ	k	R_o	F	σ	V_P	V_S	FFI	Goodness of Fit	Regression Coefficient	F-Test
GROUP VI = IMPERIAL VALLEY CORE AND CHIPS													
VI, 1	-1.71E+1	9.74E+0									0.868	0.932	242.4
VI, 1	8.85E+0		-2.04E-1								0.869	0.932	244.6
VI, 1	1.02E+0							1.24E+0			0.921	0.960	432.5
VI, 2	-5.29E-1	8.16E-1						1.14E+0			0.922	0.960	211.4
VI, 2	3.19E+0		-6.05E-2					9.08E-1			0.931	0.965	242.1
VI, 2	1.48E+0			-1.39E-3				1.17E+0			0.927	0.963	229.3
VI, 2	9.37E-1				-4.65E-4			1.27E+0			0.923	0.961	214.8
VI, 2	9.41E-1					-2.15E-5		1.27E+0			0.923	0.960	214.2
VI, 2	-1.06E-1						1.04E-3	1.45E+0			0.930	0.965	241.0
VI, 3	4.03E+0		-8.97E-2		-1.25E-3			8.21E-1			0.940	0.969	181.0
VI, 3	4.02E+0		-9.95E-2			-6.13E-5		8.22E-1			0.939	0.969	179.7
VI, 3	1.96E+0		-5.41E-2				9.24E-4	1.13E+0			0.938	0.968	176.7
VI, 3	1.76E-2			-2.38E-3			1.58E-3	1.43E+0			0.946	0.972	203.4
VI, 3	1.62E+0				-1.93E-1	9.64E-3		1.15E+0			0.968	0.984	349.8
VI, 4	-2.19E+0	3.95E+0			-3.69E-1	1.84E-2	-2.74E-3				0.964	0.982	229.3
VI, 4	7.22E-1	4.70E-1			-1.92E-1	9.59E-3		1.09E+0			0.968	0.984	255.7
VI, 4	2.45E+0		-2.58E-2		-1.78E-1	8.83E-3		1.03E+0			0.969	0.984	263.9
VI, 4	2.23E+0			-1.97E-3	-2.06E-1	1.02E-2		1.04E+0			0.980	0.990	406.4
VI, 4	3.71E+0				-2.95E-1	1.47E-2	-1.61E-3	7.66E-1			0.977	0.989	362.1

