

PW3DL

An Algorithm for Magnetotelluric Modeling of
Three-Dimensional Structures in Layered Earths

by

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ABSTRACT

PW3DL is a non-interactive algorithm for magnetotelluric modeling of three-dimensional (3-D) bodies in layered earths. It is based on the theory of integral equations. Green's functions appropriate to the particular layered earth are computed through cubic interpolation of Hankel transform tables constructed at the outset of each model run. ~~Limitations include the inability to model outcropping bodies and the inability to model structures at high induction numbers.~~ However, the user has the option of simulating structures with either two ^{or} or no vertical planes of symmetry. Although developed on a PRIME 400 computer, only minor modifications should be necessary for implementation on other systems.

→ outcropping bodies can be modelled if care is taken such that receiver positions are located on an interior cell corner.

In general receivers should be

- 1) centered on cell face
- 2) centered on cell edge
- 3) on corner of cell within body

Never put a receiver on an exterior cell boundary

INTRODUCTION

PW3DL is a non-interactive algorithm for magnetotelluric modeling of three-dimensional (3-D) bodies in layered earths. It was developed from previous code by G. W. Hohmann and S. C. Ting at the University of Utah which simulated 3-D bodies in half-spaces. PW3DL presently runs on the PRIME 400 of the University of Utah Research Institute, ^{✓ AND ON THE CRAY X-MP AT SDSC & THE} although implementation on other ^{MAX 785} systems should be quite straightforward.

METHOD OF SOLUTION

The theory of this algorithm is the method of integral equations (Hohmann, 1975; Ting and Hohmann, 1981; Wannamaker et al., 1984a). Inhomogeneity is replaced by an equivalent current distribution which is approximated by rectangular pulse basis functions. The rectangular cells in turn are comprised of cubic subcells. A matrix equation is constructed using the electric tensor Green's function appropriate to the particular layered earth, and it is solved for the vector current in each rectangular cell. Subsequently, scattered fields are found by integrating the electric and magnetic Green's functions over the scattering current.

Efficient evaluation of the tensor Green's functions is a major consideration in reducing computation time. The six electric and five magnetic Hankel transforms defining the secondary Green's functions possess complicated kernels involving time-consuming recursion and complex exponentiation (Wannamaker et al., 1984a). We find that tabulation and interpolation of these transforms is preferable to direct Hankel transformation using linear filters. By interpolating from a series of one-dimensional transform tables, we have reduced computer time in constructing the matrix, certainly the most time-consuming part of the problem, by a factor of three to four from that displayed by Ting and Hohmann (1981). Moreover, the user has the option of modeling bodies with two, ^{ONE} or no vertical planes of symmetry.

Some limitations to the algorithm are stated at the outset. First, since pulse basis functions are employed and the integration of the secondary Green's functions is crude (Wannamaker et al., 1984a), modeling of structure using cubic subcells larger than the depth of the structure is not recommended. Second, structures which transect layer interfaces cannot be modeled, although structures may touch any layer boundaries with the exception of the

) earth's surface. Finally, simulation of low-resistivity structures at high frequencies (i.e., high induction numbers) is dangerous. Specifically, our subsectional bases have difficulty representing divergence-free (non current-gathering) components to the internal currents (see Lajoie and West, 1976). For layer-body resistivity contrasts greater than about 100, we have achieved best results for rather tabular structures like sedimentary distributions (Wannamaker et al., 1984b). It is recommended that the user perform convergence tests such as those of Ting and Hohmann (1981) to understand the minimum requirements for discretization of the structure into cells.

PROGRAM ORGANIZATION

To obtain magnetotelluric functions over a 3-D body in a layered host, six programs are run in turn. These programs have been named GMEB, MTL1, MTL2, GMEH, MTL3 and MTLQ. ~~A sample command sequence on the PRIME 400 is included with the algorithm upon distribution.~~

Program GMEB computes the 1-D table of electric field Hankel transforms and transform differences for cubic interpolation by program MTL1 in constructing the matrices. Hankel transformation in PW3DL is achieved using the filter weights of Anderson (1979) incorporating lagged convolution. The range of horizontal separation (r) in the table must be sufficient to cover all values encountered between pairs of cubic subcells throughout the body. Otherwise, cubic extrapolation (very bad) may occur. Definition of adequate ranges is given in the comment statements at the head of this program. Also, selection of basepoint values in z and z' at rectangular cell and cubic subcell depth locations is required.

Program MTL1 computes the two matrices describing the electric coupling between the rectangular cells comprising the body for two polarizations of incident plane-wave fields. The whole-space tensor Green's function for a rectangular cell is evaluated using an integro-difference approach while the secondary Green's function for such a cell is obtained by treating each cubic subcell as a dipole (Wannamaker et al., 1984a). Hankel transforms for the secondary Green's functions in turn are obtained through 1-D cubic interpolation of tabulated values, ^{provided} ~~interpolated~~ by GMEB. The Green's functions only need to be computed once for both matrices, since these matrices differ just in the assembly of these functions according to the symmetry relations between fields in cells in different quadrants. However, in case the structure being modeled has no planes of symmetry, only one matrix need be computed.

