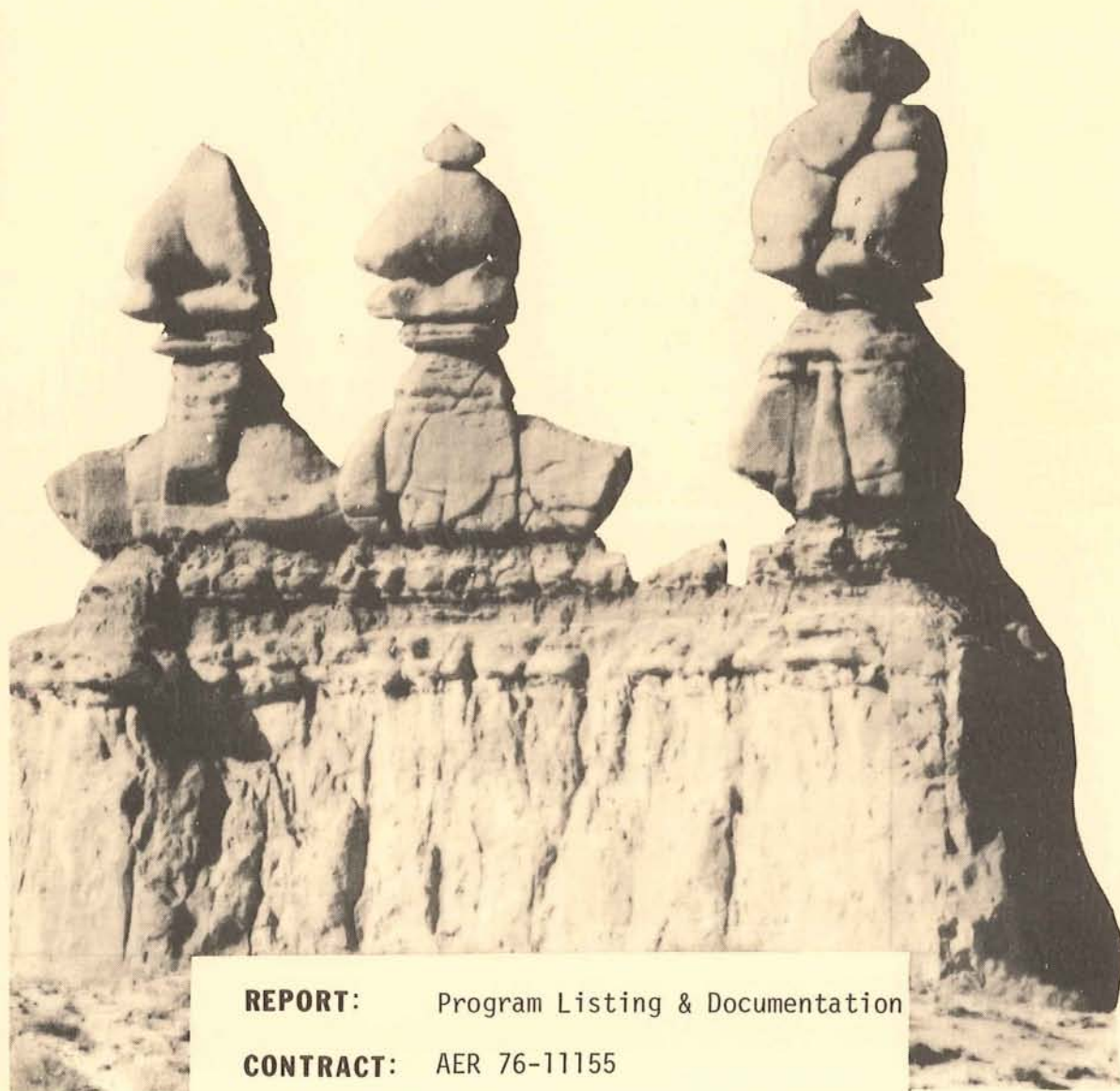


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for Solution of Geophysical Problems Governed
by the Inhomogeneous 2-D Scalar Helmholtz Equation

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Documentation of a Finite Element
Program for Solution of Geophysical
Problems Governed by the Inhomogeneous
2-D Scalar Helmholtz Equation

by

John A. Stodt

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I. Introduction

A two dimensional finite element program applicable to the numerical solution of a wide variety of geophysical problems has been developed at the University of Utah. Finite element programs to handle a number of geophysical problems were originally developed by Rijo (Rijo, 1977). Linear interpolation of the unknown field over triangular sub-domains of the region where a solution is sought was used in conjunction with the Galerkin technique to derive a system of linear equations which approximates the governing PDE. The solution of this linear system of equations gives the approximate field values at the nodes of the discretized domain.

These programs have been modified and consolidated by the author into a single program which will handle the two dimensional magnetotelluric TE and TM mode problems, as well as the infinite line source problem. In addition, the element equations obtained from the finite element technique have been re-derived and re-programmed in a sufficiently general form so that any physical problem governed by the two dimensional inhomogeneous scalar Helmholtz equation may be handled with minor modifications to the program.

II. Applications

The finite element formalism developed in part IV of this documentation is applicable to any physical problem governed by the equation

$$1) \quad \frac{\partial}{\partial x} \left(\frac{1}{k} \frac{\partial f}{\partial x} \right) + \frac{\partial}{\partial x} \left(\frac{1}{k} \frac{\partial f}{\partial z} \right) + pf = S$$

where

$k, p \equiv$ physical property factors which may be functions of position over the domain where a solution is sought

$s \equiv$ source function

$f \equiv$ unknown field for which a solution is sought

In practice, f , k , p , and s may be real or complex. Examples of some particular physical problems which are governed by this equation are given in Table 1. In Table 1 we make use of the following parameters:

$$\hat{z} = j\omega\mu$$

$$\hat{y} = \sigma + j\omega\epsilon$$

$I =$ current strength

Problem	f	k	p	S
TE-MT	E_y	\hat{z}	$-\hat{y}$	0
TM-MT	H_y	\hat{y}	$-\hat{y}$	0
Line Source	E_y	\hat{z}	$-\hat{y}$	$I\sigma(x)\delta(z)$

Table 1. Examples of physical problems governed by equation 1.

The program for which this documentation is written solves the examples of Table 1, where f , k , and p are complex quantities. Any or all of σ , μ , and ϵ may be considered piecewise constant functions of position. The DC resistivity problem, which is also governed by equation 1 after Fourier transformation of the strike direction, is not incorporated in the same program since only real numbers are required for its solution. It is more efficient computationally to develop a program utilizing only real arithmetic when dealing with problems involving only real quantities. A program based on the theory outlined in part IV of this documentation which solves the DC problem is available at the Department of Geology and Geophysics of the University of Utah as a separate program.

In closing this section, it should be pointed out that an effort was made, through appropriate structuring, to produce a program which could be easily modified to handle other physical problems governed by equation 1 once the user has gained a rudimentary knowledge of both the program and the finite element method. Sufficient theory is hopefully provided in part IV of this documentation, while knowledge of the program itself should be obtained from part III of this report in conjunction with a study of the comment statements in the program itself.

III. Documentation

A. Description of software

A flow diagram of program construction is given in figure 1. This diagram shows the sequence in which subroutines are called to do the various calculations. Also indicated are calls to the Univac 1108 system library routines where they are used. A brief description of the function of the various routines is given in comment statements in the program itself and will not be repeated here. As an aid to the programmer trying to implement the program on a different system, the write-ups on the 1108 system library subroutines are included in Appendix II.

One Univac 1108 system I/O device is used which deserves special mention. The device name is NTRAN and its use is in transferring large amounts of unformatted data from core to Fastrand drum storage and vice versa in an efficient manner. This is necessary in implementing this program on the 1108 because of the relatively limited amount of core storage available. A temporary word addressable data file is assigned in which the coefficient matrix of the linear system to be solved is stored. The Greenfield algorithm (see e.g. Swift, 1967 - Appendix 3) is then used to solve the linear system of equations, with appropriate size blocks of the coefficient matrix being transferred into core and reduced one by one during solution. This data transfer into and out of core is handled by Univac's system device NTRAN. A write up of NTRAN is included in Appendix II as an aid to the user trying to adapt the program to a different system. In this case, a suitable substitution for NTRAN must be found, unless the user's system has a large amount of core memory (150-200K) available so that execution can proceed entirely in core.

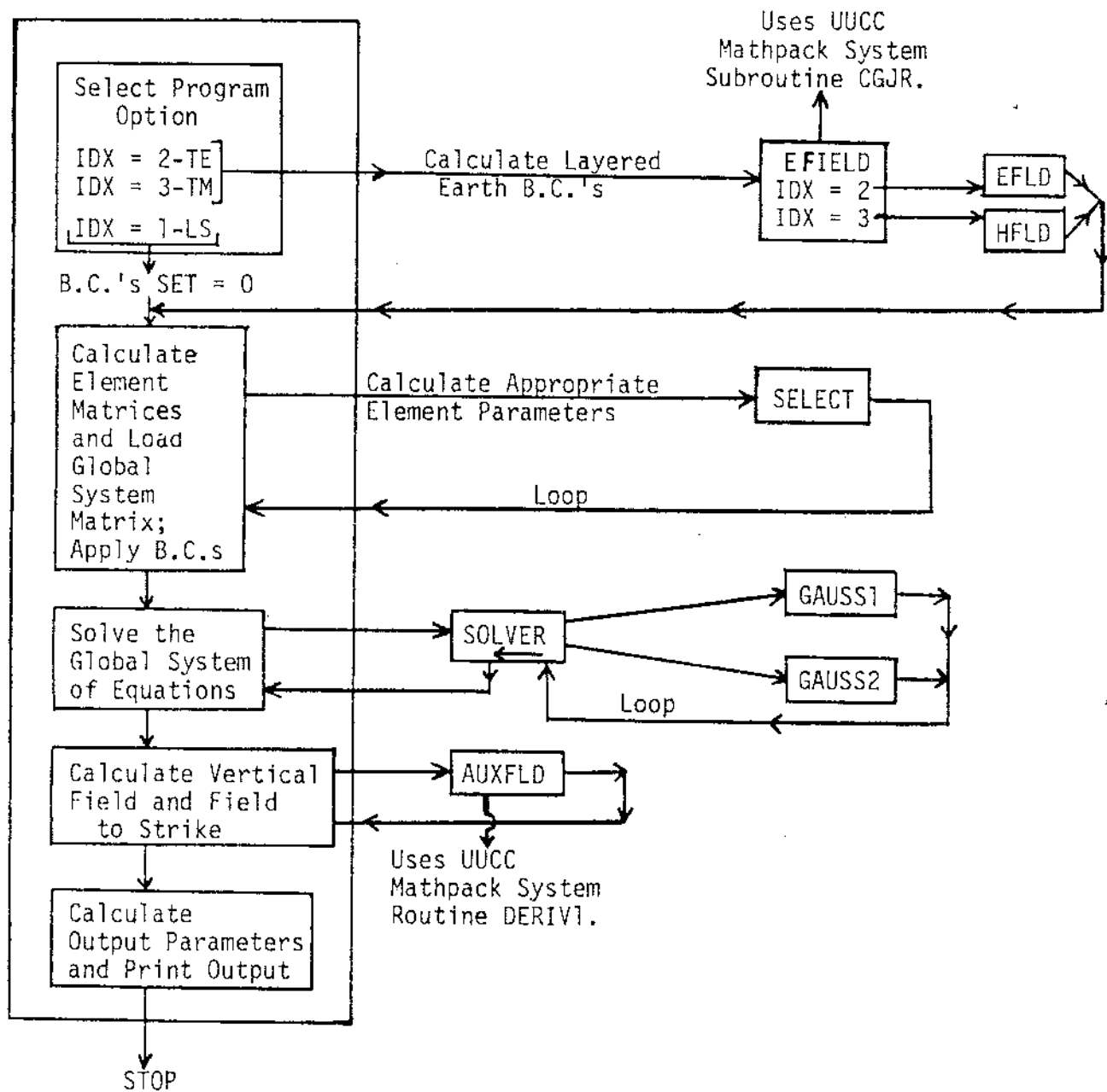


Figure 1. Flow diagram of program architecture.

B. Explanation of PARAMETERS

There are three parameters which are set during compilation of the program which must be tailored to the run. A description of these parameters, with a list of the subroutines in which they occur in PARAMETER statements follows.

IP1 = number of nodes in the z-direction. Must be greater than or equal to the actual number of nodes vertically in the mesh.

IP4 = number of nodes in the x-direction. Must be greater than or equal to the actual number of nodes horizontally in the mesh.

NLAYR = number of layers (including the half-space) of the 1-D earth model bounding the right and left sides of the mesh. This parameter is only used when calculating the boundary conditions to be applied to the sides of the mesh for MT modeling. It must be greater than or equal to the maximum number of layers on either side of the mesh. The earth models do not have to be the same on both sides of the mesh.

Subroutines in which these parameters appear:

<u>PARAMETER</u>	<u>SUBROUTINE NAME</u>
IP1	MAIN, SELECT, SOLVER, GAUSS1, GAUSS2, AUXFLD
IP4	MAIN, SELECT, SOLVER, GAUSS1, GAUSS2, AUXFLD
NLAYR	MAIN, EFIELD, EFLD, HFLD

C. Input list

Card 1: FORMAT(20I4)

IDX - Option parameter to select appropriate solution.

1 = 2-D line source (TURAM)

2 = TE Magnetotellurics (E-parallel mode)

3 = TM Magnetotellurics (H-parallel mode)

- NODEX - Number of nodes horizontally in the mesh (must be exact)
- NODEZ - Number of nodes vertically in the mesh (must be exact)
- NXX - Number of blocks of equal-sized elements horizontally.
(See figure 2 for the definition of a block which consists of a given number of elements with equal sized edges horizontally or vertically)
- NZZ - Number of blocks of equal-sized elements vertically.
- NRES - Number of conductivities in the mesh (including the air layer)
- M1 - Number of blocks vertically above $z = 0$ (air layer)
- NPRINT - 1 = just print input
0 = execute program
- LINE1 } Number of nodes horizontally from left edge of mesh to
LINE2 } where the line source(s) is/are positioned. Default value = 0. For examples, when modeling TURAM, if the effects of the return current part of the loop are not being considered, then LINE2 = 0. LINE1 gives a source current flowing in the + coordinate strike direction, while LINE2 gives a current source of equal strength flowing in the opposite direction.