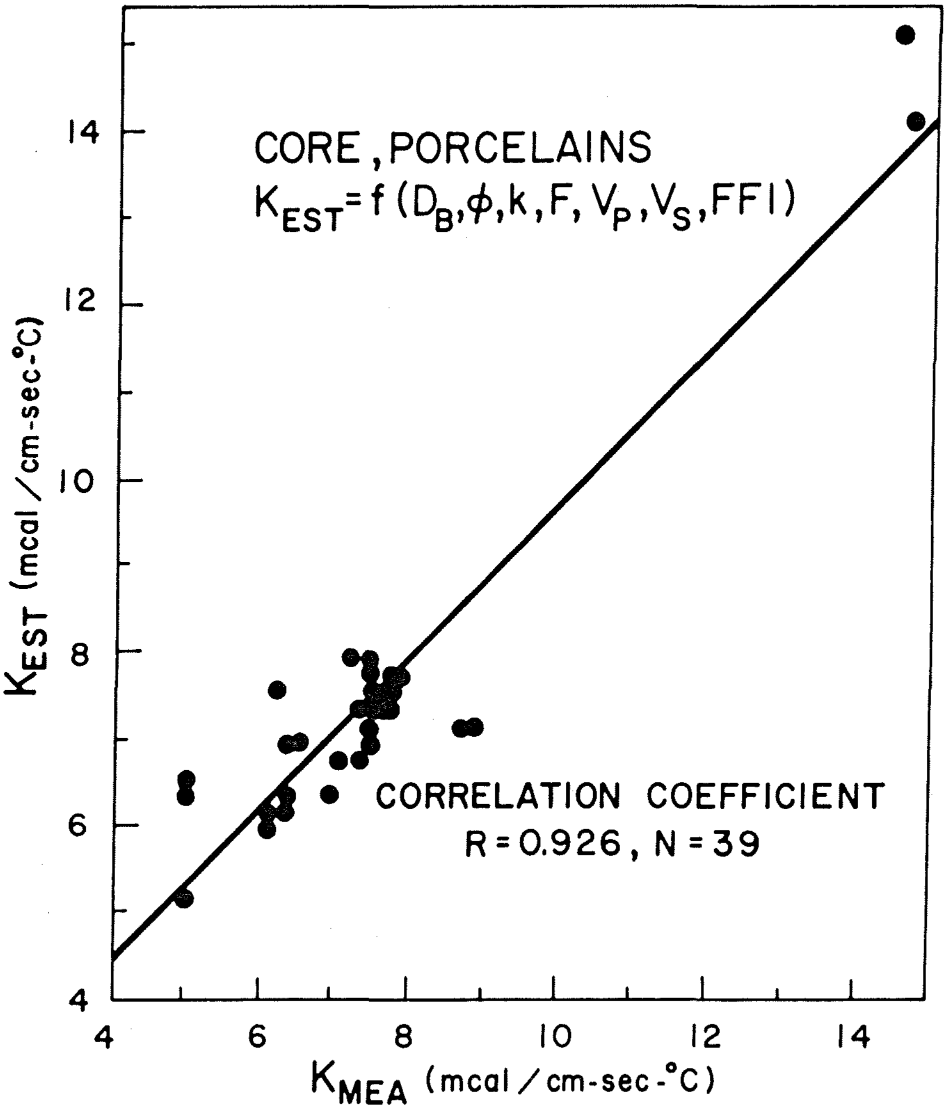

Figure 2. Measured versus predicted thermal conductivity for a multiple linear regression on cores and porcelain samples, using all of the independently measured properties.

Estimated thermal conductivity has been obtained as $K_{est} = f(\phi, V_P)$ for each of the main groups of samples, providing for the comparison of the multiple correlation coefficients. For the closely related Dunes core, R was equal to 0.893 (N = 25). With the Mesa samples added, R increased to 0.965 (N = 39), and the tests for statistical significance improved. The character of the predictive equation and the multiple correlation coefficients did not change much when other sandstones were included; however, the significance decreased. Finally consideration of all rock types resulted in a much poorer fit, with those types which were not from a similar geological environment falling in an essentially random pattern.

From this brief discussion, it can be seen that the best results were obtained with samples representing a specific geological environment. Equation (1) is the result from all 39 Imperial Valley samples:

$$K_{est} = 3.19 - 0.061\phi + 0.908\ V_P, \quad R = 0.965 \tag{1}$$

The relationships

$$K_{est} = 8.85 - 0.204\phi, \quad r = 0.932 \tag{2a}$$

and

$$K_{est} = 1.02 + 1.24\ V_P, \quad r = 0.960 \tag{2b}$$

provide reasonable thermal conductivity estimates in certain circumstances. Equation (2a) will obviously fail when thermal conductivities approach or exceed 9 mcal/cm-sec-°C, or when porosities exceed 35%, neither of which is an unusual occurrence. Equation (2b) is reasonable, but again does not apply to rocks with high thermal conductivities. For example, quartzite, with an average compressional velocity of 6.0 km/sec, would provide a predicted thermal conductivity of 8.5 compared to measured values ranging from 14 to 18 mcal/cm-sec-°C. It is noted that empirical functions pertaining to pore geometry consistently provided a better fit, but with little increase in significance.

An indication of the scatter in the data can be seen in Figure 3, which is a multiple regression plot for Equation (1). A distinct separation is indicated between core sample data and log data. Although there is no suggestion of forced linearity, it would be desirable to have samples which fell into the intermediate range. The standard deviation for this regression is ± 0.4 mcal/cm-sec-°C, which implies a reliability of approximately 10% for the predicted thermal conductivity.

DISCUSSION

Many empirical investigations have involved thermal conductivity, but only a few have been concerned with the derivation of predictive equations from several physical parameters. Since a number of these empirical studies require specific