) Program MTL2 solves the two matrices for the electric fields in the rectangular cells using the plane-wave fields incident in the layered earth as the source vectors. The solution is obtained through the method of L-U factorization (Ralston, 1965). A general matrix reduction subroutine is necessary since the coupling matrices are full and non-symmetric. If the model has no planes of symmetry, only one matrix is factored and then subjected to two forward and backward substitutions for the two polarizations.

) Program GMEH computes the 1-D tables of electric and magnetic field Hankel transforms and transform differences for cubic interpolation by program MTL3 in constructing receiver fields (Wannamaker et al., 1984a). The range of horizontal separation (r) in the table must be sufficient to cover all values of r encountered between cubic subcell-receiver point pairs throughout the 3-D model. Definition of adequate ranges is given in the comment statements at the head of this program. A set of tables is computed for each level of receiver depths (z) specified in the earth. Also, selection of basepoint values in z' at subcell depth locations is required.

) Program MTL3 calculates electric and magnetic fields at receiver points from the scattering currents within the 3-D body. Furthermore, receiver arrays at various levels of z in the earth can be calculated. The whole-space tensor Green's functions for a rectangular cell are evaluated by treating each subcell in the manner of Hohmann (1975) while the secondary Green's functions for such a cell again are obtained by treating each subcell as a dipole. The Green's functions only need to be computed once for both polarizations of incident field, since the receiver fields for the two polarizations differ just in the assembly of these functions according to the orientations of the fields in the rectangular cells.

) Program MTLQ computes MT quantities from the secondary fields at the receivers. Impedances, apparent resistivities and phases, tipper and vertical admittance are calculated in original (x-y) and principal coordinates. A page of output for each receiver location is generated in file OUTPT2. A subset of these quantities appears in file PLTFIL for plotting.

Unlike the half-space host algorithms of Hohmann (1975) and Ting and Hohmann (1981), new Hankel transform tables must be computed whenever the layering, frequencies, and ranges of r , z and z' are altered. Computation of additional receiver fields over previously computed body scattering currents can be accomplished with the existing tables and program MTL3 unless the range of r is increased.

INPUT AND OUTPUT FILES

Algorithm PW3DL requires three user-input files: GMIN1, INPT and GMIN2. These generally are created using the system editor. Also, output file generated by some of the programs may be used as input files for programs run subsequently. A complete list of input and output files as well as definitions of all input variables appear at the head of each program. All quantities in the input and output files are in MKS units.

The ultimate output files of algorithm PW3DL are OUTPT2 and PLTFIL. For OUTPT2, an explanation of the variable names printed on the output follows.

~~Detailed format is apparent from the sample runs provided.~~

File OUTPT2: Page 1.	
FREQ, NLYR, NREC	= frequency, number of layers, number of receivers
D(I)	= array of layer interface depths
RHOL(I)	= array of layer resistivities, including basal half-space
EX01-EX02-EY01-EY02	= incident electric fields at the surface for first (1) and second (2) polarizations
HX01-HX02-HY01-HY02	= incident magnetic fields at the surface for first (1) and second (2) polarizations
RHA1D, PHA1D	= apparent resistivity and impedance phase of the layered host
File OUTPT2: Pages 2 to (NREC + 1)	
Site Location X,Y	= receiver site location
EXS1-EXS2-EYS1-EYS2	= secondary E fields
HXS1-HXS2-HYS1-HYS2	= secondary H fields
HZS1-HZS2	= secondary vertical H fields
EXT1-EXT2-EYT1-EYT2	= total E fields
HXT1-HXT2-HYT1-HYT2	= total H fields
ZXXI-ZXYI-ZYXI-ZYYI	= impedance elements in original coordinates
RXYI-PXYI-RYXI-PYXI	= apparent resistivity and phase from off-diagonal impedance elements ZXYI and ZYXI, in original coords.
MAG(K) STRIKE	= incrementally rotate axes until $ K_{zy} $ is maximized