Card 2: FORMAT(8F10.0)

Y(I) - Conductivities in the mesh - A, B, C... where

A, B, C... = conductivities in the mesh

↑ ↓ ↑

0, 1, 2... = mesh code for model input (see Card 7)

F - frequency (Hz)

Card 3: FORMAT(20I4)

NX(I) - Number of equal sized intervals DELX(I) in block I

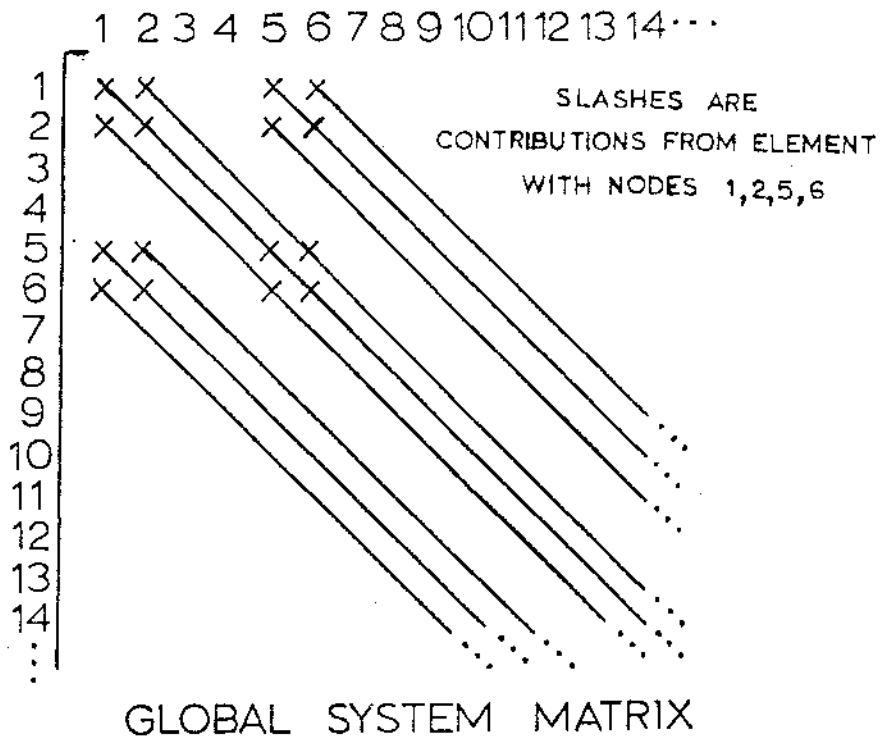
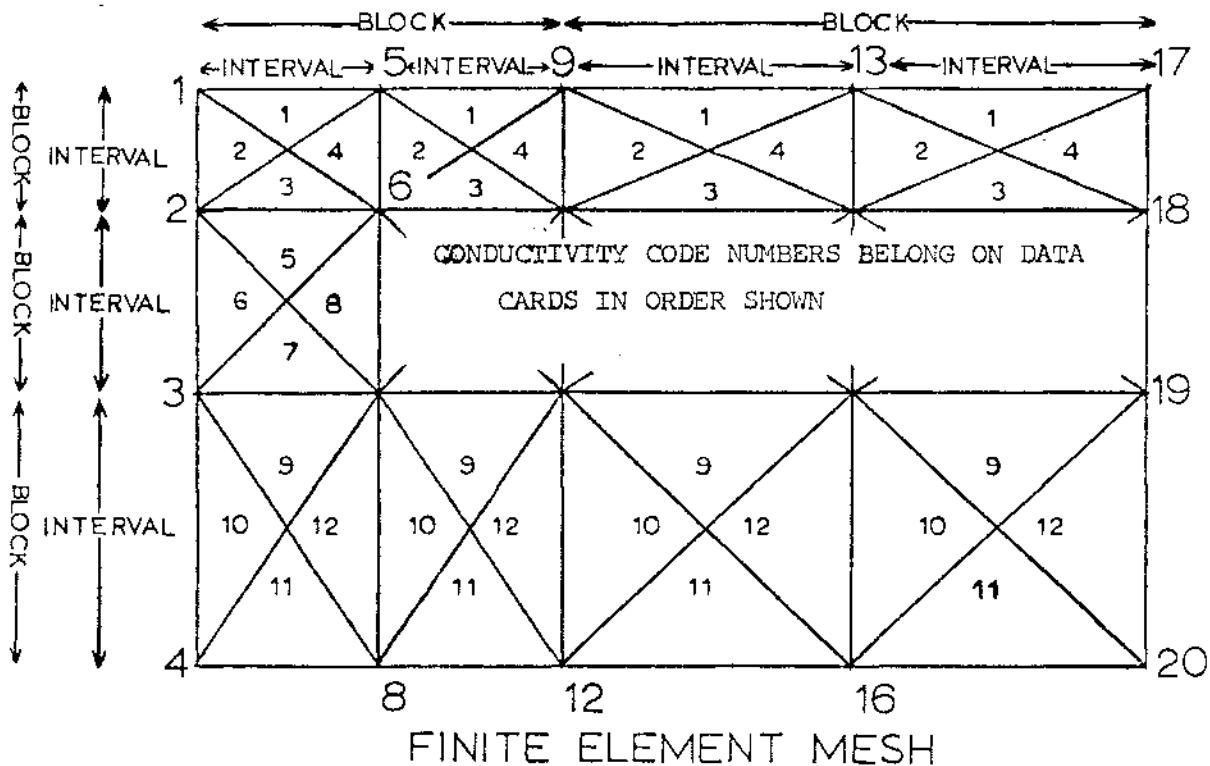


Figure 2. Finite element mesh structure, node numbering convention, conductivity code number convention. Association of node number with position in global matrix, as indicated by slashes, for element with nodes 1, 2, 5, 6.

horizontally.

Card 4: FORMAT(20I4)

DELX(I) - Size of the intervals in block I in meters

Card 5: FORMAT(20I4)

NZ(I) - Number of equal sized intervals DELZ(I) in block I
vertically.

Card 6: FORMAT(8F10.0)

DELZ(I) - Size of the intervals in block I in meters.

Card 7: FORMAT(80I1)

Model Deck - Input consists of the code numbers outlined under Card 2, with the ordering convention as illustrated in figure 2. Each interval in z comprises four data cards with conductivity code numbers appropriate to each triangular element punched on the cards in the order shown in figure 2. There are, therefore, NODEX-1 code numbers on each card. Note: The code number for the conductivity of air is always zero.

Card 8: FORMAT(8F10.0)

H(I), I=1, NLYR (NLYR=NLAYR-1) - depths from $z = 0$ to layer interfaces of 1-D boundary condition at left edge of mesh in meters.

Card 9: FORMAT(8F10.0)

P(I), I=1, NLAYR - resistivities of the successive layers on the left edge of the mesh.

Cards 10 and 11 - Same as cards 8 and 9 except these cards apply to the 1-D boundary condition at the right edge of the mesh.

D. Output list

Successive columns of output for the MT programs are:

Column

- 1 Distance from center of mesh to each node
- 2-5 Vertical field components (Re, Im, Magnitude, Phase)
- 6-9 Field components perpendicular to strike
- 10-13 Field components parallel to strike
- 14 Apparent resistivity
- 15 Negative of the phase of the impedance

Successive columns of output for the line source program are:

Column

- 1 Horizontal distance from the + line source to the nodes at $z = 0$
- 2-5 Transverse magnetic field components (Re, Im, Magnitude, Phase)
- 6-9 Vertical magnetic field components
- 10 Ratio of magnitude of transverse field to primary field
- 11 Ratio of magnitude of vertical field to primary field
- 12 Primary field

E. Notes on mesh design

Proper mesh design is important in obtaining meaningful (i.e. accurate) output from this program. Proper mesh design is an art that the programmer will gain with experience. The following rules of thumb concerning element dimensions will serve as guidelines. They are based on a unit of distance, the skin depth, defined as the distance in which the amplitude of a plane wave is attenuated by $1/e$ as it propagates through a homogeneous conducting medium. The formula is

Skin depth $\equiv \delta = 500 \sqrt{\rho/f}$ meters

where ρ = resistivity of the medium

f = frequency (Hz)

The rules of thumb are:

1) Element dimensions should not change from one element to the next by more than a factor of 3 to 5.

2) In the vicinity of a change in conductivity of the medium the element dimensions should be approximately $\delta/6$ in the medium where the element resides.

3) 2 to 3δ away from any variation in conductivity the element dimensions may be increased to the order of δ of the medium.

4) Vertical element dimensions may be increased approximately logarithmically (1, 3, 10, 30...) from the air-earth interface because of the exponential decay of the fields. The maximum vertical dimension of an element should still ideally be held to 1 to 2δ however.

5) The air layer for the TE-MT and line source problems should consist of 7 or 8 elements logarithmically increasing in vertical dimension from the air-earth interface, starting with about 10-100 m for frequencies <1 Hz and 1-10 m for frequencies >1 Hz.

6) A 1 or 2 node air layer is required computationally in this program (not theoretically) for the TM-MT case. A 2 m and 10 m layer has given good results.

7) Vertical mesh boundaries should ideally be extended 3 to 6 skin depths away from the nearest 2-D structure.

8) The bottom mesh boundary should ideally be 4 to 6 skin depths of the background conductivity from the air-earth interface.

9) When solving the line source problem, the mesh should be made "fine" in the region of the sources. A little experience will determine what "fine" is for a given problem. The mesh boundaries should be extended out to where the fields due to the source are approximately zero.

These rules of thumb will lead to inordinately large (and hence expensive) meshes much of the time. The programmer's task is then to cut corners where his experience tells him he may do so without too adversely affecting the accuracy of the results. Accuracy is checked by refining the mesh and checking convergence of the solutions. For information on convergence rates for linear approximation of the field see e.g. Strang and Fix (1973).

F. Notes on execution on the University of Utah Univac 1108 system

The first step in using the program is to compile the appropriate subroutines with the correct parameters as explained in III. B. The program then needs to be mapped into an absolute element for execution. The UUC Mathpack system library (UUC*MSLIB.) must be made available during this operation, since this is the system file where subroutines CGJR and DERIV1 reside. After compilation, a typical map sequence will be as follows:

```
@PREP, QUALIFIER*FILENAME.
@MAP, IS_^^^,QUALIFIER*FILENAME.ABSOLUTE
^IN^ QUALIFIER*FILENAME.MAIN
^LIB^ QUALIFIER*FILENAME.,UUC*MSLIB.
```

In this sequence, QUALIFIER*FILENAME. is the user's file in which the elements of the program reside. The LIB statement is crucial since this is the statement which makes the system routines in UUC*MSLIB. available to the program. ABSOLUTE is the name of the absolute element which is being created and stored in the file QUALIFIER*FILENAME.

The program is now ready to execute, given the proper data as described in III.C. A temporary word addressable data file must be assigned at execution for storage of the coefficient matrix while the solution is progressing. Data are stored on and retrieved from this file during execution through the use of NTRAN (See Appendix II).

A typical execution sequence would be:

```
@ASG,T,1., D///FILESIZE
      Data Deck
@XQT,OF,QUALIFIER*FILENAME.ABSOLUTE
@FIN
```

The first control card assigns the temporary (T option) data file. The number 1 is the unit number with which the program has been set up to identify this file. The D option makes the file word addressable. If there were no D option here, the file would be assigned as sector addressable. FILESIZE is the number of words of storage assigned to the temporary file. The formula for determining file size is:

$$\text{FILESIZE} = 2 * (\text{NBAND}^2 + \text{NNODE} * \text{NBAND})$$

where $\text{NBAND} = \text{IP1} + 2$

$$\text{NNODE} = \text{IP1} * \text{IP4}$$

We see, for example, that a 30 X 50 node problem would require the following storage:

$$2 * (32^2 + 30 * 50 * 32) = 98048 \text{ words}$$

The F option on the execution statement deserves mention. This option is used to suppress the counting of underflows which occur during execution. Since the program usually generates a large number of underflows, this option can produce a considerable savings in execution time. Overflows and divide checks will still be flagged. Neither of these should occur during normal

execution. Re-check the input carefully for proper format and consistency should these occur, as this is the usual source of error.

IV. Theory

This section will be concerned with a brief description of the application of the finite element method to the solution of equation 1. More detailed analysis may be found in Rijo (1977) and, e.g. Huebner (1975). Incorporation of boundary conditions and source parameters is also discussed as is the calculation of auxiliary field components which are obtained by appropriate manipulation of the field values obtained from the finite element solution.

A. Brief description of the finite element method.

The application of the finite element method to the solution of equation 1 hinges on the derivation of element matrix equations from the governing differential equation. We use the following technique. Re-write equation 1 in operational form:

$$2) \quad Lf = S$$

$$\text{where} \quad L \equiv \partial/\partial x(1/k \partial/\partial x) + \partial/\partial z(1/k \partial/\partial z) + p$$

Now approximate f by piecewise linear functions defined over triangular subregions e of the domain over which a solution is being sought--see figure 3.

We have

m = total number of triangular subdomains

$$3) \quad \tilde{f} = \sum_{e=1}^m \tilde{f}^e$$

where

$$4) \quad \tilde{f}^e = \alpha_1 + \alpha_2 X + \alpha_3 Z$$

Using 4, we obtain an equation for each of the field values at nodes i, j, k of the triangular region.

$$5) \quad \tilde{f}_n = \alpha_1 + \alpha_2 X_n + \alpha_3 Z_n$$

When these three equations are solved for the α 's and the results substituted into equation 4 we obtain

$$6) \quad \tilde{f}^e = N_i^e \tilde{f}_i + N_j^e \tilde{f}_j + N_k^e \tilde{f}_k$$

ORIGINAL EQUATION:

$$L F = S \quad L \equiv \frac{\partial}{\partial x} \left(\frac{1}{k} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{1}{k} \frac{\partial}{\partial z} \right) + P$$

APPROXIMATE F BY SOME \tilde{F} :

$$\text{THEN } L \tilde{F} - S = \epsilon$$

SUPPOSE F VARIES LINEARLY OVER
TRIANGULAR REGIONS e .

$$\text{THEN } \tilde{F}^e = \alpha_1 + \alpha_2 X + \alpha_3 Z$$

OR, IN TERMS

OF NODAL
VALUES

$$\tilde{F}_n = \alpha_1 + \alpha_2 X_n + \alpha_3 Z_n$$

$$n = i, j, k$$

FINALLY:

$$\tilde{F}^e = N_i^e \tilde{F}_i + N_j^e \tilde{F}_j + N_k^e \tilde{F}_k$$

$$N_i^e = \frac{1}{2\Delta} (a_i + b_i X + c_i Z)$$

$$a_i = x_j z_k - x_k z_j$$

$$b_i = z_j - z_k$$

$$c_i = x_k - x_j$$

N_j^e, N_k^e are obtained through cyclic permutation of
the subscripts i, j, k .

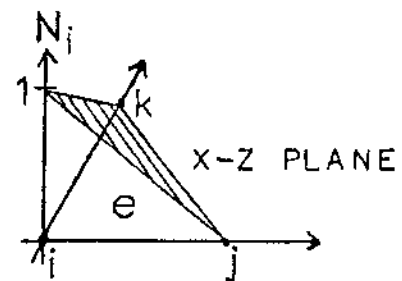
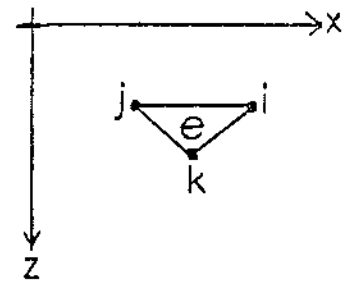


Figure 3. Derivation of linear approximation of an unknown field over a triangular sub-region e .

where N_n^e , $n = i, j, k$ is defined in figure 3.

N_j^e and N_k^e are obtained from the formula for N_i^e by cyclic permutation of the subscripts i, j, k . The N_n^e are a local, linearly independent, complete set of basis functions in which the unknown linear variation of the field is expanded over the triangular sub-domain e . Having defined the form of the approximation over these sub-domains, we can now substitute 3 into 2 and obtain an expression for the error of approximation, ϵ .

$$7) \quad \tilde{L}f - s \equiv \epsilon$$

We now wish to minimize in some sense this approximation error. One way to do this is to force the inner product of the error with the basis functions to be zero over the region where the local basis is defined, i.e.

$$8) \quad \langle N_n^e, \epsilon \rangle = \iint_e N_n^e \epsilon dx dz \equiv 0 \quad n = i, j, k$$

Mathematically, this states that the error of approximation be orthogonal to the weight functions N_n^e over the sub-domain e . In our scheme, the basis functions and weight functions are the same so that the norm of the approximation error ϵ is minimized by this technique (Harrington, 1967). By carrying out the integrations appropriately (see Rijo, 1977, or Huebner, 1975) we obtain the matrix equations in figure 4. In evaluating 8 an integration by parts is performed which results in a line integral term around element boundaries. This term is associated with the Neumann boundary condition. Since we apply Dirichlet boundary conditions to all mesh edges for the three problems which are being considered, we disregard this term.

If the finite element mesh is set up in a regular fashion so that four triangular elements combine to form a quadrilateral element, we can reduce the size of the global system of equations by 20% by eliminating the unknown field value associated with node five of the quadrilateral element in figure 4.

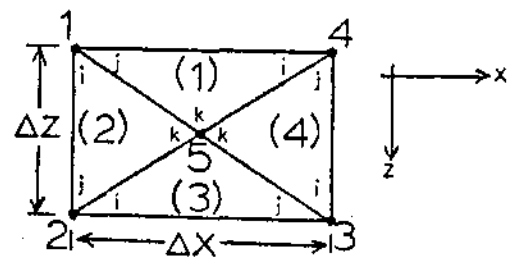
WE WISH TO MINIMIZE IN SOME SENSE THE ERROR ϵ .

LET'S TAKE $\langle N_n^e \epsilon \rangle = \iint_e N_n^e \epsilon dx dz = 0$
 $n = i, j, k$

CARRYING OUT THE INTEGRATIONS:

$$\left\{ \frac{-1}{4K\Delta} \begin{bmatrix} B_i^2 \cdot C_i^2 & B_i B_j \cdot C_i C_j & B_i B_k \cdot C_i C_k \\ & B_j^2 \cdot C_j^2 & B_j B_k \cdot C_j C_k \\ & & B_k^2 \cdot C_k^2 \end{bmatrix} + \frac{P\Delta}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \right\} \begin{bmatrix} \tilde{F}_i \\ \tilde{F}_j \\ \tilde{F}_k \end{bmatrix} = \begin{bmatrix} S_i \\ S_j \\ S_k \end{bmatrix}$$

Δ = Area of triangular element
 IT'S MORE EFFICIENT TO DEAL WITH QUADRILATERAL ELEMENTS



THE COEFFICIENT MATRIX FOR THIS ELEMENT HAS THE FORM

$$\begin{bmatrix} A & B & C & D \\ E & F & G \\ H & I & J \\ K & L \\ M \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Figure 4. Matrix equations associated with a triangular element; formation of quadrilateral element and associated matrix equation from combination of four triangular elements.