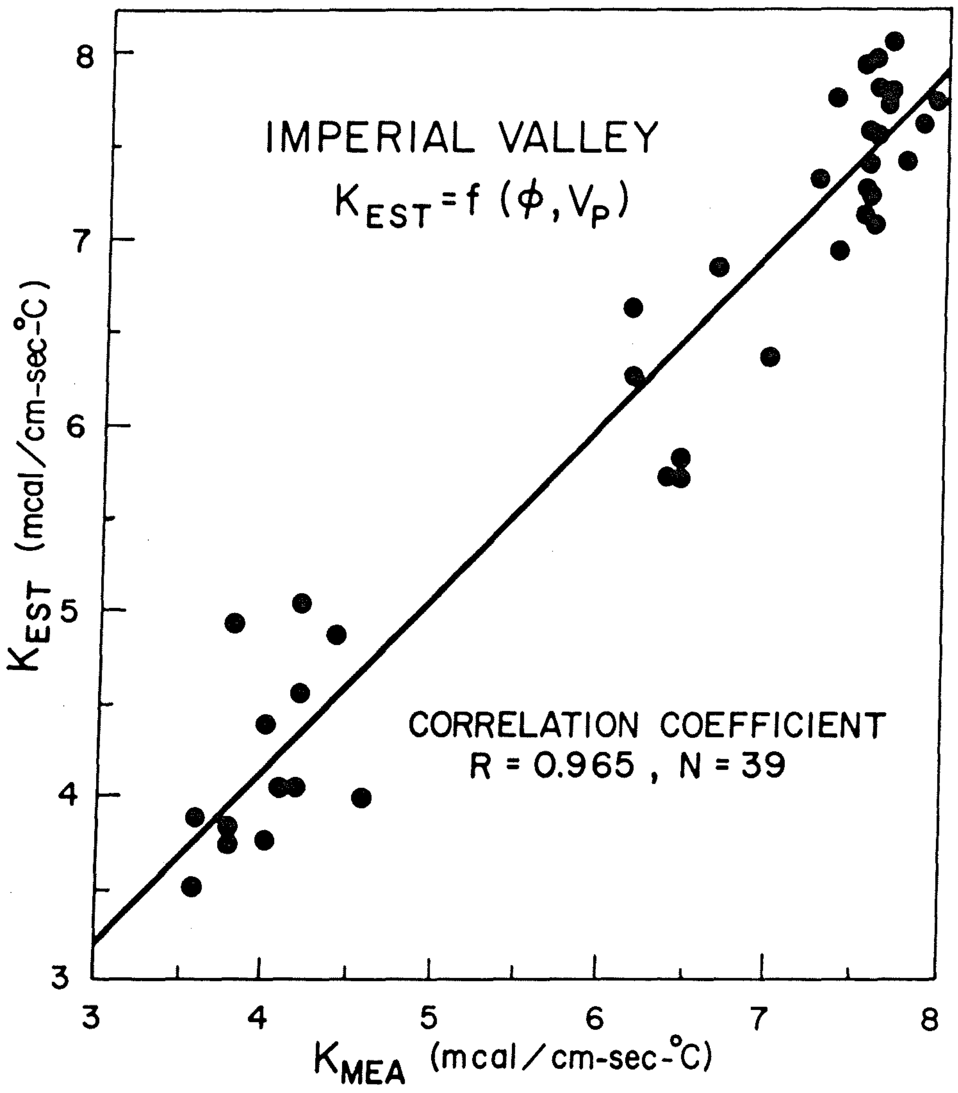

Figure 3. Measured versus predicted thermal conductivity for a multiple linear regression on the Imperial Valley samples, using Equation (1).

data or constants which are not usually available, they were disregarded. The following are published relationships which did not require additional laboratory measurements beyond geophysical borehole log analysis to apply.

One of the earliest studies (Thornton, 1919), using data on insulators from ice to wood, resulted in the relation

$$K = V_P^2 \; D_B^2 \; (10^{-14}) \qquad (3)$$

where velocity V_P is in cm/sec, the saturated bulk density D_B in g/cm^3 and thermal conductivity K in cal/cm-sec-°C. Dakhnov and Kjakonov (1952), used rock values from the literature to provide

$$K = D_B \; \frac{3.1}{4680} \qquad (4)$$

With the same approach, for classes of feldspathic, salt and other rock types, Karl (1965) obtained

$$K = 0.8 \times 10^{-8} \; V_P \qquad (5a)$$

$$K = 2.0 \times 10^{-8} \; V_P \qquad (5b)$$

and

$$K = 1.3 \times 10^{-8} \; V_P \qquad (5c)$$

respectively. Tikhomirov (1968) examined dry as well as partially saturated individual samples of many rock types, and combined the results into one equation,

$$K = 1.30 \exp \, (0.58 D_D + 0.40 S_w) \qquad (6)$$

where S_w is the fractional water content and D_D is the bulk density in the dry state. Using core from a wide region of the Siberian lowlands, Moiseyenko and coworkers (1970), derived the relation

$$K = [1.17 + 0.83 \, (3.42 - 0.055\phi)] 10^{-3} \qquad (7)$$

where the term in parentheses is for the dry conductivity, and ϕ is the porosity in percent. For a group of rock forming silicate minerals, Horai (1971) obtained

$$V_P = 6.07 + 0.15 \, K \qquad (8a)$$

and

$$V_S = 3.37 + 0.08 \, K \qquad (8b)$$

where the thermal conductivities are in mcal/cm-sec-°C, and the compressional and shear velocities in km/sec. (It should be noted that if these equations are solved for K in the velocity range of normal rocks, however, they return meaningless negative thermal conductivities). In an experiment with unconsolidated sands, Somerton, et al., (1974), found that

$$K' = 0.735 - 0.0130\phi + 0.363\, K'_s \sqrt{S_w} \qquad (9)$$

where the prime will imply a result in BTU/ft-hr-°F, and K'_s is the thermal conductivity of the solid or component grains. Of most direct interest for the present study is the work by Anand, et al., (1973) which yields for dry sandstones

