

8/30/78 - Errata - "Documentation of a Finite Element Program for Solution of Geophysical Problems Governed by the Inhomogeneous 2-D Scalar Helmholtz Equation" by John A. Stodt

Page 2 - Table I

Problem

TM-MT  $p = -\hat{y}$  should read  $p = -\hat{z}$

Line Source  $S = I\sigma(x)\delta(z)$  should read  $S = I\delta(x)\delta(z)$

Page 3 - The second sentence should be replaced by "Any or all of  $\sigma$ ,  $\mu$ , and  $\epsilon$ , may be considered piecewise constant functions of position, with suitable modifications to the input. The program is currently set up for conductivity variation."

Page 4 - Paragraph 2

The statement "The Greenfield algorithm (see e.g. Swift, 1967 - Appendix 3) is then used to solve the linear system of equations," should be replaced by "A modification of Gaussian elimination applicable to the solution of symmetric banded systems (see Cook, 1974, p. 47) is then used to solve the linear system of equations.". The actual method implemented in this program does not make use of Greenfield's factorization scheme.

Reference: Cook, Robert D., 1974. Concepts and Applications of Finite Element Analysis. John Wiley and Sons, Inc., New York, 402 p.

Page 18 - Figure 4

The node numbering for the quadrilateral element is inconsistent with the global numbering scheme described in figure 2. Note this correction and the correction to the form of the coefficient matrix in the revised figure 4.

Page 20 - Figure 5

Rows 3 and 4 and Columns 3 and 4 of the 5 x 5 matrix must be interchanged to make it compatible with the corrected node numbering scheme of figure 4. Note also the modification to the reduced 4 x 4 matrix.

Page 25 - After re-examining the auxiliary field calculations for the TM mode problem, I see no reason to include the one or two node air layer for this problem as long as the derivatives are calculated one node below the air earth-interface. One can experiment with this by setting  $M1=0$  for no air layer.

Program listing - I have discovered an error in the program listing in the section where the element matrices are loaded into the global matrix. The programming error will not affect results from the program unless different conductivities are specified in triangles 1 and 3 of the quadrilateral element of figure 4. This would occur, for example, if one were trying to model a sloping interface. Corrections to the code are required on

pages 37 and 41 of the listing. These corrected pages are provided, with the corrected lines of code highlighted. Sample runs for a sloping contact for both the TE and TM magnetotelluric problems are included from both the uncorrected and corrected versions of the program.

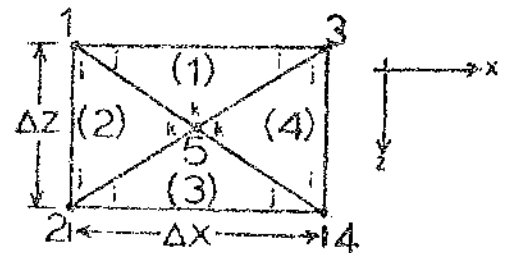
WE WISH TO MINIMIZE IN SOME SENSE THE ERROR  $\epsilon$ .

LET'S TAKE  $\langle N_n^e \epsilon \rangle = \iint_e N_n^e \epsilon dx dz = 0$   
 $n = i, j, k$

CARRYING OUT THE INTEGRATIONS:

$$\left\{ \frac{-1}{4K\Delta} \begin{bmatrix} b_i^2 + c_i^2 & b_i b_j + c_i c_j & b_i b_k + c_i c_k \\ b_j^2 + c_j^2 & & \\ b_k^2 + c_k^2 & & \end{bmatrix} + \frac{P\Delta}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \right\} \begin{bmatrix} F_i \\ F_j \\ F_k \end{bmatrix} = \begin{bmatrix} S_i \\ S_j \\ S_k \end{bmatrix}$$

IT'S MORE EFFICIENT TO DEAL WITH QUADRILATERAL ELEMENTS



THE COEFFICIENT MATRIX FOR THIS ELEMENT HAS THE FORM

$$\begin{bmatrix} A & B & C & D \\ E & F & G \\ H & I & J \\ K & L \\ M \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Figure 4. Matrix equations associated with a triangular element; formation of quadrilateral element and associated matrix equation from combination of four triangular elements.

$$\begin{array}{ccccccc}
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & -\frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & 0 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) \\
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & 0 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) & 0 \\
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) \\
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) \\
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) \\
 \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} P_1 & \frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} P_2 & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_2} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2) & \frac{1}{4} \left( \frac{\Delta X}{\Delta Z} + \frac{\Delta Z}{\Delta X} \right) \frac{1}{K_1} + \frac{\Delta X \Delta Z}{48} (P_1 + P_2)
 \end{array}$$