This is done at the element level through a process known as static condensation (Huebner, 1975). When the triangular element matrix equations are combined additively in the appropriate manner a 5×5 system of equations is obtained. The coefficient matrix of this system has the form indicated symbolically in figure 4, and is written out explicitly in figure 5. The process of static condensation consists of partitioning the 5×5 matrix associated with the quadrilateral element into a 2×2 system as shown in figures 4 and 5, and then eliminating the field variable associated with the internal node. This is achieved through appropriate manipulation of the 2×2 system as indicated in figure 5. The resulting 4×4 system involves only field values at the external nodes of the quadrilateral element. The coefficient matrix of this system is written symbolically in figure 5. This is the matrix which is actually programmed and then loaded additively into the global system coefficient matrix in the appropriate locations, as illustrated in figure 2. The appropriate locations in the global matrix are determined by the node numbering scheme. Our node numbering scheme is also shown in figure 2. Notice that the flexibility to assign different physical property factors k , p in each triangular region of the quadrilateral element has been retained, so that the discretization is still over the triangular regions, not over the quadrilateral regions.

B. Incorporation of sources and boundary conditions

The incorporation of the infinite line source of current strength I for the line source problem is an easy matter with the finite element method. The term of the source vector associated with the node in the finite element mesh where the source is located is assigned a number equal to $+I$ if current is flowing in the $+$ coordinate direction, and a number equal to $-I$ if current is flowing in the $-$ coordinate direction. This result is easily

$$\begin{array}{r}
 \left. \begin{array}{l}
 \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) \quad - \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} - \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 \\
 \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_2 K_3} + \frac{\Delta X \Delta Z}{24} (P_2 + P_3) \quad \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} - \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_3} + \frac{\Delta X \Delta Z}{48} P_3 \\
 \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_3 K_4} + \frac{\Delta X \Delta Z}{24} (P_3 + P_4) \quad - \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} - \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_4} + \frac{\Delta X \Delta Z}{48} P_4 \\
 \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2)
 \end{array} \right\} \text{SYMMETRIC} \\
 \left. \begin{array}{l}
 \frac{1}{4} \left(\frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) \\
 - \frac{\Delta X}{\Delta Z} \frac{(K_1 + K_3)}{K_1 K_3} - \frac{\Delta Z}{\Delta X} \frac{(K_2 + K_4)}{K_2 K_4} \\
 + \frac{\Delta X \Delta Z}{24} (P_1 + P_2 + P_3 + P_4)
 \end{array} \right\}
 \end{array}$$

STATIC CONDENSATION:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

ELIMINATE X_2 :

$$[K_{11} - K_{12} K_{22}^{-1} K_{21}] X_1 = [R_1 - K_{12} K_{22}^{-1} R_2]$$

OR:

$$[\tilde{K}] X = \tilde{R}$$

WHERE

$$[\tilde{K}] = \begin{bmatrix} A - \frac{D^2}{M} & B - \frac{DG}{M} & -\frac{JD}{M} & C - \frac{LD}{M} \\ L - \frac{G^2}{M} & F - \frac{JG}{M} & -\frac{LG}{M} & \\ H - \frac{J^2}{M} & I - \frac{LJ}{M} & & \\ K - \frac{L^2}{M} & & & \end{bmatrix}$$

Figure 5. Explicit form of the 5 x 5 coefficient matrix associated with the quadrilateral element of figure 4. Process of static condensation to reduce this matrix to a 4 x 4.

derived, see Rijo (1977). I is arbitrarily assigned a value of 1 in the program. It is possible to assign a + and a - source, to simulate both long wires of a TURAM loop for example. The other terms of the source vector are zero.

When solving the TE or TM magnetotelluric problem, the entire source vector is set to zero, simulating a source at infinity. The source is introduced in the problem by applying a constant field value at the top of the mesh as a boundary condition.

Dirichlet boundary conditions are applied at all external mesh boundaries when solving the problems of Table 1. The method used in the program to incorporate these conditions once the global system has been formed is illustrated in figure 6. The following boundary conditions are applied:

Line Source problem: Homogeneous (zero) boundary conditions are specified at all external boundaries, with the assumption that the source(s) is/are located sufficiently far from the mesh edges so that the fields are approximately zero there.

TE-MT problem: A homogeneous boundary condition is applied at the bottom edge of the mesh. At each side of the mesh, fields due to a normally incident plane wave over a layered earth structure (one side may differ from the other) are calculated. The amplitude of the incident E-field is arbitrarily chosen as unity at the earth's surface, oriented in the + x (strike) direction. The resultant E-fields from this calculation are applied as boundary conditions on the sides of the mesh. The field values at the upper left and upper right corners of the mesh are then extended to the center of the mesh to give the constant field

Suppose we want to fix $f_1, f_3 = \beta_1, \beta_3$

$$\begin{bmatrix} k_{11} \times 10^{15} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} \times 10^{15} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} \beta_1 k_{11} \times 10^{15} \\ S_2 \\ \beta_3 k_{33} \times 10^{15} \\ S_4 \end{bmatrix}$$

e.g. $k_{11} \times 10^{15} \underline{f_1} + k_{12}f_2 + k_{13}f_3 = \underline{\beta_1} \times k_{11} \times 10^{15}$

or $f_1 = \beta_1$

since $k_{11} \times 10^{15} > k_{1j} \quad j = 2, 3, 4$

Figure 6. Method of incorporating Dirichlet boundary conditions in the global system of equations.

at the top of the mesh which simulates the source for this problem.

TM-MT problem: The boundary conditions for this mode are applied in the same manner as for the TE mode, except that H-fields from the layered earth calculations are used. The source for the layered earth problem is again a normally incident plane wave with unit amplitude incident E-field, this time oriented in the + x (dip) direction. Notice the change in coordinate convention here. This is documented in the comment cards in the program. The reason for the change in convention is due to the manner in which the side boundary fields are calculated (see Appendix I).

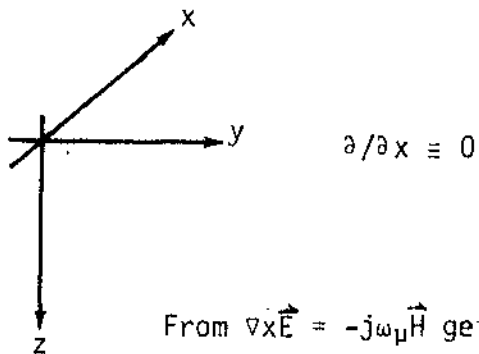
C. Calculation of auxiliary fields

The only field component we obtain with the finite element solution directly is the field (E or H depending on the problem) in the strike direction. Since other field components are required to calculate the parameters usually desired for interpretation, we must devise a numerical scheme for obtaining these from the field component in the strike direction. The auxiliary fields in this program are obtained through direct application of Maxwell's equations, as illustrated in figure 7. Note the different coordinate conventions for the three problems and the effects they have on the auxiliary field calculations. The derivatives indicated are evaluated numerically by fitting a piecewise polynomial to the mesh field values and then differentiating these analytically. This operation is performed by Univac 1108 system subroutine DERIV1 (see Appendix II).

The auxiliary field calculation for the TM-MT problem is approximately constant at the earth's surface, so that the Dirichlet boundary condition could be applied at the air-earth interface in the finite element solution. However, system subroutine DERIV1 will not calculate a derivative

TE-MT and LINE SOURCE ($e^{+j\omega t}$ time dependence)

Coordinate System



Non-zero field components

$$E_x, H_y, H_z$$

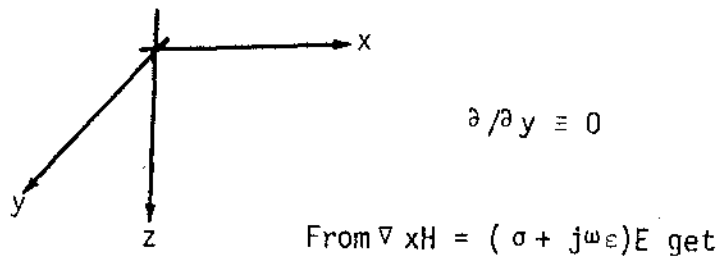
$$H_x = 0$$

$$H_y = -1/j\omega\mu \partial E_x / \partial z$$

$$H_z = +1/j\omega\mu \partial E_x / \partial y$$

TM-MT ($e^{+j\omega t}$ time dependence)

Coordinate System



Non-zero field components

$$H_y, E_x, E_z$$

$$E_y = 0$$

$$E_x = -1/(\sigma + j\omega\epsilon) \partial H_y / \partial z$$

$$E_z = +1/(\sigma + j\omega\epsilon) \partial H_y / \partial x$$

Figure 7. Calculation of auxiliary field components from the strike direction field obtained with the finite element solution.

very accurately near the end of a set of interpolated data points. Because of this, a one or two node air layer is incorporated in this problem, and the derivatives are calculated one node below the air-earth interface. For this reason, the first increment in z below the air-earth interface is made very small, say one to five meters.

References:

- Harrington, R. F., 1967. Matrix methods for field problems: Proc. of the IEEE, vol. 55, no. 2, pp. 136-149.
- Huebner, K. H., 1975. The Finite Element Method for Engineers: John Wiley & Sons, Inc., New York. 500 p.
- Rijo, L., 1977. Modeling of electric and electromagnetic data: Ph.D. thesis, University of Utah.
- Strang, G. and Fix, F. J., 1973. An Analysis of the Finite Element Method: Prentice-Hall, Inc., Englewood Cliffs, N. J., 306 p.
- Swift, C. M., Jr., 1967, A magnetotelluric investigation of an electrical conductivity anomaly in the southwestern United States: Ph.D. thesis, M.I.T.

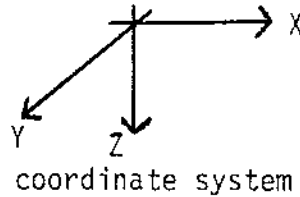
Appendix I - Calculation of the Layered Earth Fields for the Boundary

Condition for the Magnetotelluric Problem

Conventions:

$$e^{+j\omega t}$$

time dependence



coordinate system

	$E_i, H_i \uparrow \uparrow$	E_r, H_r	$k_1^2 = \omega^2 \epsilon_0 \mu_0$
\uparrow	$\overleftarrow{h_1}$	$E_{m_1}, H_{m_1} \uparrow \uparrow$	$k_2^2 = \omega^2 \mu_0 \epsilon_0 - j\omega \mu_0 \sigma_1$
h_2	$\overleftarrow{h_2}$	$E_{m_2}, H_{m_2} \uparrow \uparrow$	$k_3^2 = \omega^2 \mu_0 \epsilon_0 - j\omega \mu_0 \sigma_2$
\downarrow		$E_t, H_t \uparrow$	$k_4^2 = \omega^2 \mu_0 \epsilon_0 - j\omega \mu_0 \sigma_3$

Writing down the solutions in each layer, we have

$$1) \quad E_i = E_0 e^{-jk_1 z} \quad H_i = \frac{k_1}{\omega \mu_0} E_i$$

$$E_r = E_1 e^{+jk_1 z} \quad H_r = \frac{-k_1}{\omega \mu_0} E_r \quad z \leq 0$$

$$2) \quad E_{m_1} = (E_2^+ e^{-jk_2 z} + E_2^- e^{+jk_2 z}) \quad 0 < z \leq h_1$$

$$H_{m_1} = \frac{k_2}{\omega \mu_0} (E_2^+ e^{-jk_2 z} - E_2^- e^{+jk_2 z})$$

$$3) \quad E_{m_2} = (E_3^+ e^{-jk_3 z} + E_3^- e^{+jk_3 z}) \quad h_1 < z \leq h_2$$

$$H_{m_2} = \frac{k_3}{\omega \mu_0} (E_3^+ e^{-jk_3 z} - E_3^- e^{+jk_3 z})$$

$$4) \quad E_t = E_4^+ e^{-jk_4 z} \quad z > h_2$$

$$H_t = \frac{k_4}{\omega \mu_0} E_t$$

Assume $E_0 = 1$.Then $E|_{z=0} = 1 + E_1$

Let

$$Z_j \equiv \frac{\omega \mu_j}{k_j}$$

$$Z_{ij} \equiv \frac{Z_i}{Z_j} = \frac{\mu_i k_j}{\mu_j k_i} = \frac{k_j}{k_i} \quad \text{if } \mu_j = \mu_i$$

Apply continuity conditions on tangential E, H and then solve for the amplitude coefficients E_i^+ and E_i^- , $i = 1, 4$.

We obtain:

$$\begin{aligned} -E_1 + E_2^+ + E_2^- &= E_0 && \text{at } z = 0 \\ E_1 + Z_{12}[E_2^+ - E_2^-] &= E_0 \end{aligned}$$

$$\begin{aligned} E_2^+ A^{-1} + E_2^- A - E_3^+ B^{-1} - E_3^- B &= 0 \\ E_2^+ A^{-1} - E_2^- A - Z_{23}[E_3^+ B^{-1} - E_3^- B] &= 0 && \text{at } z = h_1 \end{aligned}$$

$$\begin{aligned} E_3^+ C^{-1} + E_3^- C - E_4^+ D &= 0 \\ E_3^+ C^{-1} - E_3^- C - Z_{34} E_4^+ D &= 0 && \text{at } z = h_2 \end{aligned}$$

where

$$A = e^{jk_2 h_1}; B = e^{jk_3 h_1}; C = e^{jk_3 h_2}; D = e^{jk_4 h_2}$$

Re-writing in matrix notation:

$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & Z_{12} & -Z_{12} & 0 & 0 & 0 \\ 0 & A^{-1} & A & -B^{-1} & -B & 0 \\ 0 & A^{-1} & -A & -Z_{23}B^{-1} & Z_{23}B & 0 \\ 0 & 0 & 0 & C^{-1} & C & -D^{-1} \\ 0 & 0 & 0 & C^{-1} & -C & -Z_{34}D^{-1} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2^+ \\ E_2^- \\ E_3^+ \\ E_3^- \\ E_4^+ \end{bmatrix} = \begin{bmatrix} E_0 \\ E_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This system of equations is set up and solved in subroutine EFIELD. Any number of layers may be handled by changing parameter NLAYR. Complex functions EFLD(Z) and HFLD(Z) implement equations 1 to 4 using the amplitude coefficients from EFIELD.

Appendix II - UUCS Univac 1108 System Subroutines

A. NTRAN

Ref. Univac 1108 Series Fortran V Library-Programmer's Reference,
UP - 7896, Univac Division, Sperry Rand Corp., 1971.

Highlights:

- 1) Drum storage is FASTRAND drum storage at the UUCS.
- 2) With FASTRAND storage, the D option must be used when assigning a file (see section III.F) to make the file word addressable, as required by the finite element program. If no D option is used, the file is automatically assigned as sector addressable; 1 sector equals 28 words. The D option allows FASTRAND files to be manipulated in the same manner as drum files.
- 3) Several operations may be grouped in a single NTRAN call.
- 4) It is important to remember that NTRAN provides for parallel processing. This means that NTRAN will perform user specified operations on a unit (file or tape) while at the same time allowing continued execution of the user's program. Operation 22 provides control over the parallel processing by stopping execution of the program until all NTRAN operations specified before operation 22 are complete. This prevents a program from using a file which is not yet ready, for example.
- 5) The CALL statement for NTRAN has the form

```
CALL NTRAN(UNIT,sequence of operations)
```

where UNIT is an integer constant or variable specifying the logical unit. The UNIT number for the program is currently set equal to 1, as described in section III.F. The sequence of operations is any list of I/O operations to be performed in order on the specified unit. An operation consists of a group of arguments. The first argument of an operation identifies the type of operation and is followed by parameters for the operation; these are fixed in number and order of occurrence by the type of operation. When referencing a FASTRAND file which has been assigned as word addressable, the current address for that file is the starting address only if the file has never been referenced in the current run. If the file was referenced before, the current address is the current address before the last CALL to the file plus the number of words transmitted or positioned in that CALL.

- 6) In order to check the status of transmission, each block of main storage in a FORTRAN program which is used for I/O has a block status word (an integer variable) associated with it; the name of the status word being specified in the argument list of an operation. When NTRAN is

called, the status words in the list of operations are set to -1, which indicates incomplete transmission, and the operations are then started in order for execution. When an interrupt occurs in an operation, the status word for that operation is set to a value which indicates the nature of completion, with the following possibilities:

STATUS WORD	NATURE OF COMPLETION
+N	normal completion; N equals the number of words transmitted.
-2	abnormal completion; attempt to read or write past an end of file marker.
-3	hardware errors, parity and character count errors (tape), or illegal unit specified. Legal units are all properly identified files and tapes.
-4	transmission aborted (previous operation had -2 or -3 status).