$$K'_D = 0.340\, D_D - 0.032\phi + 0.53k^{0.10} + 0.013F - 0.031 \qquad (10)$$

and for saturated samples,

$$K' = K'_D \left[1.0 + 0.30\left(\frac{K'_L}{K'_G} - 1.0\right)^{0.33} + 4.57\left(\frac{\phi}{100-\phi}\ \frac{K'_L}{K'_D}\right)^{0.48m} \left(\frac{D_B}{D_D}\right)^{-4.3} \right] \qquad (11)$$

where the permeability k is in millidarcies; K, ϕ, and D are the thermal conductivity, porosity and density, respectively, with subscripts D, L, and G for dry rock, saturating liquid and gas (air), respectively; m, an empirical parameter, is the cementation factor of Archie's formula

$$F = \frac{A}{\phi^m} \qquad (12)$$

with A another empirical parameter.

Most of the relationships presented above are deficient since they are not based on sets of variables measured on the same samples; instead values from the literature which are related by rock type were considered. The investigators have noted this problem and recommend that multiple measurements on the same samples be a future goal. Investigators who have made multiple measurements often note that velocity should have been a useful parameter, but was not available. When correlations are based on values for dry rock, the reductions for saturated samples tend to be involved and are not always effective for prediction. Results using different equations usually do not agree. Finally, there are not enough studies with their initial data published to determine limitations, areas of overlap, or reliability of extension to other samples.

A comparison of the results of empirical equations described above with our results is listed in Table 3. None of the equations are completely satisfactory although it must be expected that Equation (1), which is partially based on these samples, will give the best fit. Thornton (1919), used many materials besides rock to obtain Equation (3), and as noted in Table 3, values range widely. Equation (4) yields exceedingly low values, probably because Dakhnov and Kjakonov (1952), used bulk densities of nonporous rocks. Of these published relationships, reasonable agreement is provided by Equation (5c) from Karl (1965), although it is consistently low by about 20%. Equation (6) derived by Tikhomirov (1968), from

TABLE 3 - COMPARISON OF PREDICTION EQUATIONS

Sample Identification	Thermal Conductivities in mcal/cm-sec-°C Equation numbers for empirical relations								
	3	4	5c	6[1]	7	9[1]	11[1,2] F_{mea}/F_{est}	1	Measurement[3]
UCR-1[4] (Dunes)	16	1.6	6.6	7.7	3.6	9.2	42/15	7.2	7.5
115-A[4] (Dunes)	16	1.7	6.6	7.9	3.6	9.3	52/18	7.3	7.2
BER[4] (Berea)	8.1	1.6	4.9	6.6	3.2	8.8	11/11	5.6	7.6
BOI-2[4] (Boise)	4.8	1.4	4.2	5.5	2.6	8.2	9.8/8.5	4.4	6.2
6-220[5] (Mesa)	1.4	1.4	2.4	5.6	3.0	7.1	9.1/9.1	3.5	3.6
5-1680[5] (Mesa)	4.9	1.5	4.0	6.2	2.9	7.0	7.8/8.7	4.6	4.2

[1] Requires assumed--dry density $D_D = D_B - 0.01\phi$, saturation $S_w = 1.0$, and/or solid conductivity $K_S' = 4.5$ BTU/ft-hr-°F if sample is predominately quartz and 3.5 if significant clay in sample (Values based on discussion of Somerton, et al., 1974).

[2] Requires an assumption for gas conductivity (air) $K_G = 0.055$, liquid conductivity (sea water) $K_L = 1.4$, and $m = 1.73$; values taken respectively from Ingersoll, et al., (1954), Ratcliffe (1960), and Timur, et al., (1972). Since the measured values of formation factor F_{mea} for the thermally altered samples are quite high, a comparison is also made using F_{est} values estimated from the relation $F=1.13V^{-1.73}$, $V=0.01\phi$ of Timur, et al., (1972).

[3] Thermal conductivity value obtained from measurement of the sample in a divided bar apparatus. Note that the measured thermal conductivity for the Berea Sandstone is almost half the value of 12.4 reported by Anand, et al., (1973), whereas the Boise conductivity is only 15% lower than their value of 7.4.

[4] Core samples.

[5] Unconsolidated material.

many consolidated rock types, does not appear to allow for unconsolidated material. Equation (9) of Somerton, et al., (1974) with the assumed solid conductivities used, provides poor agreement, although it is derived for unconsolidated sands. Equation (7) of Moiseyenko, et al., (1970) from core samples, smooths out to consistently low values. Finally, Equation (11) by Anand, et al., (1973) tends to give unreasonably high values especially when applied to the non-reservoir type samples.

None of these relations furnish completely satisfactory predictions over the range of interest, and none are expected to give satisfactory results in the Imperial Valley geological environment. Therefore, we return to the equation derived herein, that is, Equation (1).