SYMMETRIC

$$\frac{1}{4} \left( \frac{\Delta Z}{\Delta X} + \frac{\Delta X}{\Delta Z} \right) \frac{(K_1 + K_2)}{K_1 K_2} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2)$$

$$-\frac{\Delta X (K_1 + K_2)}{\Delta Z (K_1 K_2)} - \frac{\Delta Z (K_2 + K_1)}{\Delta X (K_2 K_1)} + \frac{\Delta X \Delta Z}{24} (P_1 + P_2 + P_2 + P_1)$$

STATIC CONDENSATION:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

ELIMINATE  $X_2$ :

OR:

$$[K_{11} - K_{12} K_{22}^{-1} K_{21}] X_1 = [R_1 - K_{12} K_{22}^{-1} R_2]$$

$$[\tilde{K}] X_1 = \tilde{R}$$

WHERE

$$[\tilde{K}] = \begin{bmatrix} A - \frac{D^2}{M} & B - \frac{GD}{M} & C - \frac{JD}{M} & -\frac{LD}{M} \\ E - \frac{G^2}{M} & \frac{JG}{M} & F - \frac{LG}{M} & \\ & H - \frac{J^2}{M} & I - \frac{LJ}{M} & \\ & & & K - \frac{L^2}{M} \end{bmatrix}$$

Figure 5. Explicit form of the 5 x 5 coefficient matrix associated with the quadrilateral element of figure 4. Process of static condensation to reduce this matrix to a 4 x 4.

## Appendix III - Program Listing

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SIUDT*PWL51(1),MAIN1
 1      C      THIS PROGRAM SOLVES THE TWO DIMENSIONAL LINE SOURCE AND TE AN
 2      C      MAGNETOTELLURIC PROBLEM ACCORDING TO WHETHER THE INPUT PARAME
 3      C      IDX=1,2, OR 3. THE FINITE ELEMENT METHOD IS USED WITH LINEAR
 4      C      BASIS FUNCTIONS, MKS UNITS AND EXP(+J*WT) TIME DEPENDENCE IS
 5      C      ASSUMED. THE + COORDINATE DIRECTIONS FOR THE L.S. AND TE PROB
 6      C      ARE X-NORTH, Y-EAST, Z-DOWN. FOR THE TM PROBLEM THEY ARE X-EA
 7      C      Y-SOUTH, Z-DOWN, THE STRIKE DIRECTION IS ASSUMED N-S, THE ORI
 8      C      IS AT THE LEFT EDGE OF THE MESH AT THE AIR-EARTH INTERFACE.
 9      C
10      C      PARAMETER IP1=31,IP4=50,IP6=30
11      C      S,IP2=IP1*IP4,IP3=IP1+2,IP5=2*IP3
12      C      PARAMETER NLAYR=3
13      C      PARAMETER NLYR=NLAYR-1,NK=NLAYR+1,MDIM=2*NLAYR
14      C      COMPLEX CK(4),CP(4),CK12,CK13,CK14,CK23,CK24,CK34,CP12,CP23,CP
15      C      S,CP14,CP1234,A11,A12,A13,A15,A22,A24,A25,A33,A34,A35,A44,A45,A
16      C      S,E11,G11,C11,H11,E22,H22,C22,E33,G33,E44,BC(IP4),
17      C      S,SC1(IP4),S1(IP2,5),S2(IP5,IP3),S(IP3,IP3),R(IP2),R1(IP5),ZERO
18      C      COMPLEX XE(MDIM),XK(NK),DUM(MDIM,MDIM),EFLD,HFLD
19      C      DIMENSION H(NLYR),P(NLAYR)
20      C      COMMON/BLK2/DUM,XE,XK
21      C      COMMON/BLK3/H,W
22      C      COMMON/BLK1/S,S2,R,R1,NT1,NF1,NCOL1,NS1,NEG1
23      C      COMMON/BLK4/WU,WE,RHO,CK,CP
24      C      COMMON/BLK5/XX,ZZ,BC,BC1,NTART,NODEX,NODEX1,NODEZ
25      C      COMMON/BLK6/IDX,L,M
26      C      INTEGER NPT(IP1,4,IP4),NX(IP6),NZ(IP6)
27      C      REAL DELTAX(IP4),DELTAZ(IP1),DELX(IP6),DELZ(IP6),RHO(IP1,4,IP
28      C      S,Y(10),XX(IP4),ZZ(IP4),KE(IP4),AIE(IP4)
29      C
30      C      *****
31      C
32      C      ZERO=(0.,0.)
33      C      ELARGE=10E+15
34      C      READ 1 IDX,NODEX,NODEZ,NXX,NZZ,NRES,M1,NPRINT,LINE1,LINE2
35      C      READ 4 (Y(I),I=1,NRES),F          @INPUT MESH CONDUCTIVITIES
36      C      READ 1 (NX(I),I=1,NXX)
37      C      READ 4 (DELX(I),I=1,NXX)
38      C      READ 1 (NZ(I),I=1,NZZ)
39      C      READ 4 (DELZ(I),I=1,NZZ)
40      C      NODEX1=NODEX-1
41      C      NODEZ1=NODEZ-1
42      C      DO 29 I=1,NODEX1
43      C      DO 29 L=1,4
44      C      READ 4B (NPT(I,L,J),J=1,NODEX1)
45      C      CONTINUE
46      C      PRINT 20 IDX
47      C      PRINT 410 NODEX,NODEZ,NXX,NZZ,NRES,M1,NPRINT,LINE1,LINE2
48      C      PRINT 420
49      C      PRINT 300 (Y(I),I=1,NRES),F
50      C      PRINT 430
51      C      PRINT 1 (NX(I),I=1,NXX)
52      C      PRINT 440
53      C      PRINT 300 (DELX(I),I=1,NXX)
54      C      PRINT 450
55      C      PRINT 1 (NZ(I),I=1,NZZ)
56      C      PRINT 460