Notice that status -2 or -3 disables a unit for further NTRAN operations.

7) The following NTRAN I/O operations are used in the finite element program:

a) Write

The argument group is: 1,N,B,L

N = integer constant or variable specifying the number of words to be transmitted.

B = variable name from which data is to be written.

L = status word.

b) Read

The argument group is: 2,N,B,L

N = integer constant or variable which specifies the length of the main storage block which will receive the data.

B = variable into which data is to be read.

L = status word.

c) Position Drum

The argument group is: 6,N

N = integer constant or variable, positive or negative, which is added to the current drum address to form a new current drum address.

d) Rewind

The argument group is: 10

e) Wait and Unstack

The argument group is: 22

This operation causes a wait in NTRAN until all previous operations for the specified unit are complete before stacking any further operations or returning to the user's program. It also removes any operation which has caused an abnormal or error status and is still stacked against the specified unit, thereby enabling a file or tape which has been previously disabled, as described in 6).

The user must not change any argument of an argument group before the function is completed; i. e. before the status word for an operation has been changed from -1 to another value. All NTRAN operations are executed in sequence; completion of an operation, whether successful or unsuccessful, implies completion of all preceding operations.

B. DERIV1

Ref. Large Scale Systems Math-Pack-Programmer's Reference, UP-7542, Rev. 1, Univac Division, Sperry Rand Corp., 1970.

DERIV1 is a function subprogram which approximates the first derivative of a tabulated single valued function at one of the tabulated points. A parabolic approximation is used in which the tabulated function $f(x)$ is approximated by a parabola passing through three pivotal points. The derivative of the parabola is taken as the approximate value of the derivative of $f(x)$. The mathematical basis for the method is as follows:

Using a Taylor's series expansion of $f(x+h)$ we obtain the following expressions for $f(x_i+ah)$ and $f(x_i-h)$:

$$f(x_i+ah) = f_i + ahf'_i + \frac{a^2h^2}{2} f''_i + \dots$$

$$f(x_i-h) = f_i - hf'_i + \frac{h^2}{2} f''_i - \dots$$

where $f_i = f(x_i)$.

An approximate expression for f'_i is obtained by subtraction of the bottom equation from the top one followed by elimination of f''_i between the two to get

$$f'_i = \frac{1}{a(a+1)h} f(x_i+ah) - (1+a^2)f_i - a^2f(x_i-h) + e$$

where e represents the error. The error goes to zero as h^2 for any a .

The procedure for use of DERIV1 is as follows:

Calling statement: VAR = DERIV1(X,Y,N,XX,\$K)

where

DERIV1. is the name of the function and contains the approximation to the derivative at point XX upon return to the calling program. It is a floating point variable, as is XX. XX must equal an element of the X array, but must not equal X(1) or X(N).

X is an array of N independent variable values which may be unequally spaced, but must be stored monotonically increasing. X is a floating point array; N is an integer.

Y is an array of N dependent variable values. The elements of Y must be stored in correspondence to the elements of the X array. Y is also a floating point array.

K is a statement in the calling program to which control returns when one of the following conditions holds:

- 1) XX is not equal to an element of the X array.
- 2) $XX = X(1)$
- 3) $XX = X(N)$
- 4) overflow occurs in the computation of the first derivative approximation.

C. CGJR

Ref. Large Scale Systems Math-Pack Programmer's Reference, UP-7542, Rev. 1, Univac Division, Sprey Rand Corp., 1970.

CGJR is a subroutine which solves simultaneous equations, computes a determinant, inverts a matrix, or does any combination of these three operations, by using a Gauss-Jordan elimination technique with column pivoting.

The procedure for using CGJR is as follows:

Calling statement: CALL CGJR(A,NC,NR,N,MC,\$K,JC,V)

where

- A is the matrix whose inverse or determinant is to be determined. If simultaneous equations are solved, the last MC-N columns of the matrix are the constant vectors of the equations to be solved. On output, if the inverse is computed, it is stored in the first N columns of A. If simultaneous equations are solved, the last MC-N columns contain the solution vectors. A is a complex array.
- NC is an integer representing the maximum number of columns of the array A.
- NR is an integer representing the maximum number of rows of the array A.
- N is an integer representing the number of rows of the array A to be operated on.
- MC is the number of columns of the array A, representing the coefficient matrix if simultaneous equations are being solved; otherwise it is a dummy variable.
- K is a statement number in the calling program to which control is returned if an overflow or singularity is detected.
 - 1) If an overflow is detected, JC(1) is set to the negative of the last correctly completed row of the reduction and control is then returned to statement number K in the calling program.
 - 2) If a singularity is detected, JC(1) is set to the number of the last correctly completed row, and V is set to (0.,0.) if the determinant was to be computed. Control is then returned to statement number K in the calling program.

JC is a one dimensional permutation array of N elements which is used for permuting the rows and columns of A if an inverse is being computed. If an inverse is not computed, this array must have at least one cell for the error return identification. On output, JC(1) is N if control is returned normally.

V is a complex variable. On input REAL(V) is the option indicator, set as follows:

1. invert matrix
2. compute determinant
3. do 1. and 2.
4. solve system of equations
5. do 1. and 4.
6. do 2. and 4.
7. do 1., 2. and 4.

Notes on usage of row dimension arguments N and NR:

The arguments N and NR refer to the row dimensions of the A matrix. N gives the number of rows operated on by the subroutine, while NR refers to the total number of rows in the matrix as dimensioned by the calling program. NR is used only in the dimension statement of the subroutine. Through proper use of these parameters, the user may specify that only a submatrix, instead of the entire matrix, be operated on by the subroutine.

Appendix III - Program Listing

```

STODT*PWLS1(1),MAIN
  1   C   THIS PROGRAM SOLVES THE TWO DIMENSIONAL LINE SOURCE AND TE AND TM
  2   C   MAGNETOTELLURIC PROBLEM ACCORDING TO WHETHER THE INPUT PARAMETER
  3   C   IDX=1,2, OR 3. THE FINITE ELEMENT METHOD IS USED WITH LINEAR
  4   C   BASIS FUNCTIONS. MKS UNITS AND EXP(+J $\omega$ T) TIME DEPENDENCE IS
  5   C   ASSUMED. THE + COORDINATE DIRECTIONS FOR THE L.S. AND TE PROBLEMS
  6   C   ARE X-NORTH, Y-EAST, Z-DOWN. FOR THE TM PROBLEM THEY ARE X-EAST,
  7   C   Y-SOUTH, Z-DOWN. THE STRIKE DIRECTION IS ASSUMED N-S. THE ORIGIN
  8   C   IS AT THE LEFT EDGE OF THE MESH AT THE AIR-EARTH INTERFACE.
  9   C
 10   C   PARAMETER IP1=27,IP4=57,IP6=30
 11   C   $,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
 12   C   PARAMETER NLAYR=3
 13   C   PARAMETER NLYR=NLAYR-1,NK=NLAYR+1,MDIM=2*NLAYR
 14   C   COMPLEX CK(4),CP(4),CK12,CK13,CK14,CK23,CK24,CK34,CP12,CP23,CP34,
 15   C   $CP14,CP1234,A11,A12,A14,A15,A22,A23,A25,A33,A34,A35,A44,A45,A55,
 16   C   $E11,G11,C11,H11,E22,H22,C22,E33,G33,E44,BC(IP4),
 17   C   $BC1(IP4),S1(IP2,5),S2(IP5,IP3),S(IP3,IP3),R(IP2),R1(IP5),ZERO
 18   C   COMPLEX XE(MDIM),XK(NK),DUM(MDIM,MDIM),EFLD,HFLD
 19   C   DIMENSION H(NLYR),P(NLAYR)
 20   C   COMMON/BLK2/DUM,XE,XK
 21   C   COMMON/BLK3/H,W
 22   C   COMMON/BLK1/S,S2,R,R1,NT1,NF1,NCOL1,NS1,NEQ1
 23   C   COMMON/BLK4/WU,WE,RHO,CK,CP
 24   C   COMMON/BLK5/XX,ZZ,BC,BC1,NTART,NODEX,NODEX1,NODEZ
 25   C   COMMON/BLK6/IDX,L,M
 26   C   INTEGER NPT(IP1,4,IP4),NX(IP6),NZ(IP6)
 27   C   REAL DELTAX(IP4),DELTAZ(IP1),DELX(IP6),DELZ(IP6),RHO(IP1,4,IP4)
 28   C   I,Y(10),XX(IP4),ZZ(IP4),RE(IP4),AIE(IP4)
 29   C
 30   C   *****
 31   C
 32   C   ZERO=(0.,0.)
 33   C   ELARGE=10E+15
 34   C   READ 1  IDX,NODEX,NODEZ,NXX,NZZ,NRES,M1,NPRINT,LINE1,LINE2
 35   C   READ 4  (Y(I),I=1,NRES),F           @INPUT MESH CONDUCTIVITIES
 36   C   READ 1  (NX(I),I=1,NXX)
 37   C   READ 4  (DELX(I),I=1,NXX)
 38   C   READ 1  (NZ(I),I=1,NZZ)
 39   C   READ 4  (DELZ(I),I=1,NZZ)
 40   C   NODEX1=NODEX-1
 41   C   NODEZ1=NODEZ-1
 42   C   DO 29 I=1,NODEZ1
 43   C   DO 29 L=1,4
 44   C   READ 48 (NPT(I,L,J),J=1,NODEX1)
 45   C   CONTINUE
 46   C   PRINT 20  IDX
 47   C   PRINT 410 NODEX,NODEZ,NXX,NZZ,NRES,M1,NPRINT,LINE1,LINE2
 48   C   PRINT 420
 49   C   PRINT 300 (Y(I),I=1,NRES),F
 50   C   PRINT 430
 51   C   PRINT 1  (NX(I),I=1,NXX)
 52   C   PRINT 440
 53   C   PRINT 300 (DELX(I),I=1,NXX)
 54   C   PRINT 450
 55   C   PRINT 1  (NZ(I),I=1,NZZ)
 56   C   PRINT 460

```

```

57      PRINT 300 (DELZ(I),I=1,NZZ)
58      PRINT 470
59      C
60      C      THIS SECTION PRINTS OUT THE CODED MESH-IF THE MESH IS TOO LARGE
61      C      TO FIT ACROSS THE PAGE, JUST THE CENTER 43 ELEMENTS ARE PRINTED.
62      C
63      M=1
64      L2=NODEX1
65      IF(NODEX1.LE.42)GO TO 502
66      M=1+(NOUEX1-42)/2
67      L2=M+42
68      502 DO 501 I=1,NODEZ1
69          PRINT 62 (NPT(I,1,J),J=M,L2)
70          PRINT 63 (NPT(I,2,J),NPT(I,4,J),J=M,L2)
71      501 PRINT 62 (NPT(I,3,J),J=M,L2)
72      C
73      C      THIS SECTION PROPERLY POSITIONS THE AIR-EARTH INTERFACE.
74      C
75      NTART=1
76      IF(IDX.GT.2.AND.M1.EQ.0)GO TO 220
77      C
78      C      THIS PORTION NUMBERS THE NOUES PROPERLY IF THERE IS AN AIR LAYER.
79      C      NOTE--AT PRESENT THE TM MODE ALSO REQUIRES AN AIR LAYER TO AVOID
80      C      PROBLEMS ASSOCIATED WITH THE CALCULATION OF THE E-PERPENDICULAR
81      C      FIELD THROUGH NUMERICAL DIFFERENTIATION OF THE MESH VALUES
82      C      IN THE Z-DIRECTION.
83      C
84      DO 103 I=1,M1
85      103 NTART=NTART+NZ(I)
86          L=NTART
87          ZZ(L)=0.
88          DO 710 I=M1,1,-1
89              INI=NZ(I)
90              DO 710 J=1,INI
91                  ZZ(L-1)=ZZ(L)-DELZ(I)
92      710 L=L-1
93          L=NTART
94          M11=M1+1
95          DO 711 I=M11,NZZ
96              INI=NZ(I)
97              DO 711 J=1,INI
98                  ZZ(L+1)=ZZ(L)+DELZ(I)
99      711 L=L+1
100         GO TO 230
101      C
102      C      THIS PORTION WILL NUMBER THE NOUES PROPERLY FOR NO AIR LAYER.
103      C
104      220 L=NTART
105          ZZ(L)=0.
106          DO 240 I=1,NZZ
107              INI=NZ(I)
108              DO 240 J=1,INI
109                  ZZ(L+1)=ZZ(L)+DELZ(I)
110          L=L+1
111      240 CONTINUE
112      230 XE0=8.854333*1.0E-12
113          XMU0=3.14159265*4.0E-07

```

```

114          W=6.283185*F
115          IF (IDX,NE.1) GO TO 231
116      C
117      C      THIS SECTION SETS THE MESH BOUNDARY CONDITIONS FOR THE L.S.
118      C      PROBLEM. THE SOURCE(S) IS ASSUMED INCLUDED IN THE MESH.
119      C
120          DO 232 J=1,NODEX
121          BC(J)=ZERO
122          BC1(J)=ZERO
123      232  CONTINUE
124          PRINT 610
125          GO TO 233
126      C
127      C      THIS SECTION CALCULATES THE MESH BOUNDARY CONDITIONS FOR THE
128      C      TE AND TM PROBLEMS. A LAYERED EARTH BOUNDARY OF UP TO 9 LAYERS
129      C      + A HALF-SPACE (PARAMETER NLAYR=10) CAN BE ACCOMODATED. THE
130      C      BOUNDARY FIELDS ARE CALCULATED FOR AN INCIDENT E-FIELD AMPLITUDE
131      C      OF 1 AT THE EARTHS SURFACE (NOT AT THE TOP OF THE MESH)
132      C      POLARIZED IN THE + X DIRECTION. THIS IS THE REASON FOR THE CHANGE
133      C      IN COORDINATE CONVENTION IN CALCULATING THE TE AND TM MODES. THE
134      C      LEFT BOUNDARY IS CALCULATED FIRST AND MAY DIFFER FROM THE RIGHT.
135      C
136      231  DO 701 J=1,2
137          READ 4 (H(I),I=1,NLYR)      @INPUT INTERFACE DEPTHS FROM Z=0.
138          READ 4 (P(I),I=1,NLAYR)    @INPUT BOUNDARY RESISTIVITIES.
139          IF (J.EQ.1) PRINT 480
140          IF (J.EQ.2) PRINT 490
141          PRINT 300 (H(I),I=1,NLYR)
142          PRINT 300 (P(I),I=1,NLAYR)
143          XK(1)=CMPLX(W*SQRT(XEO*XMUO),.0)
144          DO 700 I=2,NK
145          XK(I)=CSGRT(CMPLX(0.,-W*XMUO/P(I-1)))
146      700  CONTINUE
147      C
148          CALL EFIELD
149      C
150          DO 702 I=1,NODEZ
151          Z=ZZ(I)
152          IF (IDX.EQ.3) GO TO 250
153          IF (J-1) 703,704,703
154      250  IF (J-1) 803,804,803
155          704 BC(I)=EFLO(Z)
156          GO TO 702
157          703 BC1(I)=EFLO(Z)
158          GO TO 702
159          804 BC(I)=HFLO(Z)
160          GO TO 702
161          803 BC1(I)=HFLO(Z)
162          702 CONTINUE
163          701 CONTINUE
164          IF (NPRINT.GE.1) GO TO 1009
165          PRINT 495
166      C
167      C      *****
168      C
169      C      THIS SECTION DETERMINES ELEMENT DIMENSIONS AND PARAMETERS NEEDED
170      C      IN SOLVING THE GLOBAL SYSTEM OF EQUATIONS FROM THE INPUT.

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```

171      C
172      233  NBAND=NODEZ+2
173          NNODE=NODEX*NODEZ
174          NOB=NNODE-NBAND
175          NBANI=NBAND-1
176          K=1
177          DO 28 I=1,NXX
178              NXI=NX(I)
179              DO 28 L=1,NXI
180                  DELTAX(K)=DELX(I)
181          28  K=K+1
182              K=1
183              DO 38 I=1,NZZ
184                  NZI=NZ(I)
185                  DO 38 L=1,NZI
186                      DELTAZ(K)=DELZ(I)
187          38  K=K+1
188      C
189      C      ASSOCIATE CONDUCTIVITIES WITH THE MESH INPUT CODE.
190      C
191          DO 27 J=1,NODEZ1
192              DO 27 L=1,NODEX1
193                  DO 27 M=1,4
194                      K=NPT(J,M,L)
195          27  RHO(J,M,L)=Y(K+1)
196      C
197      C      ZERO OUT THE SOLUTION VECTOR AND THE GLOBAL MATRIX.
198      C
199          DO 16 I=1,NNODE
200              R(I)=ZERO
201              DO 16 K=1,5
202          16  S1(I,K)=ZERO
203      C
204      C      THIS SECTION CALCULATES A 4 X 4 ELEMENT MATRIX FOR A RECTANGULAR
205      C      ELEMENT MADE UP OF 4 TRIANGULAR ELEMENTS. THE 5 X 5 MATRIX
206      C      OBTAINED FROM PROPER COMBINATION OF THE FOUR 3 X 3 TRIANGULAR
207      C      ELEMENT MATRICIES IS REDUCED TO A 4 X 4 MATRIX THROUGH A PROCESS
208      C      CALLED STATIC CONDENSATION. THIS ELIMINATES THE DEGREE OF FREEDOM
209      C      ASSOCIATED WITH THE INTERNAL NODE OF THE RECTANGLE, THUS REDUCING
210      C      THE SIZE OF THE GLOBAL SYSTEM MATRIX.
211      C
212          L1=0.
213          WU=W*XMUO
214          WE=W*XEO
215          DO 13 L=1,NODEX1
216              L1=L1+1
217              DO 13 M=1,NODEZ1
218                  DELZX=DELTAZ(M)/(2.*DELTAX(L))
219                  DELXZ=DELTAX(L)/(2.*DELTAZ(M))
220                  A=-(DELZX+DELXZ)/2.
221                  B=(DELZX-DELXZ)/2.
222                  C=DELTAX(L)*DELTAZ(M)/48.
223      C
224          CALL SELECT
225      C
226          CK12=(CK(1)+CK(2))/(CK(1)*CK(2))
227          CK13=(CK(1)+CK(3))/(CK(1)*CK(3))