APPLICATION

The ideal end result of this study is to determine thermal conductivity from common borehole logging parameters. While empirical relations have been derived, they are of little value unless they can predict reasonable thermal conductivities. The results of an initial attempt have been included together with an interpretation for the 300 to 700 meter interval of a borehole from the Mesa Geothermal Field. Casing at 310 meters and a convective thermal regime below 670 meters determine the limits of useful investigation. Equation (1) has been applied.

The logs from which hand-digitized versions of Figure 4 were made, and a computer evaluation generated from them, provided the basic data for the predicted thermal conductivities. Measured values were taken from drill chips of the unconsolidated sediments, using a cell apparatus in the divided bar (Sass, et al., 1971). Problems exist in the accuracy of this method, e. g., the effect of drilling mud, the uncertainty in depth, and the incompleteness of sampling. An ideal relationship with excellent measurements throughout could not be expected to provide exact agreement. As a minimum, however, both predicted and measured values should reflect similar changes with depth, with reasonable explanations for the differences.

From Figure 5, showing the deviations in repeatability for cell measurements (the line arbitrarily passes through the first measurement), with the differences of sampling depth, it can be seen that agreement between measured and predicted trends with depth is quite good. However, there is a distinct difference in the mean value using the two methods, from a mean of near 4.5 compared to about 3.5 mcal/cm-sec-°C. Field washing of the samples, which is certain to have depleted the clay content, probably caused the shift in the mean value of the two methods. Since computer evaluation of the geophysical logs indicates a relatively high content of clay (about 30%) in this shallow interval, higher measured values are expected.

For the deep zone, the shift is reversed from a mean near 4.2 to about 4.7 mcal/cm-sec-°C. This is partly a cored section from which samples were taken