MAG(Y) STRIKE	= incrementally rotate axes until $ Y_{zx} $ is maximized
KZXI-KZYI	= tipper elements in original coords.
KZXM-PKZX-KZYM-PKZY	= magnitude and phase of tipper elements in original coords.
KZXMY-KZYMY-TIPH	= tipper elements for MAG(K) strike direction and the tipper itself = $[K_{zx} ^2 + K_{zy} ^2]^{1/2}$
MKZXMK-PKZXMK-MKZYMK-PKZYMK	= magnitude and phase of tipper elements for MAG(K) strike direction
YZXI-YZYI	= vertical admittance elements in original coords.
YZXM-PYZX-YZYM-PYZY	= magnitude and phase of vertical admittance elements in original coords.
YZXMY-YZYMY-TIPE	= vertical admittance elements for MAG(Y) strike direction and the "electric tipper" itself = $[Y_{zx} ^2 + Y_{zy} ^2]^{1/2}$
MYZXMY-PYZXMY-MYZYMY-PYZYMY	= magnitude and phase of vertical admittance elements for MAG(Y) strike direction
ZXXMK-ZXYMK-ZYXMK-ZYYMK	= impedance elements for MAG(K) strike
RXYMK-PXYMK-RYXMK-PYXMK	= apparent resistivity and phase from ZXYMK and ZYXMK for MAG(K) strike
ZXXMY-ZXYMY-ZYXMY-ZYYMY	= impedance elements for MAG(Y) strike
RXYMY-PXYMY-RYXMY-PYXMY	= apparent resistivity and phases from ZXYMY and ZYXMY for MAG(Y) strike
MAG(Z) STRIKE	= axis obtained by maximizing $ Z_{xy} $
PHZ(Z) STRIKE	= axis obtained by maximizing $ \phi_{xy} $
ZXXMZ-ZXYMZ-ZYXMZ-ZYYMZ	= impedance elements for MAG(Z) strike
RXYMZ-PXYMZ-RYXMZ-PYXMZ	= apparent resistivity and phases from ZXYMZ and ZYXMZ for MAG(Z) strike
SKEW-ELIP-SKWM-ELPM	= complex skew and ellipticity, and their magnitudes
ZXXPZ-ZXYPZ-ZYXPZ-ZYYPZ	= impedance elements for PHZ(Z) strike
RXYPZ-PXYPZ-RYXPZ-PYXPZ	= apparent resistivity and phases from ZXYPZ and ZYXPZ for PHZ(Z) strike

File PLTFIL is primarily a convenient input file to user-generated plotting programs. It is a subset of OUTPT2 and contains, for each receiver

location,

X,Y	- receiver location
RXYI,PXYI,RYXI,PYXI	- off-diagonal apparent resistivities and impedance phases in original coordinates
RXXI,PXXI,RYYI,PYYI	- on-diagonal apparent resistivities and impedance phases in original coordinates
KZXM,PKZX,KZYM,PKZY	- magnitude and phase of tipper elements in original coordinates
YZXM,PYZX,YZYM,PYEZ	- magnitude and phase of vertical admittance elements in original coordinates
TIPH,TIPE	- magnitude of tipper and "electric tipper"
RK,RY,RZ,RP	- MT strike directions

The formats for reading this file along with the variable names as they appear

PLTFIL
in ~~OUTPTZ~~ are

X,Y	(2F10.1)
RXYI,PXYI,RYXI,PYXI	(2(E9.2,2X,F4.1))
RXXI,PXXI,RYYI,PYYI	(2(E9.2,2X,F4.1))
KZXM,PKZX,KZYM,PKZY	(2(E9.2,2X,F4.1))
YZXM,PYZX,YZYM,PYZY	(2(E9.2,2X,F4.1))
TIPH,TIPE	(E9.2,6X,E9.2)
RK,RY,RZ,RP	(4F10.1)
(RZ=MAG(Z) STRIKE, RP=PHZ(Z) STRIKE and RK=MAG(K) STRIKE, RY=MAG(Y) STRIKE)	

NOTE

AN ADDITIONAL OUTPUT FILE, OUTPTZ, IS
GENERATED WHICH GIVES THE STATION LOCATION
AND OFF DIAGONAL RESISTIVITIES, REFL &
COMPLEX, AND CARTESIAN COORDINATES

REFERENCES

- Anderson, W. L., 1979, Numerical integration of related Hankel transforms of orders 0 and 1 by adaptive digital filtering: Geophysics, v. 44, p. 1287-1305.
- Hohmann, G. W., 1975, Three-dimensional induced polarization and electromagnetic modeling: Geophysics, v. 40, p. 309-324.
- Hohmann, G. W., and Ting, S. C., 1978, Three-dimensional magnetotelluric modeling: Earth Science Lab. rep. 7, Salt Lake City, 48 p.
- Lajoie, J. J., and West, G. F., 1976, The electromagnetic response of a conductive inhomogeneity in a layered earth: Geophysics, v. 41, p. 1133-1156.
- Ting, S. C., and Hohmann, G. W., 1981, Integral equation modeling of three-dimensional magnetotelluric response: Geophysics, v. 46, p. 182-197.
- Wannamaker, P. E., Hohmann, G. W., and SanFilipo, W. A., 1984a, Electromagnetic modeling of three-dimensional bodies in layered earths using integral equations: Geophysics, v. 49, p. 60-74.
- Wannamaker, P. E., Hohmann, G. W., and Ward, S. H., 1984b, Magnetotelluric responses of three-dimensional bodies in layered earths: Geophysics, v. 49(9), ~~1517-1534~~ p. 1517-1534.