```

```

228      CK14=(CK(1)+CK(4))/(CK(1)*CK(4))
229      CK23=(CK(2)+CK(3))/(CK(2)*CK(3))
230      CK24=(CK(2)+CK(4))/(CK(2)*CK(4))
231      CK34=(CK(3)+CK(4))/(CK(3)*CK(4))
232      CP12=CP(1)+CP(2)
233      CP23=CP(2)+CP(3)
234      CP34=CP(3)+CP(4)
235      CP14=CP(1)+CP(4)
236      CP1234=CP12+CP34
237      C
238      C      THESE ARE THE ELEMENTS OF THE 5 X 5 MATRIX.
239      C
240      A11=A*CK12+2.*C*CP12
241      A12=-B/CK(2)+C*CP(2)
242      A14=B/CK(1)+C*CP(1)
243      A15=DELXZ/CK(1)+DELZX/CK(2)+C*CP12
244      A22=A*CK23+2.*C*CP23
245      A23=B/CK(3)+C*CP(3)
246      A25=DELXZ/CK(3)+DELZX/CK(2)+C*CP23
247      A33=A*CK34+2.*C*CP34
248      A34=-B/CK(4)+C*CP(4)
249      A35=DELXZ/CK(3)+DELZX/CK(4)+C*CP34
250      A44=A*CK14+2.*C*CP14
251      A45=DELXZ/CK(1)+DELZX/CK(4)+C*CP14
252      A55=2.*(C*CP1234-DELXZ*CK13-DELZX*CK24)
253      C
254      C      THESE ARE THE ELEMENTS OF THE 4 X 4 CONDENSED MATRIX.
255      C
256      E22=A11-A15*A15/A55
257      G11=A12-A15*A25/A55
258      H22=-A15*A35/A55
259      C22=A14-A15*A45/A55
260      E11=A22-A25*A25/A55
261      C11=A23-A25*A35/A55
262      H11=-A25*A45/A55
263      E33=A33-A35*A35/A55
264      G33=A34-A35*A45/A55
265      E44=A44-A45*A45/A55
266      C
267      C      THIS SECTION LOADS THE ELEMENT MATRICIES INTO THE GLOBAL MATRIX.
268      C
269      S1(L1,1)=S1(L1,1)+E11
270      S1(L1,2)=S1(L1,2)+G11
271      S1(L1,4)=S1(L1,4)+C11
272      S1(L1,5)=S1(L1,5)+H11
273      L2=L1+1
274      S1(L2,1)=S1(L2,1)+E22
275      S1(L2,3)=S1(L2,3)+H22
276      S1(L2,4)=S1(L2,4)+C22
277      L3=L1+NODEZ
278      S1(L3,1)=S1(L3,1)+E33
279      S1(L3,2)=S1(L3,2)+G33
280      L4=L3+1
281      S1(L4,1)=S1(L4,1)+E44
282      13 L1=L1+1
283      IF(IDX.NE.1)GO TO 14
284      C

```

```

285 C     HERE THE SOURCE TERM(S) IS APPLIED AT THE APPROPRIATE NODE(S) FOR
286 C     THE L.S. PROBLEM. A CURRENT AMPLITUDE OF 1 IS ASSUMED.
287 C
288     LINE11=NODEZ*(LINE1-1)+NTART
289     LINE22=NODEZ*(LINE2-1)+NTART
290     R(LINE11)=CMPLX(1.,0.)
291     IF(LINE2.NE.0)R(LINE22)=-R(LINE11)
292 14    NJI=NNODE-NODEZ1
293     NJ1=NJI-1
294     L4=NODEX/2
295     L2=L4*NODEZ
296     L1=L2+1
297 C
298 C     THIS SECTION APPLIES THE BOUNDARY CONDITIONS TO THE MESH EDGES.
299 C     *****
300 C     TOP LEFT HALF OF MESH GETS VALUE AT TOP LEFT CORNER.
301 C
302     DO 12 I=1,L1,NODEZ
303     S1(I,1)=S1(I,1)*ELARGE
304 12    R(I)=BC(1)*S1(I,1)
305     L3=L1+NODEZ
306 C
307 C     TOP RIGHT HALF OF MESH GETS VALUE AT TOP RIGHT CORNER.
308 C
309     DO 1212 I=L3,NJI,NODEZ
310     S1(I,1)=S1(I,1)*ELARGE
311 1212 R(I)=BC1(1)*S1(I,1)
312 C
313 C     BOTTOM LEFT HALF OF MESH GETS VALUE AT BOTTOM LEFT CORNER.
314 C
315     DO 11 I=NODEZ,L2,NODEZ
316     S1(I,1)=S1(I,1)*ELARGE
317 11    R(I)=BC(NODEZ)*S1(I,1)
318     L2=L2+NODEZ
319 C
320 C     BOTTOM RIGHT HALF OF MESH GETS VALUE AT BOTTOM RIGHT CORNER.
321 C
322     DO 1111 I=L2,NNODE,NODEZ
323     S1(I,1)=S1(I,1)*ELARGE
324 1111 R(I)=BC1(NODEZ)*S1(I,1)
325 C
326 C     LEFT AND RIGHT SIDES GET LEFT AND RIGHT BOUNDARY VALUES.
327 C
328     DO 9 I=2,NODEZ1
329     S1(I,1)=S1(I,1)*ELARGE
330     R(I)=S1(I,1)*BC(I)
331     J=NJ1+I
332     S1(J,1)=S1(J,1)*ELARGE
333 9    R(J)=S1(J,1)*BC1(I)
334 C
335 C     THIS SECTION LOADS THE APPROPRIATE SIZE BLOCKS OF THE GLOBAL
336 C     MATRIX UNTO FASTRAND DRUM THROUGH THE USE OF THE UNIVAC 1108 I=0
337 C     DEVICE NTRAN. ONLY A SMALL PIECE OF THE GLOBAL SYSTEM WHICH IS
338 C     BEING WORKED ON AT ANY ONE TIME DURING THE SOLUTION OF THE
339 C     SYSTEM OF EQUATIONS IS STORED IN CORE.
340 C
341     L=1

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```

342         L3=NBAND
343         L2=1+NBAND/L3
344         L4=2*IP3*IP3
345         DO 900 J=1,L2.
346         DO 901 I=1,L3
347             S(I,1)=S1(L,1)
348             S(I,2)=S1(L,2)
349             S(I,NODEZ)=S1(L,3)
350             S(I,NBAN1)=S1(L,4)
351             S(I,NBAND)=S1(L,5)
352     901     L=L+1
353     900     CALL NTRAN(1,1,L4,S,IERR0,22)
354     C
355     CALL SOLVER(NODEZ,NODEX,NBAND)
356     C
357     C     THIS SECTION REORDERS THE ARRAY LABELING FOR THE X AND Z
358     C     COORDINATES OF THE NODES SO THAT THE UCC MATHPACK NUMERICAL
359     C     DERIVATIVE ROUTINES CAN BE USED IN SUBROUTINE AUXFLD.
360     C
361         L=2
362         XX(1)=.0
363         DO 500 I=1,NXX
364             INI=NX(I)
365             DO 500 J=1,INI
366                 XX(L)=XX(L-1)+DELX(I)
367     500     L=L+1
368         L=2
369         ZZ(1)=0.
370         DO 505 I=1,NZZ
371             INI=NZ(I)
372             DO 505 J=1,INI
373                 ZZ(L)=ZZ(L-1)+DELZ(I)
374     505     L=L+1
375     C
376     CALL AUXFLD
377     C
378     C     THIS SECTION CHANGES THE LOCATION OF X=0 FROM THE LEFT EDGE OF
379     C     THE MESH TO ITS CENTER FOR PRINTOUT. THE UNIT OF DISTANCE IS KM
380     C     IF THE DISTANCE FROM THE EDGE OF THE MESH TO THE CENTER IS
381     C     GREATER THAN 9 KM. IF THE L.S. PROBLEM IS BEING SOLVED, X=0 IS
382     C     LOCATED AT THE NODE WHERE THE SOURCE CURRENT IS FLOWING IN
383     C     THE + DIRECTION.
384     C
385         J=NTART+NODEZ
386         NTART=(NODEX+1)/2
387         FA=1.
388         XNART=XX(NTART)
389         IF(XNART.GT.9000.)FA=1000.
390         IF(IDX.EQ.1)XNART=XX(LINE1)
391         DO 101 I=2,NODEX1
392     101     XX(I)=(XX(I)-XNART)/FA
393         IF(IDX.EQ.1)GO TO 80
394     C
395     C     CALCULATE MAGNITUDES, PHASES OF FIELD COMPONENTS AND APPARENT
396     C     RESISTIVITY AND PHASE OF THE IMPEDANCE FOR PRINTOUT WHEN SOLVING
397     C     FOR THE TE OR TM PROBLEM.
398     C

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```

399      DO 79 I=2,NODEX1
400      E11=BC(I)
401      H11=BC1(I)
402      AV=CABS(E11)
403      AH=CABS(H11)
404      A=AV/AH
405      AF=ATAN2(AIMAG(H11),REAL(H11))
406      IF(IDX.EQ.3)GO TO 77
407      ARHO=RHO(2,3,I)/AH
408      C      NOTE THIS IS THE NEGATIVE OF THE PHASE OF THE IMPEDANCE.
409      PHASE=57.29578*(KHO(2,4,I)-AF)
410      GO TO 76
411      77      ARHO=AH/RHO(2,3,I)
412      C      NOTE THIS IS THE NEGATIVE OF THE PHASE OF THE IMPEDANCE.
413      PHASE=57.29578*(AF-RHO(2,4,I))
414      78      ARHO=ARHO*ARHO/WU
415      RE11=REAL(E11)
416      IF(ABS(RE11).LT.10E-30)RE11=10E-30
417      AF=57.29578* AF
418      FA=57.29578*ATAN2(AIMAG(E11),RE11)
419      RE(I)=REAL(R(J))
420      AIE(I)=AIMAG(R(J))
421      J=J+NODEZ
422      RHODG=RHO(2,4,I)*57.29578
423      B=REAL(H11)
424      G=AIMAG(H11)
425      DELTZ4=REAL(E11)
426      DELT44=AIMAG(E11)
427      79      PRINT 45 XX(I),DELTZ4,DELT44,AV,FA,B,G,AH,AF,RE(I),AIE(I),
428      $RHO(2,3,I),RHODG,ARHO,PHASE
429      GO TO 1009
430      C
431      C      CALCULATE MAGNITUDES AND PHASES OF H-FIELD COMPONENTS, NORMALIZED
432      C      MAGNITUDES AND PRIMARY H-FIELD FOR PRINTOUT WHEN SOLVING THE
433      C      L.S. PROBLEM.
434      C
435      80      FCTR=1./FA
436      DO 89 I=2,NODEX1
437      IF(ABS(XX(I)).LT.10E-04) XX(I)=FCTR/6.283185*10E-20
438      HPRIM=FCTR/(-6.283185*XX(I))
439      HPRIM=ABS(HPRIM)
440      E11=BC(I)
441      B11=BC1(I)
442      AV=CABS(E11)
443      AH=CABS(H11)
444      DELTZ4=REAL(E11)
445      DELT44=AIMAG(E11)
446      B=REAL(H11)
447      G=AIMAG(H11)
448      AF=57.29578*ATAN2(DELT44,DELTZ4)
449      FA=57.29578*ATAN2(G,B)
450      AVNOR=AV/HPRIM
451      AHNOR=AH/HPRIM
452      89      PRINT 46 XX(I),B,G,AH,FA,DELTZ4,DELT44,AV,AF,AHNOR,AVNOR,HPRIM
453      C
454      C      *****
455      C

```

```

456 C *****
457 C
458 C
459 1 FORMAT(20I4)
460 2 FORMAT(2I4,3F10.0)
461 4 FORMAT(6F10.0)
462 20 FORMAT(1H,'IDX=1 IMPLIES INF. LINE SOURCE',/,',', 'IDX=2 IMPLIES TE M
463 MAGNETOTELLURICS',/,',', 'IDX=3 IMPLIES TM MAGNETOTELLURICS',/,',', 'PARAME
464 TER IDX=',I4)
465 45 FORMAT(1X,F8.3,12E8.3,2F8.3)
466 46 FORMAT(1X,F9.4,13(1X,E8.3))
467 48 FORMAT(80I1)
468 62 FORMAT(43(2X,I1))
469 63 FORMAT(1X,43(I1,1X,I1))
470 300 FORMAT(10E13.4)
471 410 FORMAT(1H,'NODES HORIZONTALLY=',I4,',', 'NODES VERTICALLY=',I4,/,',',
472 'BLOCKS HORIZONTALLY=',I4,',', 'BLOCKS VERTICALLY=',I4,/,',', 'NKES IN MES
473 SH=',I4,',', 'BLOCKS OF AIR=',I4,',', 'NPRINT=',I4,/,',', 'POS. OF 1ST L.S.=
474 S',I4,',', 'POS. OF 2ND L.S.=',I4)
475 420 FORMAT(1H,'MESH CONDUCTIVITIES AND FREQUENCY')
476 430 FORMAT(1H,'NUMBERS OF EQUAL SIZED INCREMENTS HORIZONTALLY')
477 440 FORMAT(1H,'SIZE OF THE INCREMENTS IN EACH HORIZONTAL BLOCK(M)')
478 450 FORMAT(1H,'NUMBERS OF EQUAL SIZED INCREMENTS VERTICALLY')
479 460 FORMAT(1H,'SIZE OF THE INCREMENTS IN EACH VERTICAL BLOCK(M)')
480 470 FORMAT(1H,'FINITE ELEMENT MESH CONDUCTIVITY CODE')
481 480 FORMAT(1H,'LEFT EDGE INTERFACE DEPTHS(M) AND RESISTIVITIES')
482 490 FORMAT(1H,'RIGHT EDGE INTERFACE DEPTHS(M) AND RESISTIVITIES')
483 495 FORMAT(1H,I1,1X,' DELX RE(FZ) IM(FZ) IFZI PHZ RE(FTR) IM(
484 FTR) IFTRI PHZ RE(FPL) IM(FPL) IFPLI PHZ AP.RES. -PHZ
485 S(2)')
486 610 FORMAT(1H,I1,1X,' DELX RE(HTR) IM(HTR) IHTRI PHZ RE(HZ
487 S) IM(HZ) IHZI PHZ IHTR/HPRI IHZ/HPRI HPR ')
488 1009 CONTINUE
489 STOP
490 END