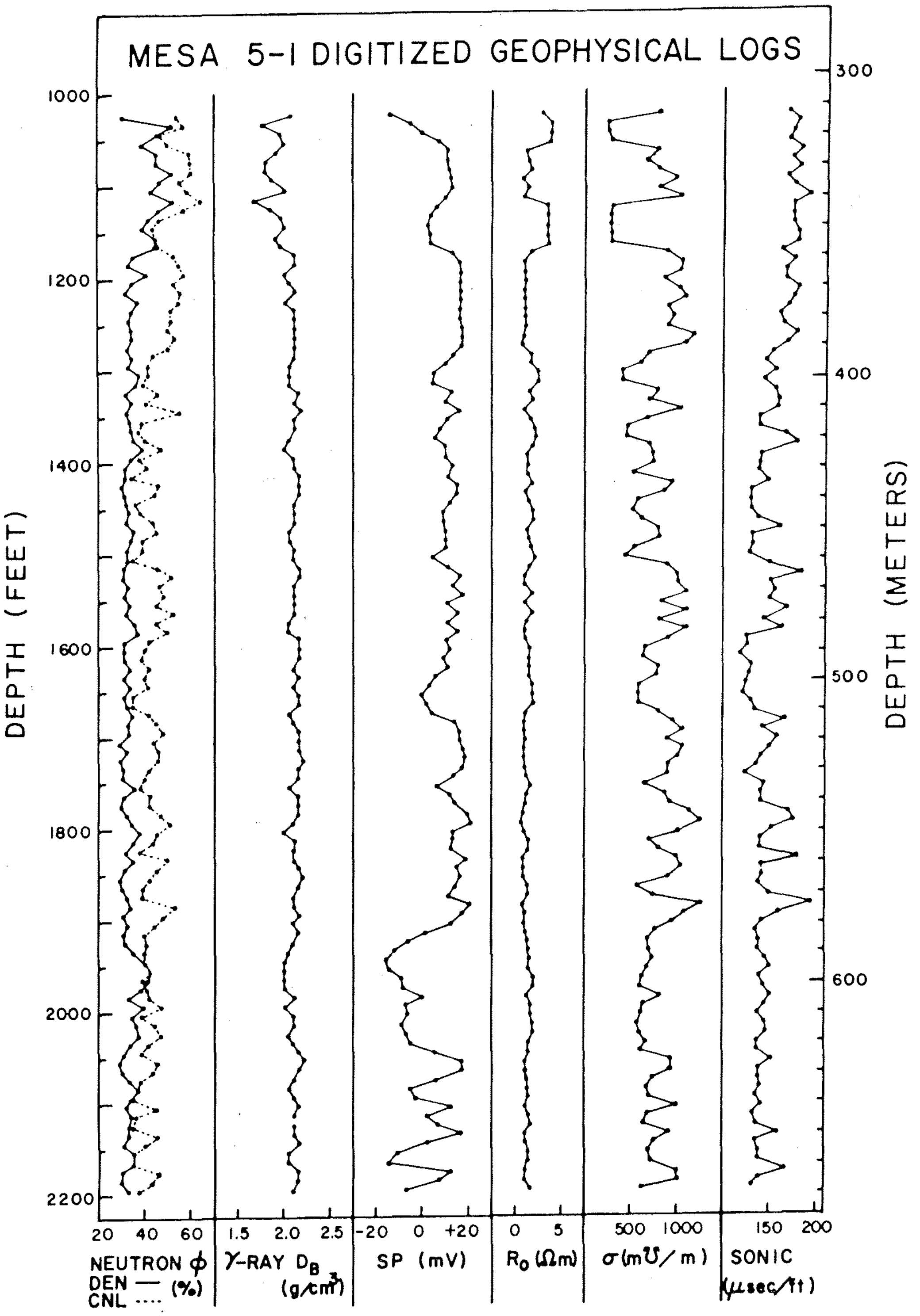

Figure 4. Digitized geophysical logs for the Mesa 5-1 geothermal well. Data points obtained from original logs by averaging over 3 meter intervals.

MM

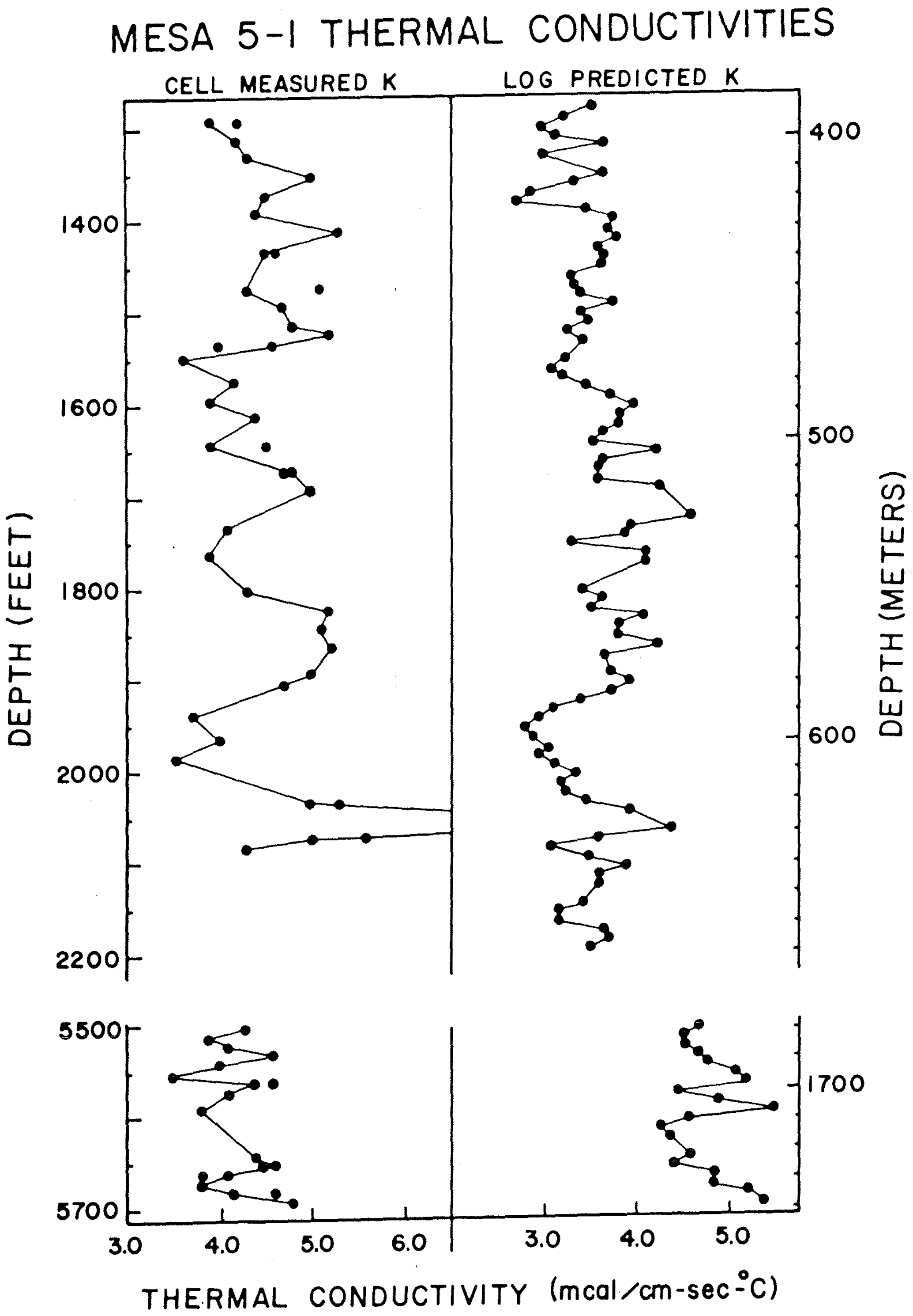

Figure 5. Thermal conductivities for the Mesa 5-1 geothermal well. Two cell data points at the same depth indicate repeated measurements. Log predictions are from Equation (1).