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228      CK14=(CK(1)+CK(4))/(CK(1)*CK(4))
229      CK23=(CK(2)+CK(3))/(CK(2)*CK(3))
230      CK24=(CK(2)+CK(4))/(CK(2)*CK(4))
231      CK34=(CK(3)+CK(4))/(CK(3)*CK(4))
232      CP12=CP(1)+CP(2)
233      CP23=CP(2)+CP(3)
234      CP34=CP(3)+CP(4)
235      CP14=CP(1)+CP(4)
236      CP1234=CP12+CP34
237
238 C      THESE ARE THE ELEMENTS OF THE 5 X 5 MATRIX,
239 C
240      A11=A*CK12+2.*C*CP12
241      A12=-B/CK(2)+C*CP(2)
242      → A13=B/CK(1)+C*CP(1)
243      A15=DELXZ/CK(1)+DELZX/CK(2)+C*CP12
244      A22=A*CK23+2.*C*CP23
245      → A24=B/CK(5)+C*CP(3)
246      A25=DELXZ/CK(3)+DELZX/CK(2)+C*CP23
247      → A33=A*CK14+2.*C*CP14
248      A34=-B/CK(4)+C*CP(4)
249      → A35=DELXZ/CK(1)+DELZX/CK(4)+C*CP34
250      → A44=A*CK34+2.*C*CP34
251      → A45=DELXZ/CK(3)+DELZX/CK(4)+C*CP34
252      A55=2.*(C*CP1234-DELXZ*CK13-DELZX*CK24)
253
254 C      THESE ARE THE ELEMENTS OF THE 4 X 4 CONDENSED MATRIX,
255 C
256      → E11=A11-A15*A15/A55
257      G11=A12-A15*A25/A55
258      → C11=A13-A15*A35/A55
259      → H11=-A15*A45/A55
260      → E22=A22-A25*A25/A55
261      → H22=-A25*A35/A55
262      → C22=A24-A25*A45/A55
263      E33=A33-A35*A35/A55
264      G33=A34-A35*A45/A55
265      E44=A44-A45*A45/A55
266
267 C      THIS SECTION LOADS THE ELEMENT MATRICES INTO THE GLOBAL MATRIX
268 C
269      S1(L1,1)=S1(L1,1)+E11
270      S1(L1,2)=S1(L1,2)+G11
271      S1(L1,4)=S1(L1,4)+C11
272      S1(L1,5)=S1(L1,5)+H11
273      L2=L1+1
274      S1(L2,1)=S1(L2,1)+E22
275      S1(L2,3)=S1(L2,3)+H22
276      S1(L2,4)=S1(L2,4)+C22
277      L3=L1+NODEZ
278      S1(L3,1)=S1(L3,1)+E33
279      S1(L3,2)=S1(L3,2)+G33
280      L4=L3+1
281      S1(L4,1)=S1(L4,1)+E44
282
283 *3 L1=L1+1
284      IF (IBX.NE.3) GO TO 14

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