```

<*>

STODT*PWLS1(1).SELECT

```

1      C      THIS SUBROUTINE CALCULATES THE APPROPRIATE COEFFICIENTS OF THE
2      C      HELMHOLTZ EQUATION, FOR INSEKTIION INTO THE ELEMENT MATRICIES
3      C      ACCORDING TO WHICH PROBLEM IS BEING SOLVED.
4      C
5      SUBROUTINE SELECT
6      PARAMETER IP1=27,IP4=57,IP6=30
7      COMPLEX CK(4),CP(4)
8      REAL RHO(IP1,4,IP4)
9      COMMON/BLK4/WU,WE,RHO,CK,CP
10     COMMON/BLK6/IDX,L,M
11     IF(IDX.EQ.1,OR.IDX.EQ.2)GO TO 10
12     IF(IDX.EQ.3)GO TO 20
13     10      DO 100 I=1,4
14             CK(I)=CMPLX(0.,WU)
15             CP(I)=-CMPLX(RHO(M,I,L),WE)
16     100     CONTINUE
17             GO TO 400
18     20      DO 200 I=1,4
19             CK(I)=CMPLX(RHO(M,I,L),WE)
20             CP(I)=-CMPLX(0.,WU)
21     200     CONTINUE
22     400     RETURN
23     END

```

<*>

STODT*PWLS1(1).SOLVER

```

1      C      THIS SUBROUTINE SOLVES THE GLOBAL SYSTEM OF EQUATIONS USING
2      C      GAUSSIAN ELIMINATION AND JACK SUBSTITUTION. THE GLOBAL MATRIX IS
3      C      DELT WITH IN NBAND X NBAND PIECES. THE SOLUTION IS RETURNED TO
4      C      MAIN IN ARRAY R(J),
5      C
6      SUBROUTINE SOLVER(N,M,NB)
7      PARAMETER IP1=27,IP4=57
8      S,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
9      COMPLEX S1,S,R,R1
10     COMMON/BLK1/S(IP3,IP3),S1(IP5,IP3),R(IP2),R1(IP5),NT1,NF1,NCOL1,NS
11     S1,NEQ1
12     CALL NTRAN(1,10,22)
13     NBNB2=2*IP3*IP3
14     NBK=NBNB2
15     CALL NTRAN(1,2,NBK,S,IERR0,22)
16     DO 8000 I=1,NB
17     R1(I)=R(I)
18     DO 8000 J=1,NB
19     8000 S1(I,J)=S(I,J)
20     NM1=N-1
21     NF1=1
22     NCOL1=NB
23     NEQ1=NB
24     NT1=NB
25     NWR2=2*NBK
26     NTOTAL = N*M
27     NNC3=NTOTAL/NB
28     NC2=2*NB
29     L=NB
30     NREST = NTOTAL-NNC3*NB
31     IF(NREST .EQ. 0) NNC3=NNC3-1
32     C
33     C      *****
34     C
35     DO 8002 K=2,NNC3
36     CALL NTRAN(1,2,NBK,S,IERR0,22)
37     DO 8003 I=1,NB
38     II=I+NB
39     L=L+1
40     R1(II)=R(L)
41     DO 8003 J=1,NB
42     8003 S1(II,J)=S(I,J)
43     CALL GAUSS1
44     CALL NTRAN(1,6,-NWR2,22)
45     LNC2=L-NC2
46     DO 8004 I=1,NB
47     II=I+NB
48     LL=LNC2+I
49     R(LL)=R1(I)
50     R1(I)=R1(II)
51     DO 8004 J=1,NB
52     S(I,J)=S1(II,J)
53     8004 S1(I,J)=S1(II,J)
54     CALL NTRAN(1,1,NBK,S,IERR0,22)
55     8002 CALL NTRAN(1,6,NBK,22)
56     IF(NREST .EQ. 0) NREST=NB

```

```

57         NBK=2*NREST*IP3
58         CALL NTRAN(1,2,NBK,S,IERR0,22)
59         DO 8005 I=1,NREST
60           II=I+NB
61           L=L+1
62           R1(II)=R(L)
63           DO 8005 J=1,NB
64     8005  S1(II,J)=S(I,J)
65           NF1=0
66           NT1=NB+NREST
67           CALL GAUSS1
68           NBKT=2*NBNB2+NBK
69           NBK=NBNB2
70           NS1=NTOTAL/NB-1
71           IF(NREST .EQ. NB) NS1=NS1-1
72           NREST=NREST+NB
73     C
74     C *****
75     C
76         CALL GAUSS2
77         NT1=NB
78         NS1=NS1-1
79         NF1=1
80         CALL NTRAN(1,6,-NBKT,22)
81     8008  IL=NS1*NB
82         CALL NTRAN(1,2,NBK,S,IERR0,22)
83         DO 8009 I=1,NB
84           IL=IL+1
85           R1(I)=R(IL)
86           DO 8009 J=1,NB
87     8009  S1(I,J)=S(I,J)
88         CALL GAUSS2
89         NS1=NS1-1
90         IF( NS1 .EQ. -1) RETURN
91         CALL NTRAN(1,6,-NWR2,22)
92         GO TO 8008
93         END

```

<*>

STOOT*PwLS1(1).GAUSS1

```

1      C      THIS SUBROUTINE PERFORMS THE FORWARD REDUCTION PART OF
2      C      GAUSSIAN ELIMINATION.
3      C
4      SUBROUTINE GAUSS1
5      PARAMETER IP1=27,IP4=57
6      $,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
7      COMPLEX B,S,A,SOLU,C
8      COMMON/BLK1/S(IP3,IP3),A(IP5,IP3),SOLU(IP2),B(IP5),NN,NF,NCOL,NS,
9      $NEG
10     MT=NCOL
11     IF (NF.EQ.0) MT=NN
12     DO 300 N=1,NN
13     B(N)=B(N)/A(N,1)
14     MM=MT-(1-NF)*(N-1)
15     IF (MM .EQ. 1) GO TO 300
16     IF (MM .GT. NCOL ) MM=NCOL
17     I=N
18     DO 275 L=2,MM
19     C=A(N,L)/A(N,1)
20     I=I+1
21     J=0
22     DO 250 K=L,MM
23     J=J+1
24     250 A(I,J)=A(I,J)-C*A(N,K)
25     B(I)=B(I)-A(N,L)*B(N)
26     275 A(N,L)=C
27     300 CONTINUE
28     RETURN
29     END

```

<*>

STODT*PWLS1(1).GAUSS2

```

1      C      THIS SUBROUTINE PERFORMS THE BACKWARD SUBSTITUTION PART OF
2      C      GAUSSIAN ELIMINATION.
3      C
4      SUBROUTINE GAUSS2
5      PARAMETER IP1=27,IP4=57
6      S,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
7      COMPLEX B,S,A,SOLU,D
8      COMMON/BLK1/S(IP3,IP3),A(IP5,IP3),SOLU(IP2),B(IP5),NN,NF,NCOL,NS,
9      SNEG
10     N=NN+1
11     DO 450 M=1,NN
12     N=N-1
13     MM=NCOL
14     LL=N+NS*NEQ
15     L=LL
16     D=B(N)
17     IF (NF .EQ. 0) MM=M
18     IF ( MM .EQ. 1) GO TO 450
19     IF (MM .GT. NCOL ) MM=NCOL
20     DO 425 K=2,MM
21     L=L+1
22     425 D=D-A(N,K)*SOLU(L)
23     450 SOLU(LL)=D
24     RETURN
25     END

```

<*>

STODT*PwLS1(1).AUXFLD

```

1      C      THIS SUBROUTINE CALCULATES THE VERTICAL FIELD AND THE FIELD
2      C      PERPENDICULAR TO STRIKE. THESE FIELDS ARE RELATED TO THE
3      C      APPROPRIATE DERIVATIVES OF THE FIELD PARALLEL TO STRIKE WHICH
4      C      WAS SOLVED FOR WITH THE FINITE ELEMENT METHOD. THE PROPER
5      C      RELATIONS ARE OBTAINED FROM MAXWELLS EQUATIONS IN THE PROPER
6      C      COORDINATE SYSTEM. THE AUXILIARY FIELD VALUES ARE RETURNED TO
7      C      MAIN IN ARRAYS BC(I) (VERTICAL FIELD) AND RC1(I) (FIELD
8      C      PERPENDICULAR TO STRIKE).
9      C
10     SUBROUTINE AUXFLD
11     PARAMETER IP1=27,IP4=57,IP6=30
12     S,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
13     COMPLEX UC,BC(IP4),BC1(IP4),R(IP2)
14     COMPLEX DUM1(IP3,IP3),DUM2(IP5,IP3),DUM3(IP5),DUM9(4),DUM10(4)
15     REAL RHO(IP1,4,IP4),XX(IP4),ZZ(IP4),RE(IP4),AIE(IP4)
16     COMMON/BLK1/DUM1,DUM2,R,DUM3,DUM4,DUM5,DUM6,DUM7,DUM8
17     COMMON/BLK4/WU,WE,RHO,DUM9,DUM10
18     COMMON/BLK5/XX,ZZ,BC,BC1,NTART,NODEX,NODEZ
19     COMMON/BLK6/IDX,LDUM,MOUM
20     IF(IDX.EQ.4)GO TO 100
21     NTAR=NTART
22     C
23     C      THIS STATEMENT PROVIDES FOR E PERPENDICULAR TO BE CALCULATED
24     C      1 NODE BELOW THE AIR-EARTH INTERFACE IN THE TM PROBLEM TO AVOID
25     C      NUMERICAL INSTABILITIES IN CALCULATION OF THE APPROPRIATE
26     C      DERIVATIVE. PUTTING THE NODE 1M BELOW THE SURFACE APPEARS TO
27     C      WORK AT THE HIGHER FREQUENCIES, THIS MUST BE DEEPENED AS
28     C      FREQUENCY DECREASES.
29     C
30     IF(IDX.EQ.3)NTAR=NTART+1
31     J=NTART
32     DO 10 I=1,NODEX
33     RE(I)=REAL(R(J))
34     AIE(I)=AIMAG(R(J))
35     RHO(2,3,I)=CABS(R(J))
36     RHO(2,4,I)=ATAN2(AIE(I),RE(I))
37     J=J+NODEZ
38     10    CONTINUE
39     DO 20 J=2,NODEX1
40     C
41     C      DERIV1 IS A UCC MATHPACK SYSTEM ROUTINE WHICH CALCULATES
42     C      DERIVATIVES FROM THE COEFFICIENTS OF A SPLINE FIT TO THE SET
43     C      OF DATA POINTS.
44     C
45     AH=DERIV1(XX,RE,NODEX,XX(J),%1009)
46     AV=DERIV1(XX,AIE,NODEX,XX(J),%1009)
47     IF(IDX.EQ.3)GO TO 70
48     UC=CMPLX(0.,WU)
49     GO TO 75
50     70    UC=CMPLX(RHO(NTART,1,J-1),WE)
51     75    BC(J)=CMPLX(AH,AV)/UC
52     20    CONTINUE
53     L1=1+NODEZ
54     DO 30 J=2,NODEX1
55     L=L1
56     DO 40 I=1,NODEZ

```

```
57         RE(I)=REAL(R(L))
58         AIE(I)=AIMAG(R(L))
59         L=L+1
60     40    CONTINUE
61         AH=DERIV1(ZZ,RE,NODEZ,ZZ(NTAR),$1009)
62         AV=DERIV1(ZZ,AIE,NODEZ,ZZ(NTAR),$1009)
63         IF(IDX.EQ.3)GO TO 80
64         UC=-CMPLX(0.,WU)
65         GO TO 85
66     80    UC=-CMPLX(RHO(NTART,1,J-1),WE)
67     85    BC1(J)=CMPLX(AH,AV)/UC
68         L1=L1+NODEZ
69     30    CONTINUE
70     1009  CONTINUE
71     100    RETURN
72         END
```

<*>

STODT*PWLS1(1).EFIELD

```

1      C      THIS SUBROUTINE CALCULATES THE AMPLITUDE COEFFICIENTS FOR THE
2      C      DOWN AND UPGOING WAVES IN EACH LAYER. THE COEFFICIENTS ARE MADE
3      C      AVAILABLE TO FUNCTIONS EFLO AND HFLO THROUGH ARRAY E(I). AN
4      C      INCIDENT E-FIELD AMPLITUDE OF 1 IN THE + X DIRECTION AT THE
5      C      AIR-EARTH INTERFACE IS ASSUMED.
6      C
7      C      SUBROUTINE EFIELD
8      C      PARAMETER NLAYR=3
9      C      PARAMETER NLYR=NLAYR-1,MDIM=2*NLAYR,NDIM=MDIM+1,NK=NLAYR+1
10     C      COMPLEX XE(MDIM,NDIM),E(MDIM),XK(NK),EXPN(NK),Z(NLAYR),
11     C      $DUM(MDIM,MDIM),TEM,TEM1,SOURCE,VAR,OPT
12     C      DIMENSION H(NLYR),JC(MDIM)
13     C      COMMON/BLK2/DUM,E,XK
14     C      COMMON/BLK3/H,W
15     C      EQUIVALENCE (E,XE(1,NDIM))
16     C      DATA TEM,TEM1/(0.,0.),(1.,0.)/
17     C
18     C      THIS IS THE UNIT AMPLITUDE INCIDENT E-FIELD SOURCE TERM. IT IS
19     C      EASILY CHANGED SO THAT SCALING MAY BE DONE FOR COMPARISON WITH
20     C      OTHER SOLUTIONS.
21     C
22     C      DATA SOURCE/(1.,0.)/
23     C      OPT=CMPLX(6.,0.)
24     C
25     C      SN=1.
26     C      L=1
27     C      L1=2
28     C      I1=1
29     C      IJ=3
30     C      J2=4
31     C
32     C      DO 10 I=1,NLYR
33     C      DO 20 J=1,2
34     C      JIDX=2*I+J-2
35     C      VAR=CMPLX(0.,-1.)*XK(I+J)*H(I)
36     C      EXPN(JIDX)=CEXP(VAR)
37     C      20 CONTINUE
38     C      10 CONTINUE
39     C
40     C      DO 30 J=1,NLAYR
41     C      Z(J)=XK(J+1)/XK(J)
42     C      30 CONTINUE
43     C
44     C      DO 40 I=1,MDIM
45     C      DO 40 J=1,NDIM
46     C      XE(I,J)=TEM
47     C      40 CONTINUE
48     C
49     C      XE(1,1)=-TEM1
50     C      XE(1,2)=TEM1
51     C      XE(1,3)=TEM1
52     C      XE(2,1)=TEM1
53     C      XE(2,2)=Z(1)
54     C      XE(2,3)=-Z(1)
55     C      XE(1,NDIM)=SOURCE
56     C      XE(2,NDIM)=SOURCE

```

```

57      C
58      DO 100 M=1,NLYR
59      DO 70 I=I1,IJ,2
60      VAR=EXPN(L)
61      DO 80 J=IJ,J2
62      DO 90 K=1,2
63      XE(J,I+K)=VAR*SN
64      VAR=1./VAR
65      JJ=J
66      KK=K
67      90  CONTINUE
68      80  CONTINUE
69      XE(JJ,I+KK)=-XE(JJ,I+KK)
70      SN=-SN
71      L=L+1
72      70  CONTINUE
73      XE(J2,J2)=XE(J2,J2)*Z(L1)
74      XE(J2,J2+1)=XE(J2,J2+1)*Z(L1)
75      L1=L1+1
76      I1=I1+2
77      IJ=IJ+2
78      J2=J2+2
79      100 CONTINUE
80      XE(MDIM-1,NDIM)=TEM
81      XE(MDIM,NDIM)=TEM
82      C  WRITE(6,300)
83      300 FORMAT(1H0,'SYSTEM OF EQUATIONS FROM THE BOUNDARY CONDITIONS')
84      C  WRITE(6,400) ((XE(I,J),J=1,7),I=1,6)
85      400 FORMAT(a(1X,7(1P2E9.2),//))
86      C
87      C  CGJR IS A UCC MATHPACK SYSTEM ROUTINE FOR APPLYING GAUSS-JORDAN
88      C  REDUCTION TO COMPLEX MATRICIES. OPT=6 SOLVES THE SYSTEM OF
89      C  EQUATIONS AND EVALUATES THE DETERMINAT OF THE COEFFICIENT MATRIX.
90      C
91      CALL CGJR(XE,NDIM,MDIM,MDIM,NDIM,$1000,JC,OPT)
92      C
93      C  WRITE(6,200)OPT
94      200 FORMAT(1H0,'DETERMINAT OF SYSTEM MATRIX=',1P2E10.2)
95      GO TO 999
96      1000 WRITE(6,500)JC(1),OPT
97      500 FORMAT(1X,'OVERFLOW OR SINGULARITY FROM SYSTEM SUBROUTINE CGJR',
98      $/,1X,'LAST CORRECTLY REDUCED ROW=',I3,5X,'LN(DET)--PARTIAL=',
99      $1P2E10.2)
100     999 RETURN
101     END