for the prediction analysis and the values measured on the core samples were about 5.0 mcal/cm-sec-°C. Deviations in this zone appear to be caused by a relative loss of sand from the drill chips reaching the surface. Core includes sections of clay cemented sand; computer evaluation of the geophysical logs indicates less clay than in the shallower section, yet the grab samples consist of a considerable amount of shale fragments. There are also shifts with depth for up to 15 m in this deep zone, but the gross patterns appear to follow closely. These shifts are almost certainly a result of the poor control on sampling and the sampled depth as depth increases.

CONCLUSIONS

In conclusion, there is every indication that useful empirical relationships for the prediction of thermal conductivity can be obtained. Application of a predictive equation to a geological sequence similar to the one from which it was derived may be reliable. While a relationship might remain useful in comparable environments, more work needs to be undertaken to determine limitations. There seems little hope of more general predictive relationships being successful; however, typical geological settings can probably be characterized.

Experimental data and an empirical equation for the Imperial Valley of Southern California have indicated satisfactory prediction of thermal conductivity in an initial application. In fact, we conclude that the indirect method of prediction may be more accurate than direct cell measurements on drill chips. This may well be true for most unconsolidated sedimentary environments.

ACKNOWLEDGMENTS

Part of this research was carried out while two of the authors (R. Goss and J. Combs) were associated with the University of California, Riverside. Among those who have been particularly helpful at various stages of the project are Shawn Biehler, Lewis Cohen, Jean Davidson, Fumiko Goss, Frank Griswold, Ross Hager, Susan Hilton and Jo Shearer. Financial support was provided by a University of California at Riverside Chancellor's Patent Fund Grant, the Chevron Oil Field Research Company, the USDI Bureau of Reclamation contract no. 14-06-300-2390, and by the National Science Foundation - Research Applied to National Needs grants no. GI-36250 and AER 72-03551.

REFERENCES

Anand, J., Somerton, W. H., and Gomaa, E., 1973, Predicting thermal conductivities of formations from other known properties: Journal of the Society of Petroleum Engineers, V. 13, p. 267-273.

Dakhnov, V. N., and Kjakonov, D. J., 1952, Thermal investigation of fissures (Russian text): Moscow; cited in Karl, 1965, section 2.1.3.

Davis, J. C., 1973, Statistics and Data Analysis in Geology: Wiley and Sons, New York, 550 pp.

Debye, P., 1914, Equation of state and quantum hypothesis, with an appendix on thermal conductivity (German text), in Lectures on the Kinetic Theory of Matter and Electricity: Von M. Planck, et al., editors, Leipzig, Teubner, p. 19-60.

Goss, R. D., 1974, Empirical Relationships between Thermal Conductivity and other Physical Parameters in Rocks: Ph.D. Thesis, University of California at Riverside, Riverside, California, 216 pp.

Horai, K., 1971, Thermal conductivity of rock-forming minerals: Journal of Geophysical Research, V. 76, p. 1278-1308.

Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C., 1954, Heat Conduction with Engineering, Geological and Other Applications: University of Wisconsin Press, Madison, Wisconsin, 323 pp.

Karl, R., 1965, Physical rock parameters, wave velocity, thermal conductivity, (German text): Freiberger Forschungshefte, Series C, V. 197, p. 7-76.

Kittel, C., 1971, Introduction to Solid State Physics: Wiley and Sons, New York, 648 pp.

Moiseyenko, V. I., Doroginitskaya, L. M., Leont'ev, Ye. I., and Sokolova, L. S., 1970, Dependence of the heat conductivity of the clastic rocks of the West Siberian Lowlands on other physical parameters (Russian text), in Methods of Determining the Thermal Properties of Rocks: Akademy Nauk SSSR Sibirsk, Otdeleniye Geologiya i Geofizika, no. 2, p. 106-110.

Ratcliffe, E. H., 1960, The thermal conductivities of ocean sediments: Journal of Geophysical Research, V. 65, p. 1535-1541.

Sass, J. H., Lachenbruch, A. H., and Munroe, R. J., 1971, Thermal conductivity of rocks from measurements on fragments and its application to heat-flow determinations: Journal of Geophysical Research, V. 76, p. 3391-3401.

for the prediction analysis and the values measured on the core samples were about 5.0 mcal/cm-sec-°C. Deviations in this zone appear to be caused by a relative loss of sand from the drill chips reaching the surface. Core includes sections of clay cemented sand; computer evaluation of the geophysical logs indicates less clay than in the shallower section, yet the grab samples consist of a considerable amount of shale fragments. There are also shifts with depth for up to 15 m in this deep zone, but the gross patterns appear to follow closely. These shifts are almost certainly a result of the poor control on sampling and the sampled depth as depth increases.