```

<***>

STODT*PWLS1(1).EFLD

```

1      C      EFLD IS A COMPLEX FUNCTION WHICH GIVES THE VALUE OF E AT DEPTH Z
2      C      USING THE AMPLITUDE COEFFICIENTS GENERATED IN SUBROUTINE EFIELD.
3      C      THIS FUNCTION REQUIRES DEPTH TO LAYER INTERFACES FROM THE
4      C      SURFACE Z=0 (NOT LAYER THICKNESSES) FOR ITS PROPER OPERATION.
5      C
6      C      COMPLEX FUNCTION EFLD(Z)
7      C      PARAMETER NLYR=3
8      C      PARAMETER NLYR=NLYR-1,MDIM=2*NLYR,NK=NLYR+1
9      C      COMPLEX E(MDIM),XK(NK),DUM(MDIM,MDIM),EXPN
10     C      DIMENSION H(NLYR)
11     C      COMMON/BLK2/DUM,E,XK
12     C      COMMON/BLK3/H,W
13     C
14     C      IF(Z.LT.0.)GO TO 10
15     C      DO 20 J=1,NLYR
16     C      IF(Z-H(J))15,15,20
17     C      15  JJ=J
18     C      GO TO 30
19     C      20  CONTINUE
20     C      EXPN=CMPLX(0.,-1.)*XK(NK)*Z
21     C      EXPN=CEXP(EXPN)
22     C      EFLD=E(MDIM)*EXPN
23     C      GO TO 40
24     C      30  EXPN=CMPLX(0.,-1.)*XK(JJ+1)*Z
25     C      EXPN=CEXP(EXPN)
26     C      EFLD=E(2*JJ)*EXPN+E(2*JJ+1)/EXPN
27     C      GO TO 40
28     C      10  EXPN=CMPLX(0.,-1.)*XK(1)*Z
29     C      EXPN=CEXP(EXPN)
30     C      EFLD=EXPN+E(1)/EXPN
31     C      40  RETURN
32     C      END

```

<*>

STGDT*PWS1(1).HFLO

```

1      C      HFLO IS A COMPLEX FUNCTION WHICH GIVES THE VALUE OF H AT DEPTH Z
2      C      USING THE AMPLITUDE COEFFICIENTS GENERATED IN SUBROUTINE EFIELD.
3      C      THIS FUNCTION REQUIRES DEPTH TO LAYER INTERFACES FROM THE
4      C      SURFACE Z=0 (NOT LAYER THICKNESSES) FOR ITS PROPER OPERATION.
5      C
6      C      COMPLEX FUNCTION HFLO(Z)
7      C      PARAMETER NLYR=3
8      C      PARAMETER NLYR=NLYR-1,MDIM=2*NLYR,NK=NLYR+1
9      C      COMPLEX E(MDIM),XK(NK),DUM(MDIM,MDIM),EXPN
10     C      DIMENSION H(NLYR)
11     C      COMMON/BLK2/DUM,E,XK
12     C      COMMON/BLK3/H,W
13     C
14     C      XMU0=3.14159 *0.0000004
15     C      IF(Z.LT.0.)GO TO 10
16     C      DO 20 J=1,NLYR
17     C      IF(Z-H(J))15,15,20
18     C      15 JJ=J
19     C      GO TO 30
20     C      20 CONTINUE
21     C      EXPN=CMPLX(0.,-1.)*XK(NK)*Z
22     C      EXPN=CEXP(EXPN)
23     C      HFLO=XK(NK)*(E(MDIM)*EXPN)/(W*XMU0)
24     C      GO TO 40
25     C      30 EXPN=CMPLX(0.,-1.)*XK(JJ+1)*Z
26     C      EXPN=CEXP(EXPN)
27     C      HFLO=XK(JJ+1)*(E(2*JJ)*EXPN-E(2*JJ+1)/EXPN)/(W*XMU0)
28     C      GO TO 40
29     C      10 EXPN=CMPLX(0.,-1.)*XK(1)*Z
30     C      EXPN=CEXP(EXPN)
31     C      HFLO=XK(1)*(EXPN-E(1)/EXPN)/(W*XMU0)
32     C      40 RETURN
33     C      END

```

<*>

Appendix IV - Sample Runs

Note: FORTRAN library function ATAN2, which is used in this program, returns a value for the inverse tangent in the range $-180,+180$. Therefore, a suitable additive adjustment to the phase as printed by the program may have to be made to obtain the correct phase values for the convention the user happens to be using.

DELX	RE(INTR)	IM(INTR)	INTR1	PHZ	RE(HZ)	IM(HZ)	INH1	PHZ	INTR/MPRI	INH/MPRI	MPR
-7.5000	-.103-05	.406-06	.111-05	.159+03	-.943-07	-.778-09	.943-07	-.180+03	.522-01	.444-02	.212-04
-4.5000	-.171-06	.169-05	.170-05	.958+02	.267-06	.179-05	.181-05	.815+02	.481-01	.512-01	.354-04
-1.5000	-.299-04	.148-04	.333-04	.154+03	-.408-04	.341-04	.532-04	.140+03	.314+00	.502+00	.106-03
-.5000	-.121-03	-.411-04	.128-03	-.161+03	-.288-03	.466-04	.292-03	.171+03	.401+00	.916+00	.318-03
-.2000	-.159-03	-.104-03	.190-03	-.147+03	-.817-03	.233-04	.817-03	.178+03	.239+00	.103+01	.796-03
-.1000	-.179-03	-.130-03	.222-03	-.144+03	-.155-02	-.225-05	.155-02	-.180+03	.139+00	.975+00	.159-02
-.0750	-.174-03	-.138-03	.221-03	-.142+03	-.203-02	-.129-04	.203-02	-.180+03	.104+00	.959+00	.212-02
-.0500	-.176-03	-.144-03	.227-03	-.141+03	-.323-02	-.272-04	.323-02	-.180+03	.715-01	.102+01	.318-02
-.0250	-.206-03	-.150-03	.255-03	-.144+03	-.646-02	-.446-04	.646-02	-.180+03	.400-01	.102+01	.637-02
-.0200	-.209-03	-.151-03	.257-03	-.144+03	-.790-02	-.487-04	.790-02	-.180+03	.324-01	.993+00	.796-02
-.0150	-.262-03	-.152-03	.303-03	-.150+03	-.107-01	-.534-04	.107-01	-.180+03	.285-01	.100+01	.106-01
-.0100	-.463-03	-.153-03	.487-03	-.162+03	-.159-01	-.586-04	.159-01	-.180+03	.306-01	.998+00	.159-01
-.0075	-.554-03	-.153-03	.574-03	-.165+03	-.210-01	-.614-04	.210-01	-.180+03	.271-01	.991+00	.212-01
-.0050	-.238-03	-.154-03	.284-03	-.147+03	-.334-01	-.645-04	.334-01	-.180+03	.891-02	.105+01	.318-01
-.0025	.154-02	-.154-03	.154-02	-.572+01	-.803-01	-.680-04	.803-01	-.180+03	.242-01	.126+01	.637-01
.0000	.225-02	-.153-03	.226-02	-.369+01	.226-03	-.724-04	.237-03	-.178+02	.226-21	.237-22	-.100+20
.0025	.153-02	-.152-03	.154-02	-.568+01	.808-01	-.768-04	.808-01	-.545-01	.242-01	.127+01	-.637-01
.0050	-.248-03	-.151-03	.289-03	-.148+03	.340-01	-.804-04	.340-01	-.136+00	.906-02	.107+01	-.318-01
.0075	-.584-03	-.149-03	.603-03	-.166+03	.220-01	-.834-04	.220-01	-.217+00	.284-01	.104+01	-.212-01
.0125	-.362-03	-.146-03	.390-03	-.158+03	.135-01	-.889-04	.135-01	-.377+00	.306-01	.106+01	-.127-01
.0175	-.228-03	-.142-03	.269-03	-.148+03	.101-01	-.936-04	.101-01	-.533+00	.296-01	.111+01	-.909-02
.0225	-.214-03	-.124-03	.248-03	-.150+03	.422-02	-.112-03	.422-02	-.153+01	.661-01	.113+01	-.374-02
.0275	-.224-03	-.104-03	.247-03	-.155+03	.254-02	-.127-03	.255-02	-.286+01	.105+00	.108+01	-.236-02
.0325	-.256-03	-.821-04	.269-03	-.162+03	.194-02	-.137-03	.194-02	-.406+01	.156+00	.113+01	-.172-02
.0375	-.332-03	-.320-04	.333-03	-.174+03	.138-02	-.144-03	.139-02	-.597+01	.298+00	.124+01	-.112-02
.0425	-.508-03	.259-04	.507-03	.177+03	.111-02	-.124-03	.111-02	-.638+01	.613+00	.135+01	-.827-03
.0475	-.697-03	.402-04	.698-03	.177+03	.987-03	-.981-04	.992-03	-.567+01	.954+00	.136+01	-.732-03
.0525	-.748-03	.383-04	.749-03	.177+03	.954-03	-.919-04	.958-03	-.550+01	.105+01	.134+01	-.715-03
.0575	-.803-03	.345-04	.803-03	.178+03	.910-03	-.868-04	.914-03	-.545+01	.115+01	.131+01	-.700-03
.0625	-.858-03	.292-04	.858-03	.176+03	.853-03	-.839-04	.857-03	-.561+01	.125+01	.125+01	-.685-03
.0675	-.909-03	.233-04	.909-03	.179+03	.782-03	-.839-04	.787-03	-.612+01	.136+01	.117+01	-.670-03
.0725	-.948-03	.188-04	.949-03	.179+03	.690-03	-.871-04	.703-03	-.711+01	.145+01	.107+01	-.656-03
.0775	-.970-03	.175-04	.970-03	.179+03	.603-03	-.925-04	.610-03	-.872+01	.151+01	.948+00	-.643-03
.0825	-.969-03	.205-04	.970-03	.179+03	.504-03	-.983-04	.514-03	-.110+02	.154+01	.815+00	-.630-03
.0875	-.948-03	.276-04	.946-03	.178+03	.411-03	-.103-03	.423-03	-.148+02	.153+01	.685+00	-.618-03
.0925	-.904-03	.371-04	.905-03	.178+03	.328-03	-.104-03	.345-03	-.176+02	.149+01	.568+00	-.606-03
.0975	-.852-03	.470-04	.853-03	.177+03	.261-03	-.102-03	.281-03	-.214+02	.143+01	.471+00	-.595-03
.1025	-.796-03	.558-04	.798-03	.176+03	.209-03	-.974-04	.231-03	-.249+02	.137+01	.395+00	-.584-03
.1075	-.740-03	.626-04	.743-03	.175+03	.171-03	-.906-04	.193-03	-.280+02	.130+01	.337+00	-.574-03
.1125	-.688-03	.672-04	.692-03	.174+03	.145-03	-.830-04	.167-03	-.298+02	.123+01	.296+00	-.563-03
.1175	-.494-03	.858-04	.498-03	.172+03	.801-04	-.504-04	.946-04	-.322+02	.962+00	.183+00	-.518-03
.1225	-.376-03	.512-04	.380-03	.172+03	.614-04	-.301-04	.684-04	-.261+02	.794+00	.143+00	-.479-03
.1275	-.381-03	.370-04	.303-03	.173+03	.587-04	-.209-04	.623-04	-.196+02	.681+00	.140+00	-.445-03
.1325	-.213-03	.175-04	.214-03	.175+03	.597-04	-.157-04	.617-04	-.148+02	.547+00	.158+00	-.391-03
.1375	-.185-03	.705-05	.165-03	.178+03	.598-04	-.165-04	.621-04	-.154+02	.475+00	.178+00	-.348-03
.1425	-.117-03	-.447-07	.117-03	-.180+03	.551-04	-.205-04	.588-04	-.204+02	.408+00	.206+00	-.285-03
.1475	-.921-04	-.363-06	.921-04	-.180+03	.471-04	-.230-04	.524-04	-.260+02	.301+00	.217+00	-.242-03
.1525	-.787-04	.128-05	.767-04	.179+03	.387-04	-.237-04	.454-04	-.314+02	.365+00	.216+00	-.210-03
.1575	-.656-04	.333-05	.656-04	.177+03	.313-04	-.231-04	.389-04	-.365+02	.354+00	.210+00	-.186-03
.1625	-.586-04	.523-05	.569-04	.175+03	.251-04	-.219-04	.333-04	-.410+02	.342+00	.200+00	-.166-03
1.0575	-.491-04	.681-05	.495-04	.172+03	.203-04	-.202-04	.286-04	-.449+02	.329+00	.190+00	-.151-03
1.2575	-.386-04	.887-05	.377-04	.166+03	.141-04	-.168-04	.219-04	-.501+02	.298+00	.173+00	-.127-03
2.2575	-.104-04	.616-05	.121-04	.149+03	.307-05	-.704-05	.768-05	-.665+02	.171+00	.109+00	-.705-04
5.2575	-.155-05	.113-05	.191-05	.144+03	.107-05	-.619-06	.628-06	-.802+02	.632-01	.207-01	-.303-04
8.2575	-.586-06	.386-06	.702-06	.147+03	.222-06	-.100-06	.103-06	-.775+02	.364-01	.534-02	-.193-04

DELX	RE(FZ)	IM(FZ)	IFZI	PHZ	RE(FTR)	IM(FTR)	IFRT	PHZ	RE(FPL)	IM(FPL)	IFPLI	PHZ	AP.RES.	-PHZ(Z)
-52.000	.740-06-	.107-04	.107-04-.860+02	.523-02-.895-04	.523-02-.968+00	.333-04	.333-04	.472-04	.449+02	10.309	45.870			
-22.000	.723-05-	.514+04	.519-04-.820+02	.550-02-.324-03	.551-02-.337+01	.367-04	.336-04	.498-04	.424+02	10.339	45.789			
-12.000	.153-04-	.968-04	.980-04-.810+02	.593-02-.665-03	.597-02-.630+01	.418-04	.343-04	.541-04	.394+02	10.390	45.694			
-10.000	.203-04-	.116-03	.118-03-.798+02	.609-02-.770-03	.614-02-.721+01	.434-04	.346-04	.555-04	.385+02	10.340	45.735			
-9.500	.226-04-	.123-03	.125-03-.795+02	.614-02-.803-03	.619-02-.745+01	.439-04	.347-04	.559-04	.383+02	10.334	45.751			
-9.600	.239-04-	.131+03	.133-03-.796+02	.618-02-.838-03	.624-02-.771+01	.444-04	.347-04	.564-04	.381+02	10.330	45.771			
-6.500	.238-04-	.140-03	.142-03-.804+02	.624-02-.875-03	.630-02-.799+01	.449-04	.348-04	.568-04	.378+02	10.323	45.793			
-6.000	.222-04-	.152-03	.154-03-.817+02	.629-02-.917-03	.636-02-.829+01	.455-04	.349-04	.574-04	.375+02	10.310	45.815			
-7.500	.190-04-	.167-03	.168-03-.835+02	.635-02-.963-03	.642-02-.862+01	.461-04	.350-04	.579-04	.372+02	10.287	45.830			
-7.000	.143-04-	.188-03	.189-03-.856+02	.642-02-1.01-02	.650-02-1.07-02	.468-04	.351-04	.585-04	.369+02	10.252	45.827			
-6.500	.803-05-	.217-03	.217-03-.877+02	.650-02-1.13-02	.659-02-1.21-02	.476-04	.351-04	.592-04	.364+02	10.200	45.785			
-6.600	.405-05-	.256-03	.256-03-.891+02	.662-02-1.17-02	.670-02-1.27-02	.485-04	.352-04	.599-04	.359+02	10.132	45.672			
-5.500	.315-05-	.315+03	.315-03-.894+02	.672-02-1.20-02	.683-02-1.31+02	.496-04	.352-04	.608-04	.353+02	10.052	45.447			
-5.000	.117-04-	.394+03	.394-03-.883+02	.687-02-1.27-02	.698-02-1.35+02	.510-04	.352-04	.620-04	.346+02	9.970	45.057			
-4.500	.388-04-	.500-03	.501-03-.856+02	.705-02-1.33-02	.717-02-1.47+02	.527-04	.353-04	.634-04	.338+02	9.909	44.447			
-4.000	.982-04-	.636-03	.644-03-.812+02	.727-02-1.38-02	.740-02-1.57+02	.556-04	.355-04	.654-04	.329+02	9.904	43.574			
-3.500	.210-03-	.806-03	.832-03-.754+02	.753-02-1.49-02	.766-02-1.65+02	.580-04	.360-04	.681-04	.320+02	10.008	42.429			
-3.000	.399-03-	.100-02	.108-02-.683+02	.784-02-1.36-02	.795-02-1.58+02	.613-04	.371-04	.717-04	.312+02	10.290	41.058			
-2.500	.702-03-	.121-02	.140-02-.599+02	.817-02-1.26-02	.827-02-1.47+02	.657-04	.392-04	.765-04	.308+02	10.843	39.582			
-2.000	.115-02-	.139-02	.181-02-.504+02	.850-02-1.07-02	.857-02-1.35+02	.709-04	.427-04	.827-04	.311+02	11.805	38.208			
-1.500	.180-02-	.149-02	.234-02-.397+02	.878-02-.772-03	.881-02-.502+01	.767-04	.483-04	.906-04	.322+02	13.399	37.225			
-1.000	.264-02-	.143-02	.301-02-.205+02	.899-02-.397-03	.890-02-.256+01	.827-04	.569-04	.103-03	.345+02	16.111	37.085			
-0.750	.316-02-	.150-02	.341-02-.224+02	.892-02-1.196-03	.883-02-1.28+01	.854-04	.626-04	.106-03	.362+02	18.230	37.495			
-0.500	.374-02-	.106-02	.389-02-.158+02	.862-02-.257-04	.862-02-1.171+00	.878-04	.693-04	.112-03	.383+02	21.334	38.472			
-0.375	.406-02-	.894+03	.415-02-.124+02	.844-02-.336-04	.844-02-.228+00	.888-04	.732-04	.115-03	.395+02	23.514	39.273			
-0.250	.440-02-	.693+03	.445-02-.895+01	.820-02-.540-04	.820-02-.378+00	.896-04	.773-04	.118-03	.408+02	26.387	40.437			
-0.125	.476-02-	.445-03	.476-02-.534+01	.780-02-.618-05	.786-02-.450-01	.901-04	.819-04	.122-03	.422+02	30.373	42.292			
0.000	.499-02-	.254+03	.499-02-.292+01	.740-02-.278-03	.740-02-.215+01	.904-04	.867-04	.125-03	.438+02	36.304	45.957			
0.125	.490-02-	.275-03	.496-02-.318+01	.682-02-.476-03	.684-02-.399+01	.906-04	.917-04	.129-03	.453+02	45.029	49.329			
0.250	.460-02-	.396-03	.482-02-.471+01	.651-02-.649-03	.654-02-.569+01	.910-04	.965-04	.133-03	.467+02	52.061	52.388			
0.375	.465-02-	.481+03	.467-02-.592+01	.625-02-.719-03	.627-02-.659+01	.914-04	.101-03	.136-03	.479+02	59.898	54.489			
0.500	.450-02-	.555-03	.454-02-.703+01	.599-02-.754-03	.604-02-.717+01	.919-04	.106-03	.140-03	.490+02	68.133	56.158			
0.750	.425-02-	.688-03	.431-02-.920+01	.563-02-.784-03	.568-02-.793+01	.932-04	.114-03	.147-03	.508+02	85.275	59.752			
1.000	.403-02-	.794+03	.411-02-.111+02	.530-02-.791-03	.542-02-.838+01	.947-04	.122-03	.155-03	.523+02	103.213	60.688			
1.500	.365-02-	.939-03	.377-02-.144+02	.501-02-.766-03	.506-02-.870+01	.981-04	.138-03	.169-03	.545+02	141.028	63.200			
2.000	.331-02-	.103-02	.347-02-.172+02	.477-02-.715-03	.482-02-.852+01	.102-03	.151-03	.182-03	.560+02	181.245	64.517			
2.500	.302-02-	.108-02	.321-02-.197+02	.460-02-.663-03	.465-02-.807+01	.108-03	.164-03	.195-03	.570+02	223.180	65.090			
3.000	.277-02-	.111-02	.298-02-.218+02	.440-02-.587-03	.452-02-.746+01	.111-03	.175-03	.207-03	.577+02	266.020	65.191			
3.500	.255-02-	.113-02	.279-02-.238+02	.439-02-.592-03	.442-02-.676+01	.115-03	.180-03	.218-03	.582+02	309.008	64.991			
4.000	.236-02-	.114-02	.262-02-.257+02	.432-02-.459-03	.435-02-.606+01	.119-03	.195-03	.229-03	.585+02	351.517	64.602			
4.500	.219-02-	.114-02	.247-02-.275+02	.427-02-.400-03	.429-02-.536+01	.124-03	.204-03	.239-03	.587+02	393.052	64.097			
5.000	.204-02-	.114-02	.234-02-.292+02	.425-02-.346-03	.425-02-.467+01	.128-03	.213-03	.248-03	.588+02	433.237	63.522			
5.500	.190-02-	.114-02	.222-02-.308+02	.421-02-.296-03	.422-02-.402+01	.133-03	.220-03	.257-03	.589+02	471.823	62.910			
6.000	.178-02-	.113-02	.211-02-.324+02	.419-02-.250-03	.420-02-.341+01	.137-03	.228-03	.266-03	.589+02	508.627	62.282			
6.500	.167-02-	.112-02	.201-02-.339+02	.418-02-.208-03	.418-02-.284+01	.142-03	.234-03	.274-03	.588+02	543.559	61.652			
7.000	.157-02-	.112-02	.193-02-.354+02	.417-02-.169-03	.418-02-.232+01	.146-03	.241-03	.282-03	.587+02	576.571	61.029			
7.500	.148-02-	.110-02	.185-02-.368+02	.417-02-.133-03	.418-02-.183+01	.151-03	.247-03	.289-03	.586+02	607.672	60.419			
8.000	.139-02-	.109-02	.177-02-.381+02	.416-02-.101-03	.418-02-.136+01	.155-03	.252-03	.296-03	.584+02	636.886	59.825			
8.500	.131-02-	.108-02	.170-02-.394+02	.418-02-.709-04	.419-02-.970+00	.159-03	.258-03	.303-03	.583+02	664.281	59.250			
9.000	.124-02-	.107-02	.164-02-.407+02	.419-02-.434-04	.419-02-.592+00	.164-03	.263-03	.310-03	.581+02	689.916	58.694			
9.500	.117-02-	.105-02	.158-02-.419+02	.421-02-.180-04	.421-02-.245+00	.168-03	.268-03	.316-03	.579+02	713.879	58.159			
10.000	.111-02-	.104-02	.152-02-.430+02	.422-02-.551-05	.422-02-.748-01	.172-03	.272-03	.322-03	.577+02	736.244	57.643			
12.000	.921-03-	.973-03	.134-02-.466+02	.429-02-.836-04	.430-02-.112+01	.188-03	.288-03	.344-03	.569+02	811.160	55.765			
22.000	.403-03-	.680-03	.790-03-.594+02	.472-02-.248-03	.472-02-.248+01	.251-03	.330-03	.415-03	.527+02	976.458	50.206			
52.000	.146-04-	.238-03	.239-03-.865+02	.529-02-.499-04	.529-02-.541+00	.324-03	.334-03	.466-03	.459+02	981.200	45.368			

DELA	RE(FZ)	IM(FZ)	FEZI	PHZ	RE(FTR)	IM(FTR)	IFTR	PHZ	RE(FPL)	IM(FPL)	IFPL	PHZ	AP.FES.	-PHZ(Z)
-52.000	.537-14	.274-14	.603-14	.153+03	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.956-03	9.980	44.990	44.990	
-22.000	.262-12	.184-14	.262-12	.180+03	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.985	44.985	44.985	
-12.000	.768-12	.427-14	.768-12	.180+03	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.979	45.003	45.003	
-10.000	.166-10	.381-14	.166-10	.131-01	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.967	45.016	45.016	
-9.560	.467-11	.254-14	.407-11	.357-01	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.963	45.012	45.012	
-9.000	.175-11	.364-14	.175-11	.119-01	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.958	45.004	45.004	
-8.500	.233-11	.688-16	.233-11	.219-02	.333-04	.333-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.954	44.985	44.985	
-8.000	.582-11	.133-15	.582-11	.180+03	.333-04	.332-04	.471-04	.450+02	.531-02-.885-07	.531-02-.955-03	9.951	44.956	44.956	
-7.500	.582-12	.266-16	.582-12	.262-02	.333-04	.332-04	.471-04	.449+02	.531-02-.885-07	.531-02-.955-03	9.952	44.914	44.914	
-7.000	.928-26	.178-16	.178-16	.900+02	.334-04	.332-04	.471-04	.449+02	.531-02-.885-07	.531-02-.955-03	9.961	44.858	44.858	
-6.500	.494-26	.888-17	.638-17	.900+02	.334-04	.332-04	.471-04	.446+02	.531-02-.885-07	.531-02-.955-03	9.981	44.793	44.793	
-6.000	.000	.000	.000	.000	.335-04	.332-04	.472-04	.447+02	.531-02-.885-07	.531-02-.955-03	10.020	44.732	44.732	
-5.500	.000	.000	.000	.000	.337-04	.333-04	.474-04	.447+02	.531-02-.885-07	.531-02-.955-03	10.082	44.692	44.692	
-5.000	.000	.000	.000	.000	.338-04	.335-04	.476-04	.447+02	.531-02-.885-07	.531-02-.955-03	10.173	44.733	44.733	
-4.500	.000	.000	.000	.000	.339-04	.338-04	.479-04	.449+02	.531-02-.885-07	.531-02-.955-03	10.292	44.896	44.896	
-4.000	.000	.000	.000	.000	.339-04	.342-04	.482-04	.453+02	.531-02-.885-07	.531-02-.955-03	10.432	45.272	45.272	
-3.500	.000	.000	.000	.000	.337-04	.349-04	.485-04	.460+02	.531-02-.885-07	.531-02-.955-03	10.566	45.967	45.967	
-3.000	.494-26	.888-17	.888-17	.900+02	.331-04	.356-04	.487-04	.471+02	.531-02-.885-07	.531-02-.955-03	10.639	47.107	47.107	
-2.500	.494-26	.688-17	.888-17	.900+02	.319-04	.365-04	.485-04	.488+02	.531-02-.885-07	.531-02-.955-03	10.556	48.830	48.830	
-2.000	.582-12	.169-15	.582-12	.180+03	.297-04	.371-04	.475-04	.513+02	.531-02-.885-07	.531-02-.955-03	10.156	51.275	51.275	
-1.500	.233-11	.613-15	.233-11	.151-01	.262-04	.369-04	.452-04	.546+02	.531-02-.885-07	.531-02-.955-03	9.200	54.574	54.574	
-1.000	.194-10	.165-13	.194-10	.180+03	.210-04	.346-04	.405-04	.508+02	.531-02-.885-07	.531-02-.955-03	7.358	58.810	58.810	
-.750	.698-11	.647-13	.698-11	.179+03	.175-04	.320-04	.365-04	.613+02	.531-02-.885-07	.531-02-.955-03	5.977	61.273	61.273	
-.500	.643-10	.172-11	.683-10	.179+03	.130-04	.276-04	.308-04	.639+02	.531-02-.885-07	.531-02-.955-03	4.252	63.856	63.856	
-.375	.279-10	.728-11	.289-10	.165+03	.114-04	.244-04	.269-04	.651+02	.531-02-.885-07	.531-02-.955-03	3.260	65.057	65.057	
-.250	.529-10	.442-10	.769-10	.351+02	.904-05	.202-04	.221-04	.659+02	.531-02-.885-07	.531-02-.957-03	2.201	65.898	65.898	
-.125	.279-09	.271-09	.389-09	.136+03	.660-05	.145-04	.160-04	.656+02	.531-02-.885-07	.531-02-.943-03	1.147	65.588	65.588	
.000	.316-07	.316-07	.447-07	.135+03	.414-05	.405-05	.579-05	.444+02	.531-02-.955-07	.531-02-.103-02	.151	44.404	44.404	
.125	.316-05	.316-05	.447-05	.135+03	.390-03	.381-03	.551-03	.437+02	.531-02-.878-06	.531-02-.948-02	1364.126	43.750	43.750	
.250	.258-07	.271-07	.375-07	.134+03	.399-03	.379-03	.550-03	.435+02	.531-02-.888-06	.531-02-.957-02	1361.043	43.482	43.482	
.375	.582-08	.446-08	.733-08	.375+02	.398-03	.373-03	.546-03	.432+02	.531-02-.885-06	.531-02-.955-02	1339.190	43.253	43.253	
.500	.386-09	.166-08	.113-08	.696+02	.390-03	.370-03	.542-03	.431+02	.531-02-.885-06	.531-02-.955-02	1320.072	43.112	43.112	
.750	.532-09	.397-10	.583-09	.391+01	.392-03	.364-03	.535-03	.429+02	.531-02-.885-06	.531-02-.955-02	1288.744	42.912	42.912	
1.000	.128-08	.101-10	.128-08	.450+00	.399-03	.360-03	.570-03	.428+02	.531-02-.885-06	.531-02-.955-02	1263.271	42.798	42.798	
1.500	.466-09	.446-12	.466-09	.180+03	.384-03	.354-03	.522-03	.427+02	.531-02-.885-06	.531-02-.955-02	1225.268	42.687	42.687	
2.000	.524-09	.135-12	.524-09	.148-01	.380-03	.350-03	.516-03	.427+02	.531-02-.885-06	.531-02-.955-02	1197.488	42.663	42.663	
2.500	.349-09	.712-14	.349-09	.117-02	.376-03	.347-03	.512-03	.427+02	.531-02-.885-06	.531-02-.955-02	1176.620	42.672	42.672	
3.000	.233-09	.355-13	.233-09	.190+03	.373-03	.344-03	.508-03	.427+02	.531-02-.885-06	.531-02-.955-02	1159.443	42.720	42.720	
3.500	.233-09	.284-13	.233-09	.700-02	.371-03	.343-03	.505-03	.428+02	.531-02-.885-06	.531-02-.955-02	1145.540	42.779	42.779	
4.000	.233-09	.426-13	.233-09	.180+03	.368-03	.341-03	.502-03	.428+02	.531-02-.885-06	.531-02-.955-02	1134.264	42.839	42.839	
4.500	.000	.000	.000	.000	.360-03	.340-03	.500-03	.429+02	.531-02-.885-06	.531-02-.955-02	1124.761	42.902	42.902	
5.000	.000	.000	.000	.000	.360-03	.340-03	.498-03	.430+02	.531-02-.885-06	.531-02-.955-02	1116.432	42.973	42.973	
5.500	.000	.000	.000	.000	.363-03	.339-03	.497-03	.430+02	.531-02-.885-06	.531-02-.955-02	1109.623	43.035	43.035	
6.000	.791-21	.142-13	.142-13	.900+02	.362-03	.338-03	.495-03	.431+02	.531-02-.885-06	.531-02-.955-02	1103.516	43.102	43.102	
6.500	.000	.000	.000	.000	.361-03	.338-03	.494-03	.432+02	.531-02-.885-06	.531-02-.955-02	1098.132	43.167	43.167	
7.000	.000	.000	.000	.000	.359-03	.336-03	.493-03	.432+02	.531-02-.885-06	.531-02-.955-02	1093.530	43.227	43.227	
7.500	.000	.000	.000	.000	.358-03	.337-03	.492-03	.433+02	.531-02-.885-06	.531-02-.955-02	1089.498	43.284	43.284	
8.000	.000	.000	.000	.000	.358-03	.337-03	.491-03	.433+02	.531-02-.885-06	.531-02-.955-02	1085.854	43.341	43.341	
8.500	.000	.000	.000	.000	.357-03	.337-03	.491-03	.434+02	.531-02-.885-06	.531-02-.955-02	1082.534	43.396	43.396	
9.000	.233-09	.284-13	.233-09	.700-02	.350-03	.337-03	.490-03	.434+02	.531-02-.885-06	.531-02-.955-02	1079.698	43.445	43.445	
9.500	.128-08	.135-12	.128-08	.180+03	.350-03	.337-03	.489-03	.435+02	.531-02-.885-06	.531-02-.955-02	1076.967	43.497	43.497	
10.000	.235-08	.289-12	.235-08	.180+03	.350-03	.337-03	.489-03	.435+02	.531-02-.885-06	.531-02-.955-02	1074.881	43.536	43.536	
12.000	.302-09	.881-13	.302-09	.167-01	.352-03	.337-03	.487-03	.437+02	.531-02-.885-06	.531-02-.955-02	1066.990	43.706	43.706	
22.000	.471-10	.674-13	.471-10	.821-01	.347-03	.337-03	.484-03	.441+02	.531-02-.885-06	.531-02-.955-02	1051.778	44.138	44.138	
52.000	.272-11	.860-13	.272-11	.178+03	.342-03	.338-03	.481-03	.446+02	.531-02-.885-06	.531-02-.955-02	1040.224	44.615	44.615	