CONCLUSIONS

In conclusion, there is every indication that useful empirical relationships for the prediction of thermal conductivity can be obtained. Application of a predictive equation to a geological sequence similar to the one from which it was derived may be reliable. While a relationship might remain useful in comparable environments, more work needs to be undertaken to determine limitations. There seems little hope of more general predictive relationships being successful; however, typical geological settings can probably be characterized.

Experimental data and an empirical equation for the Imperial Valley of Southern California have indicated satisfactory prediction of thermal conductivity in an initial application. In fact, we conclude that the indirect method of prediction may be more accurate than direct cell measurements on drill chips. This may well be true for most unconsolidated sedimentary environments.

ACKNOWLEDGMENTS

Part of this research was carried out while two of the authors (R. Goss and J. Combs) were associated with the University of California, Riverside. Among those who have been particularly helpful at various stages of the project are Shawn Biehler, Lewis Cohen, Jean Davidson, Fumiko Goss, Frank Griswold, Ross Hager, Susan Hilton and Jo Shearer. Financial support was provided by a University of California at Riverside Chancellor's Patent Fund Grant, the Chevron Oil Field Research Company, the USDI Bureau of Reclamation contract no. 14-06-300-2390, and by the National Science Foundation - Research Applied to National Needs grants no. GI-36250 and AER 72-03551.

REFERENCES

Anand, J., Somerton, W. H., and Gomaa, E., 1973, Predicting thermal conductivities of formations from other known properties: Journal of the Society of Petroleum Engineers, V. 13, p. 267-273.

Dakhnov, V. N., and Kjakonov, D. J., 1952, Thermal investigation of fissures (Russian text): Moscow; cited in Karl, 1965, section 2.1.3.

Davis, J. C., 1973, Statistics and Data Analysis in Geology: Wiley and Sons, New York, 550 pp.

Debye, P., 1914, Equation of state and quantum hypothesis, with an appendix on thermal conductivity (German text), in Lectures on the Kinetic Theory of Matter and Electricity: Von M. Planck, et al., editors, Leipzig, Teubner, p. 19-60.

Goss, R. D., 1974, Empirical Relationships between Thermal Conductivity and other Physical Parameters in Rocks: Ph.D. Thesis, University of California at Riverside, Riverside, California, 216 pp.

Horai, K., 1971, Thermal conductivity of rock-forming minerals: Journal of Geophysical Research, V. 76, p. 1278-1308.

Ingersoll, L. R., Zobel, O. J., and Ingersoll, A. C., 1954, Heat Conduction with Engineering, Geological and Other Applications: University of Wisconsin Press, Madison, Wisconsin, 323 pp.

Karl, R., 1965, Physical rock parameters, wave velocity, thermal conductivity, (German text): Freiberger Forschungshefte, Series C, V. 197, p. 7-76.

Kittel, C., 1971, Introduction to Solid State Physics: Wiley and Sons, New York, 648 pp.

Moiseyenko, V. I., Doroginitskaya, L. M., Leont'ev, Ye. I., and Sokolova, L. S., 1970, Dependence of the heat conductivity of the clastic rocks of the West Siberian Lowlands on other physical parameters (Russian text), in Methods of Determining the Thermal Properties of Rocks: Akademy Nauk SSSR Sibirsk, Otdeleniye Geologiya i Geofizika, no. 2, p. 106-110.

Ratcliffe, E. H., 1960, The thermal conductivities of ocean sediments: Journal of Geophysical Research, V. 65, p. 1535-1541.

Sass, J. H., Lachenbruch, A. H., and Munroe, R. J., 1971, Thermal conductivity of rocks from measurements on fragments and its application to heat-flow determinations: Journal of Geophysical Research, V. 76, p. 3391-3401.

Somerton, W. H., Keese, J. A., and Chu, S. L., 1974, Thermal behavior of unconsolidated oil sands: *Journal of the Society of Petroleum Engineers*, V. 14, p. 513-521.

Thornton, W. M., 1919, The thermal conductivity of solid insulators: cited *in Karl*, 1965, Sect. 2.2.

Tikhomirov, V. M., 1968, The thermal conductivity of rocks and its relationship to density, moisture content, and temperature (Russian text): *Neftianoe Khoziaistvo*, V. 46, p. 36-40.

Timur, A., Hempkins, W. B., and Worthington, A. E., 1972, Porosity and pressure dependence of formation resistivity factor for sandstones: *Transactions of the Canadian Well Logging Society*, Fourth Formation Evaluation Symposium, Sec. D.